1. The following problems are in 2D. **Leave answers in matrix form, but completely fill out each matrix.** You're given 2 line segments: L1 goes from (1,1) to (2,2), and L2 goes from (-1,1) to (-1,3).
   (a) What is the transformation, M1, to be applied to L1 such that it becomes a unit length along the X-axis? That is, L1' goes from (0,0) to (1,0). Hint: M1 is a composite of 3 elementary transformations. What are they?

   (b) What is the transformation, M2, to be applied to L2 such that it becomes a unit length along the X-axis? That is, L2' goes from (0,0) to (1,0). Hint: M2 is a composite of 3 elementary transformations. What are they?

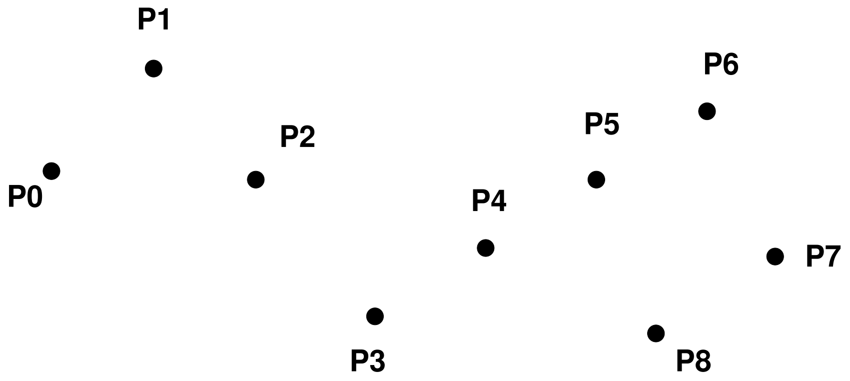
   (c) What is the transformation, M, to be applied to L1 such that it becomes L2? That is, transforming (1,1) by M produces (-1,1), and transforming (2,2) by M produces (-1,3). Hint: you need to somehow combine the results from parts a and b.

2. **Bezier Curve**

The blending functions of Bezier curves are defined by products of different powers of t and $(1-t)$ and coefficients that are given by Pascal's triangle. Thus for a cubic Bezier curve, the blending functions are: $(1-t)^3$, $3t(1-t)^2$, $3t^2(1-t)$ and t^3.

(a) Find the blending functions for a *quadratic* Bezier curve.

(b) Given the points: $P_0..P_8$, sketch the piecewise quadratic Bezier curves through this set of points. $P_1..P_3$ are collinear, and $P_3..P_6$ are collinear.



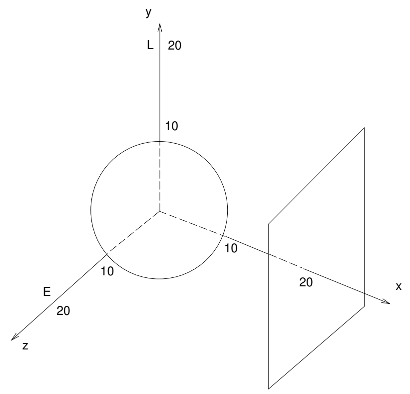
(c) What is the continuity at P_2?

(d) Sketch/overlay the convex hulls of the individual quadratic Bezier curves.

3. **Shading**

Assume your world contains a sphere centered at the origin, with a light source above it and a wall on its right (see figure below). The table below gives the specifications about position, lighting and surface property.

Object	Physical	Ambient	Diffuse	Lighting
sphere	center(0,0,0) radius = 10	(0.1, 0, 0)	(0.9, 0, 0)	
plane	x = 20	(0.2, 0.2, 0.2)	(0.8, 0, 0.8)	
light	at (0, 20, 0)			(1, 1, 1)
eye	at (0, 0, 20)			



(a) If the eye is looking at the point (7.07, 0, 7.07) on the sphere, what is the equation of the light ray from the eyes? Express in parametric form.

(b) What is the surface normal of the sphere at (7.07, 0, 7.07)? Normalize your answer.

(c) What is the reflection vector from this intersection point? Show your calculations and normalize the reflection vector.

(d) What is the intersection of the reflection vector with the $x = 20$ plane? Show your work.

(e) What is ambient and diffuse color of the $x = 20$ plane as seen by the reflection vector?

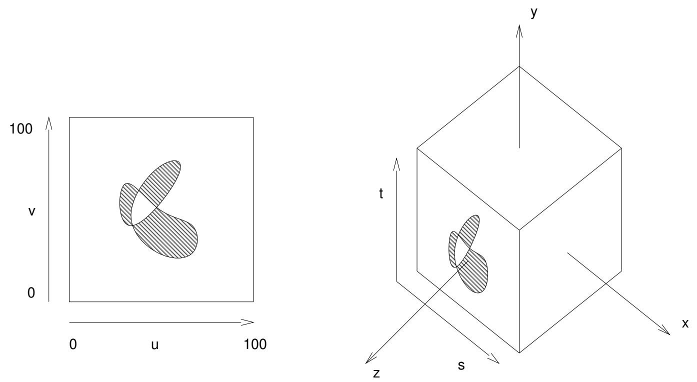
(f) What is the net amount of light as seen by the reflection vector?

(g) What is the ambient and diffuse color of the sphere at the point (7.07, 0, 7.07)? What is the net amount of light at this point?

(h) The net amount of light seen by the reflection vector can be treated as another light source for the intersection point. What is the net amount of light seen by the eyes looking at the intersection point?

4. **Texture Mapping**

You have an 101 x 101 pixel image that you want to use as a texture map. This image is going to be pasted on each face of a cube with dimensions +/- 1 in x,y,z. For example, if we just consider the front face, looking down the z-axis, the texure is to be pasted in the "natural" position (see figure below).



(a) What is the parametric equation of the front face? Hint: It's linear. Also assume that both s and t range from -1 to +1, rather than the usual 0 to 1. Write the $x(s,t), y(s,t), z(s,t)$ separately.

(b) What is the mapping from texture space to the front face? That is, given a point in texture space, where does that point get pasted on the front polygon? Hint: the left hand side of your equations would be s and t; you also need to define u and v over your texture space.

(c) What is the inverse mapping from the polygon to texture space? That is, given a point on the polygon, you need to find the corresponding point on the texture image. Hint: rewrite your equations above with u and v on the left hand side.

(d) Which texture image point would get pasted to the point (0.5, 0.5, 1) on the front polygon?

5. **Shading**

   Unlike Gouraud shading where colors are interpolated, Phong shading interpolates normals. The linearly interpolated normals at each point are then used in shading calculations. Using a scanline algorithm, assume that the normals at the edges of the current scanline are N1 and N2 with components $[N1_x, N1_y, N1_z]$ and $[N2_x, N2_y, N2_z]$ respectively. Calculate the normals at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ way through the span. Express answers in terms of N1 and N2 components.