AMS 10/10A, Homework 3

Problems for Section 1.3

Problem 1. For each of the vector equations below, write down its equivalent linear system.

$$x_1 \begin{bmatrix} 3 \\ 0 \\ 1 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 9 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 2 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 0 \\ -5 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Problem 2. For each of the linear systems below, write down its equivalent vector equation.

$$\begin{cases} 2x_1 + 3x_2 - x_3 &= 1 \\ 4x_1 + 7x_2 + x_3 &= 3 \\ 7x_1 + 10x_2 - 4x_3 &= 4 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 + x_3 &= 13 \end{cases}$$

Problem 3. Determine if b is a linear combination of v_1, v_2 and v_3.

$$b = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Problem 4. Let

$$b = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -5 \\ -8 \\ 1 \end{bmatrix}$$

For what value(s) of h is vector b in $span\{v_1, v_2\}$?

Problem 5. Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 4 \\ 1 & 5 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

Let W be the set of all linear combinations of the columns of A.

5.1. Is b in W? Why?

5.2. Show that the third column of A is in W.

Problems for Section 1.4

Problem 6. Compute the following products

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -5 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 \\ 1 \\ -1 \\ 5 \end{bmatrix}.$$

Problem 7. Use the definition of Ax to write the matrix equation as a vector equation, or vice versa.

$$\begin{bmatrix} -1 & 4 & 7 & -3 \\ 2 & 9 & -6 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

Problem 8. Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

How many rows of A contain a pivot position?
Does the equation $Ax = b$ have a solution for each b in $\mathbb{R}^4$?

Problem 9. Let $v_1 = \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Does $\{v_1, v_2, v_3\}$ span $\mathbb{R}^3$? Why or why not?

Problem 10. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ where n is smaller than m?

Problem 11. Mark each statement True or False.

11.1. If a matrix A has a row of all zeros, then its reduced echelon form also contains a row of all zeros.

11.2. If the reduced echelon form of matrix A has a row of all zeros, then matrix A contains a row of all zeros.

11.3. An example of a linear combination of vectors v_1 and v_2 is the vector $\frac{1}{3}v_1$.

11.4. Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x.

11.5. The equation $Ax = b$ is consistent if the augmented matrix $[A\ b]$ has a pivot position in every row.