AMS 10/10A, Homework 2

Problem 1. Do the three lines $x_1 + 2x_2 = 5$, $3x_1 - 2x_2 = 1$, and $2x_1 + 4x_2 = 10$ have a common point of intersection? If yes, find the point; if no, explain the reason.

Problem 2. Write each linear system below in matrix form. Then solve each linear system by elementary row operations.

$$\begin{cases} 2x_1 + 3x_2 - x_3 &= 1 \\ 4x_1 + 7x_2 + x_3 &= 3 \\ 7x_1 + 10x_2 - 4x_3 &= 4 \end{cases}$$

$$\begin{cases} 3x_1 + 3x_2 + x_3 &= -4.5 \\ x_1 + x_2 + x_3 &= 0.5 \\ -2x_1 - 2x_2 &= 5 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 + x_3 &= 13 \\ 4x_1 + 8x_2 - 2x_3 &= 9 \end{cases}$$

Problem 3. Identify all matrices list below that are not in echelon form.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & -4 & 1 \\ 2 & 0 & 0 \\ 1 & -3 & 3 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & -7 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Problem 4. Find the value(s) of α such that the linear system below has (a) no solution, (b) unique solution

$$\begin{cases} x_1 + 2x_2 + 2x_3 &= 1 \\ x_2 + \alpha x_3 &= 1 \\ -x_1 + x_2 + \alpha x_3 &= \alpha \end{cases}$$

Problem 5. For each of the matrices listed below, find the reduced echelon forms and identify all pivot positions.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 2 & -9 & -9 \\ 8 & 8 & 4 & 4 \\ -2 & -2 & 6 & 6 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 4 \\ 1 & 5 & -3 \end{bmatrix}$$

Problem 6. Solve each of the following linear systems by finding the reduced echelon form of its augmented matrix.

$$\begin{cases} 6x_1 - 6x_2 + 6x_3 &= 6 \\ 2x_1 + 4x_2 - 6x_3 &= 12 \\ 10x_1 - 5x_2 + 5x_3 &= 30 \end{cases}$$

$$\begin{cases} 2x_1 - x_2 + 3x_3 &= 3 \\ 4x_1 - x_2 + x_3 &= 3 \\ -2x_1 + 2x_2 + 5x_3 &= 1 \\ 6x_1 + x_2 - x_3 &= 5 \end{cases}$$

Problem 7. Solve each linear system whose augmented matrix is given below

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right], \qquad B = \left[\begin{array}{ccc|c} 2 & 2 & 5 & 1 \\ 0 & 8 & 0 & 4 \\ -2 & -2 & 6 & 2 \end{array}\right]$$

Problem 8. Let the augmented matrix of a linear system be

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & \alpha & 2\alpha \\ 1 & \alpha & 2 & -2 \end{array}\right]$$

Find the value(s) of α for which the linear system

1. has three basic variables;
2. has two basic variables and one free variable;
3. is inconsistent.

Problem 9. Mark each statement below **True** or **False**

9.1. If an augmented matrix has 8 columns and 6 rows, then the associated linear system has 8 equations and 6 unknown variables.

9.2. Elementary row operations on an augmented matrix never change the solution set of the associated linear system of equations.

9.3. An inconsistent linear system can have a solution.

9.4. A matrix may be row reduced to two different matrices in echelon form, using different sequences of row operations.

9.5. If one row in an echelon form of an augmented matrix is $[0, 0, 0, -3, 0]$, then the associated linear system of equations is inconsistent.