AMS 10: Review for the Midterm Exam

The scope of the midterm exam is up to and includes Section 2.1 in the textbook (homework sets 1-4). Below we highlight some of the important items.

Complex numbers

The Cartesian form:

$a+bi$

Complex conjugate: ?

Arithmetics:

Example: $\dfrac{a_1+b_1 i}{a_2+b_2 i}$

How to calculate this division?

The polar form:

$r(\cos\theta+\sin\theta i)$

The absolute value: ?

The argument: ?

Conversion between $a+bi$ and $r(\cos\theta+\sin\theta i)$

How to calculate the absolute value given the Cartesian form?

How to calculate the argument given the Cartesian form?

The exponential form:

$re^{\theta i}=r(\cos\theta+\sin\theta i)$

Arithmetics in the exponential form:

Example: $\dfrac{r_1 e^{\theta_1 i}}{r_2 e^{\theta_2 i}}$

How to calculate this division?

Example: $\left(\sqrt{2}e^{\frac{\pi}{3}i}\right)^{11}$

How to write out the exponential form of $\left(\sqrt{2}e^{\frac{\pi}{3}i}\right)^{11}$?

How to write out the Cartesian form of $\left(\sqrt{2}e^{\frac{\pi}{3}i}\right)^{11}$ after we obtain its exponential form?

Roots of polynomials

The fundamental theorem of algebra

How to factor a real polynomial?

The n-th roots of a complex number (there are *n* of them)

Example: The 7-th roots of (-2 + 2i)

How to write out **ALL** of the 7-th roots of (-2 + 2i) in the exponential form?

How to write out **ALL** of the 7-th roots of (-2 + 2i) in the Cartesian form after we obtain their exponential forms?

Chapter 1

Elementary row operations

What are the 3 kinds of elementary row operations?

Row reduction algorithm

Forward phase: row reduction to echelon form

Backward phase: row reduction to reduced echelon form

What is the definition of echelon form?

What is the definition of reduced echelon form?

Theorem 1 (Chapter 1):

Row equivalence to reduced echelon form

Pivot positions

What is a pivot position?

Pivot columns

What is a pivot column?

Basic variables

How do we identify basic variables?

Free variables

How do we identify free variables?

Theorem 2 (Chapter 1)

Existence and uniqueness of solution of $A\vec{x}=\vec{b}$

Solution set in parametric form

How to find the solution set of $A\vec{x}=\vec{0}$?

How to find the solution set of $A\vec{x}=\vec{b}$?

Matrix equation, vector equation and linear system

Theorem 3 (Chapter 1)

Equivalence of matrix equation, vector equation and linear system

Row-vector rule for computing $A\vec{x}$

Theorem 4 (Chapter 1): 4 statements are equivalent to each other

a. $A\vec{x}=\vec{b}$ is consistent for every $\vec{b}$ in $\mathbb{R}^m$.

b.

c.

d. Matrix A has a pivot position in every row.

Theorem 5 (Chapter 1):

Properties of matrix-vector multiplication

Theorem 6 (Chapter 1):

The solution set of $A\vec{x}=\vec{b}$

Linear independence and dependence

Theorem 7 (Chapter 1)

A necessary and sufficient condition for linear dependence

3 special cases:

*) A set of 2 vectors

*) A set containing the zero vector Theorem 9 (Chapter 1)

*) # of vectors in the set > # of entries in each vector Theorem 8 (Chapter 1)

Chapter 2:

Matrix addition and scalar multiplication

Theorem 1 (Chapter 2)

Properties of matrix addition and scalar multiplication

Matrix multiplication

Theorem 2 (Chapter 2)

Properties of matrix multiplication

Row-column rule for computing AB

The transpose of a matrix

Theorem 3 (Chapter 2)

Properties of matrix transpose

$(AB)^T = ?$

$(A_1 A_2 \cdots A_k)^T = ?$

Question 1:

Is $A\vec{x} = \vec{b}$ consistent for a particular given $\vec{b}$? How to check?

Hint: use Theorem 2 (Chapter 1)

Question 1B:

How to find pivot positions of the augmented matrix $\left[\begin{array}{c:c} A & \vec{b} \end{array}\right]$?

Hint: Row reduction to an echelon form.

