AMS 10/10A, Homework 6

Problems for Section 2.8 and 2.9

Problem 1. Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $w = \begin{bmatrix} -3 \\ -3 \\ 10 \end{bmatrix}$.

Determine if w is in the subspace spanned by v_1 and v_2.

Problem 2. Let $v_1 = \begin{bmatrix} -2 \\ 0 \\ 6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -5 \\ 5 \end{bmatrix}$, and $w = \begin{bmatrix} -6 \\ 1 \\ 17 \end{bmatrix}$.

Determine if w is in $Col\ A$, where $A = [v_1\ v_2\ v_3]$. Determine if w is in $Nul\ A$.

Problem 3. Consider matrix A given below. $Col\ A$ is a subspace in $\mathbb{R}^p$ and $Nul\ A$ is a subspace in $\mathbb{R}^q$. Write out the values of p and q.

$$A = \begin{bmatrix} -3.1 & 21 & 12 & 5 & 17 \\ -2 & 27 & -13 & 3 & -1 \\ 4 & 1 & 0 & 6 & 3 \end{bmatrix}$$

Problem 4. Find a basis for $Col\ A$ and a basis for $Nul\ A$.

$$A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix}$$

Determine which sets in Problems 5-7 are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer.

Problem 5. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Problem 6. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

Problem 7. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

Each of problems 8-9 displays a matrix A and its echelon form. In each problem,

1. find a basis for $Col\ A$,
2. state the dimension of $Col\ A$,
3. find a basis for $Nul\ A$, and
4. state the dimension of $Nul\ A$.

Problem 8.

$$A = \begin{bmatrix} 1 & 3 & 2 & -6 \\ 3 & 9 & 1 & 5 \\ 2 & 6 & -1 & 9 \\ 5 & 15 & 0 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 9.

$$A = \begin{bmatrix} 2 & 4 & -5 & 2 & -3 \\ 3 & 6 & -8 & 3 & -5 \\ 0 & 0 & 9 & 0 & 9 \\ -3 & -6 & -7 & -3 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 10. Suppose the rank of a 7×9 matrix is 5. what is the dimension of $Col\ A$? What is the dimension of $Nul\ A$?

Problem 11. Mark each statement True or False

11.1. The set of all solutions of a system of homogeneous equation with m equations and n unknowns is a subspace in $\mathbb{R}^m$.

11.2. The set of all linear combinations of columns of an $m \times n$ matrix is a subspace in $\mathbb{R}^n$.

11.3. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

11.4. Let $A = [a_1\ a_2\ a_3]$, where a_1, a_2, and a_3, are vectors in $\mathbb{R}^n$. Then the column space of matrix $[a_1\ a_2\ a_3]$ is the same as the column space of matrix $[a_3\ a_1\ a_2]$.

11.5. The columns of a singular (non-invertible) $n \times n$ matrix may still be a basis for $\mathbb{R}^n$.

Problem 12. Mark each statement True or False

12.1. The dimension of $Col\ A$ is the number of pivot columns in A.

12.2. Suppose A is an invertible $n \times n$ matrix. Then $Col\ A = \mathbb{R}^n$.

12.3. Suppose A is an invertible $n \times n$ matrix. Then $Nul\ A = \{0\}$.

12.4. The dimension of $Nul\ A$ is the number of variables in the equation $Ax = 0$.

12.5. The dimension of $Nul\ A$ is the number of basic variables in the equation $Ax = 0$.

12.6. The dimension of $Nul\ A$ is the number of free variables in the equation $Ax = 0$.

Problem 13. Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

Find a basis for $Col\ A$ and find a basis for $Nul\ A$.

Problem 14. Consider two matrices

$$A = \begin{bmatrix} 1 & -4 & 2 & 3 \\ -2 & 1 & -1 & 7 \\ 3 & -4 & 2 & -5 \\ 2 & 0 & 3 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -4 & 2 & 3 & 0 \\ -2 & 1 & -1 & 7 & 0 \\ 3 & -4 & 2 & -5 & 0 \\ 2 & 0 & 3 & -1 & 0 \end{bmatrix}$$

Notice that matrix B is constructed by appending a column of zeros to matrix A.

1. Is it true that $Col\ A = Col\ B$?
2. Is it possible that $Nul\ A = Nul\ B$?