AMS 10/10A, Homework 3 Solutions

Problem 1:

$$\begin{bmatrix} 3 & -1 & -1 \\ 0 & 9 & 2 \\ 1 & 0 & 7 \\ -5 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 0 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 & 7 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Problem 2:

$$x_1 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 7 \\ 10 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \end{bmatrix}$$

Problem 3:

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ -4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -15 \end{array}\right] \Longrightarrow b = -3v_1 + 3v_2 - 15v_3$$

Problem 4:

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ 2 & 1 & h \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & 0 & h+16 \end{array}\right] \Longrightarrow h = -16$$

Problem 5:

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 1 \\ 1 & 5 & -3 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & -8 & -4 \end{array}\right] \Longrightarrow \text{The linear system has a solution.} \Longrightarrow b \text{ is in } W.$$

The third column of A is a linear combination of all columns of A with wrights 0, 0 and 1. Therefore, it's in the set W.

Problem 6:

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 19 \\ 32 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 \\ 1 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} -59 \\ 0 \end{bmatrix}$$

Problem 7:

$$x_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 9 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -6 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 4 & 1 & 0 \\ 5 & 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

Problem 8:

$$A \sim \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A has only three pivots. Row 4 does not have a pivot. $Ax = b$ will be inconsistent for some b in $\mathbb{R}^4$. Therefore, it is not true that $Ax = b$ has a solution for every b in $\mathbb{R}^4$.

Problem 9:

$$[v_1,\ v_2,\ v_3] = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[v_1,\ v_2,\ v_3]$ has 3 pivots, one pivot for each row. Therefore, each b in $\mathbb{R}^3$ can be expressed as a linear combination of v_1, v_2 and v_3. It implies that $span\{v_1, v_2, v_3\} = \mathbb{R}^3$.

Problem 10: A set of three vectors in R^4 cannot span R^4. Reason: The matrix A consisting of these three column vectors has four rows. To have a pivot in each row, A would have to have at least four columns (one pivot column for each pivot). Therefore, a set of three vectors in R^4 cannot span R^4. In general, when $n < m$, a set of n vectors in R^m cannot span R^m.

Problem 11:

11.1. If a matrix A has a row of all zeros, then its reduced echelon form also contains a row of all zeros. **T**

11.2. If the reduced echelon form of matrix A has a row of all zeros, then matrix A contains a row of all zeros. **F**

11.3. An example of a linear combination of vectors v_1 and v_2 is the vector $\frac{1}{3}v_1$. **T**

11.4. Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x. **T**

11.5. The equation $Ax = b$ is consistent if the augmented matrix $[A\ b]$ has a pivot position in every row. **F**