Question 2:

Suppose $A\vec{x} = \vec{b}$ is consistent for the given $\vec{b}$.

How to check if $A\vec{x} = \vec{b}$ have a unique solution or infinitely many solutions?

Hint: use Theorem 2 (Chapter 1)

Question 2B:

How to identify basic variables and free variables of $A\vec{x} = \vec{b}$?

Hint: Row reduction to echelon form, identify pivot positions and …

Question 2C:

Suppose $A\vec{x} = \vec{b}$ is consistent for the given $\vec{b}$.

How to write out the solution set of $A\vec{x} = \vec{b}$?

Hint: Row reduction to reduced echelon form;

Identify basic variables and free variables;

…

Question 3:

Is $\vec{b}$ a linear combination of $\vec{a}_1, \ldots, \vec{a}_n$? How to check?

Hint: use Theorem 3 (Chapter 1) and Theorem 2 (Chapter 1)

Question 4:

Is $\vec{b}$ in $span\{\vec{a}_1, \ldots, \vec{a}_n\}$? How to check?

Hint: use Theorem 3 (Chapter 1) and Theorem 2 (Chapter 1)

Question 5:

Suppose matrix A is $m \times n$.

Is $A\vec{x} = \vec{b}$ consistent for every $\vec{b}$ in $\mathbb{R}^m$?

Hint: use Theorem 4 (Chapter 1)

Question 5B:

Suppose matrix A is 11×9.

Is it possible that matrix A has a pivot position in every row?

Hint: Can a column have more than 1 pivot position?

Question 6:

Suppose $\vec{a}_1, \ldots, \vec{a}_n$ are in $\mathbb{R}^m$.

Is $span\{\vec{a}_1, \ldots, \vec{a}_n\} = \mathbb{R}^m$? How to check?

Hint: use Theorem 4 (Chapter 1)

Question 7:

Suppose $\vec{a}_1, \ldots, \vec{a}_n$ are in $\mathbb{R}^m$.

Is every $\vec{b}$ in $\mathbb{R}^m$ a linear combination of $\vec{a}_1, \ldots, \vec{a}_n$? How to check

Hint: use Theorem 4 (Chapter 1)

Question 8:

Does $A\vec{x}=\vec{0}$ have a non-trivial solution?

Hint: use Theorem 2 (Chapter 1)

Question 8B:

Is $A\vec{x}=\vec{0}$ always consistent?

Hint: What is the trivial solution?

Question 8C:

Suppose matrix A is 11×15.

Does $A\vec{x}=\vec{0}$ have a non-trivial solution?

Hint: Check the number of free variables.

Question 9:

Is $\{\vec{a}_1,\ldots,\vec{a}_n\}$ in $\mathbb{R}^m$ linearly dependent? How to check?

Hint: Is this question related to question 8?

Question 10:

Is $\{\vec{u}_1,\vec{u}_2,\vec{0}\}$ linearly dependent?

Can we conclude anything without doing row reduction?

Hint: use Theorem 9 (Chapter 1)

Question 11:

Is $\{2\vec{u},\ 7\vec{u}\}$ linearly dependent?

Can we conclude anything without knowing $\vec{u}$?

Hint: use Theorem 7 (Chapter 1)

Question 12:

Suppose $\vec{a}_1,\ldots,\vec{a}_n$ are in $\mathbb{R}^m$.

Suppose $n > m$.

Is $\{\vec{a}_1,\ldots,\vec{a}_n\}$ linearly dependent?

Can we conclude anything without knowing $\left\{\vec{a}_1, \ldots, \vec{a}_n\right\}$?

Hint: use Theorem 8 (Chapter 1)

Question 13:

Suppose matrix A is $m \times n$.

Suppose $n > m$.

Does $A\vec{x} = \vec{0}$ have a non-trivial solution?

Hint: Examine the number of free variables, …

Question 14:

Suppose AB is well defined.

Does that necessarily imply BA is well defined?

Suppose AB = 0.

Does that imply BA = 0?

Suppose AB = AC.

Does that imply A = 0 or B = C?

Suppose matrix A is $n \times n$.

Is A^5 well defined?

Suppose A is 3×5 and B is 4×3.

Is AB well defined?

Is BA well defined?

Is A^7 well defined?

Is $(AB)^7$ well defined?