AMS 10/10A, Homework 5 Solutions

Problem 1:

$$A^{-1} = \begin{bmatrix} -\frac{1}{11} & -\frac{3}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & -1 & -0.5 \\ 2 & -1 & -1.5 \\ -0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$C \quad : \quad \text{not invertible}$$

Problem 2: When $k = 0$, or $k = 4$ or $k = -4$ the matrix is not invertible.

Problem 3: $X = CB - A$.

Problem 4:

$$D^{-1} = \begin{bmatrix} 1/d_{11} & 0 & \cdots & 0 & 0 \\ 0 & 1/d_{22} & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 1/d_{n-1,n-1} & 0 \\ 0 & 0 & \cdots & 0 & 1/d_{nn} \end{bmatrix}$$

Problem 5: The second column of A^{-1} is given by the solution of the equation

$$Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Solving this equation we get the second column of A^{-1}, which is $\begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}$.

Problem 6: By The Invertible Matrix Theorem, A is invertible implies that A^T is invertible, which implies that the columns of A^T are linearly independent.

Problem 7: Since AB is invertible, there is a square matrix C such that $C(AB) = I$. We re-write it as $(CA)B = I$, which implies that matrix B is invertible.

Similarly, since AB is invertible, there is a square matrix D such that $(AB)D = I$. We re-write it as $A(BD) = I$, which implies that matrix A is invertible.

Problem 8: Since columns of $A_{n\times n}$ are linearly independent, matrix A is invertible. Therefore, $A^2 = AA$ is invertible. By The Invertible Matrix Theorem, the columns of A^2 are linearly independent.

Problem 9: $A^{-1} = (A^T)^{-1} = (A^{-1})^T$. Therefore, A^{-1} is symmetric.

Problem 10: Since A, B and $(A+B)^{-1}$ are all invertible, matrix $(A(A+B)^{-1}B)$ is invertible and

$$\begin{aligned}\left(A(A+B)^{-1}B\right)^{-1} &= B^{-1}\left((A+B)^{-1}\right)^{-1}A^{-1} \\ &= B^{-1}(A+B)A^{-1} \\ &= (B^{-1}A+I)A^{-1} \\ &= B^{-1}+A^{-1}\end{aligned}$$

Therefore, matrix $B^{-1}+A^{-1}$ is invertible and

$$\left(B^{-1}+A^{-1}\right)^{-1} = A(A+B)^{-1}B$$

Problem 11: Mark each statement True or False

11.1. If A and B are invertible, then $A+B$ is invertible. **F**

11.2. If A is $n\times n$ and not invertible, then the linear system $Ax=b$ is inconsistent. **F**

11.3. If $(A-I)$ is invertible, then the linear system $Ax=x$ has a nonzero solution for x. **F**

11.4. If a square matrix has nonzero entries on the diagonal, then A is invertible. **F**

11.5. If A is $n\times n$, and the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$. **T**

Problem 12: Mark each statement True or False

12.1. Let A be a square matrix. If the equation $Ax=0$ has a nontrivial solution, then A is not invertible. **T**

12.2. A square matrix with two identical rows cannot be invertible. **T**

12.3. A square matrix with two identical columns cannot be invertible. **T**

12.4. A product of invertible matrices is invertible. **T**

12.5. If A and B are $n\times n$ invertible matrices, then $A^{-1}B^{-1}$ is the inverse of AB. **F**