AMS 10/10A, Homework 5

Problems for Section 2.2 and 2.3

Problem 1. Find the inverse of each of the following matrices, if is exist.

$$A = \begin{bmatrix} -2 & 3 \\ -3 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & 4 \\ -1 & 1 & 2 \\ 2 & 0 & 4 \end{bmatrix}, \qquad C = \begin{bmatrix} -5 & 1 & -4 \\ 0 & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

Problem 2. Find the value(s) of k for which the matrix $\begin{bmatrix} k^2 & 2k \\ 8 & k \end{bmatrix}$ is not invertible.

Problem 3. If A, B, and C are $n \times n$ invertible matrices, does the equation

$$C^{-1}(A + X)B^{-1} = I_n$$

have a solution for X? If so, find it.

Problem 4. Let D be a $n \times n$ diagonal matrix, i.e.,

$$D = \begin{bmatrix} d_{11} & 0 & \cdots & 0 & 0 \\ 0 & d_{22} & \cdots & 0 & 0 \\ 0 & 0 & \ddots & d_{n-1,n-1} & 0 \\ 0 & 0 & \cdots & 0 & d_{nn} \end{bmatrix}$$

with $d_{ii} \neq 0$, for all $i = 1, 2, \cdots, n$. Find the inverse of D.

Problem 5. Let

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & -1 & 2 \\ 4 & 1 & 0 \end{bmatrix}$$

Find the second column of A^{-1} (you don't need to compute other columns).

Problem 6. Let A be a $n \times n$ invertible matrix. Prove that the columns A^T are linearly independent.

Problem 7. Let A and B be $n \times n$ matrices such that AB is invertible. Prove that both A and B are invertible.

Problem 8. Let A be a $n \times n$ matrix whose columns are linearly independent. Prove that the columns of A^2 are linearly independent.

Problem 9. A square matrix A is called symmetric if $A^T = A$. Prove that if a symmetric matrix is invertible, then its inverse is also symmetric.

Problem 10. If A, B and $A + B$ are all $n \times n$ invertible matrices. Prove that $A^{-1} + B^{-1}$ is invertible and the inverse is $A(A + B)^{-1}B$.

Problem 11. Mark each statement below True or False

11.1. If A and B are invertible, then $A + B$ is invertible.

11.2. If A is $n \times n$ and not invertible, then the linear system $Ax = b$ is inconsistent.

11.3. If $(A - I)$ is invertible, then the linear system $Ax = x$ has a nonzero solution for x.

11.4. If a square matrix has nonzero entries on the diagonal, then A is invertible.

11.5. If A is $n \times n$, and the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$.

Problem 12. Mark each statement below True or False

12.1. Let A be a square matrix. If the equation $Ax = 0$ has a nontrivial solution, then A is not invertible.

12.2. A square matrix with two identical rows cannot be invertible.

12.3. A square matrix with two identical columns cannot be invertible.

12.4. A product of invertible matrices is invertible.

12.5. If A and B are $n \times n$ invertible matrices, then $A^{-1}B^{-1}$ is the inverse of AB.