AMS 10/10A, Homework 2 Solutions

Problem 1: Yes. The intersection is $x_1 = 3/2$, $x_2 = 7/4$.

Problem 2:

$$\left[\begin{array}{rrr|r} 2 & 3 & -1 & 1 \\ 4 & 7 & 1 & 3 \\ 7 & 10 & -4 & 4 \end{array}\right] \sim \left[\begin{array}{rrr|r} 2 & 3 & -1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Longrightarrow \left\{\begin{array}{rcl} x_1 & = & 4 \\ x_2 & = & -2 \\ x_3 & = & 1 \end{array}\right.$$

$$\left[\begin{array}{rrr|r} 3 & 3 & 1 & -4.5 \\ 1 & 1 & 1 & 0.5 \\ -2 & -2 & 0 & 5 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 1 & 1 & 0.5 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 1 & 0 & -2.5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \Longrightarrow \left\{\begin{array}{rcl} x_1 & = & -x_2 - 2.5 \\ x_3 & = & 3 \\ x_2 & : & free \end{array}\right.$$

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & 1 \\ 3 & 6 & 1 & 13 \\ 4 & 8 & -2 & 9 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 2 & -3 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right] \Longrightarrow \text{inconsistent, no solution}$$

Problem 3: B, C and D are not in echelon form.

Problem 4:

$$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 1 \\ 0 & 1 & \alpha & 1 \\ -1 & 1 & \alpha & \alpha \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & \alpha & 1 \\ 0 & 0 & 2-2\alpha & \alpha - 2 \end{array}\right]$$

When $\alpha = 1$, the linear system has no solution. For all $\alpha \neq 1$, the linear system has unique solution.

Problem 5: (Pivot positions are marked in red)

$$A \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 6:

$$\left[\begin{array}{rrr|r} 6 & -6 & 6 & 6 \\ 2 & 4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 7 \end{array}\right] \Longrightarrow \begin{cases} x_1 & = & 5 \\ x_2 & = & 11 \\ x_3 & = & 7 \end{cases}$$

$$\left[\begin{array}{rrr|r} 2 & -1 & 3 & 3 \\ 4 & -1 & 1 & 3 \\ -2 & -2 & 5 & 1 \\ 6 & 1 & -1 & 5 \end{array}\right] \sim \left[\begin{array}{rrr|r} 2 & -1 & 3 & 3 \\ 0 & 1 & -5 & -3 \\ 0 & 0 & 13 & 7 \\ 0 & 0 & 0 & 34/13 \end{array}\right] \Longrightarrow \text{ no solution}$$

Problem 7:

$$A \sim \left[\begin{array}{rrr|r} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right] \Longrightarrow \begin{cases} x_1 & = & -3x_2 - 5 \\ x_3 & = & 3 \\ x_2 & : & \mathit{free} \end{cases}$$

$$B \sim \left[\begin{array}{rrr|r} 1 & 0 & 0 & -15/22 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 3/11 \end{array}\right] \Longrightarrow \begin{cases} x_1 & = & -15/22 \\ x_2 & = & 1/2 \\ x_3 & = & 3/11 \end{cases}$$

Problem 8:

$$A \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & \alpha - 1 & 2\alpha + 1 \\ 0 & 0 & \alpha^2 - 2\alpha & 2\alpha^2 - \alpha \end{array}\right]$$

When $\alpha \neq 0$ and $\alpha \neq 2$, the linear system has three basic variables.
When $\alpha = 0$, the linear system has two basic variable and one free variable.
When $\alpha = 2$ the linear system is inconsistent.

Problem 9: Mark each statement True or False

9.1. If an augmented matrix has 8 columns and 6 rows, then the associated linear system has 8 equations and 6 unknown variables. **F**

9.2. Elementary row operations on an augmented matrix never change the solution set of the associated linear system of equations. **T**

9.3. An inconsistent linear system can have a solution. **F**

9.4. A matrix may be row reduced to more than one matrix in echelon form, using different sequences of row operations. **T**

9.5. If one row in an echelon form of an augmented matrix is $[0, 0, 0, -3, 0]$, then the associated linear system of equations is inconsistent. **F**