

Practice Exam for Cmps 160. Fall 2003

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NAME: _____ LOGIN: _____

Useful Information – Read This First

READ THIS FIRST.

The exam will be closed book, closed note, no calculators. I will provide scrap paper if needed.

Write on the exam any information that explains your reasoning toward the answer, if you think it may help toward part credit. Always show your work. If you just give the final answer with no indication of how you got it, you may not get credit.

Think about the problem rather than blindly throwing mathematics at it. There might be an obvious solution.

Always write matrices as we do in class, such that the transformation is applied on the left of the column vectors being transformed, i.e. $\text{Matrix2} * \text{Matrix2} * \text{Vector}$.

Hints: $\cos(90) = 0$, $\sin(90) = 1.0$, $\cos(45) = 0.707$, $\sin(45) = 0.707$, $\cos(60) = 0.5$, $\sin(60) = 0.866$.

This practice exam covers the entire course. Those questions in the following list numbered 1 are appropriate for the first exam, those numbered 2 are TENTATIVELY appropriate for the second exam, and those numbered 3 are appropriate for the third exam. (The selection for the second exam is somewhat tentative as it depends on how far we get before the test.)

	Subject	Points	Score
1	Linear Algebra		
1	Line Drawing		
1	Fill Algorithms		
1	3D Line Clipping		
1	Polygon Clipping		
1	Polygon Scan Conversion		
1	Anti-Aliasing		
1	Window-Viewport Transformation		
2	2D Geometrical Transformation		
2	3D Geometrical Transformation		
2	Into Plane		
2	Viewing Transformation		
2	Projection		
2	Viewing Pipeline Order		
2	Color and Transparency		
2	Illumination Models		
3	Texture Mapping		
3	Gouraud Shading		
3	Dithering		
3	Scanline Algorithms Revisited		
3	Parametric Curves and Surfaces		
3	Modeling Techniques		
3	Hierarchies		
3	Ray Tracing		
3	Hidden Surfaces		
3	Plane Equations		
3	Animation		
	TOTAL	100	

1 Linear Algebra (? points)

1. Find a vector of length 1 that is perpendicular (normal) to these two vectors: $(.707, 0, .707)$ and $(-.707, 0, .707)$.

ANSWER $(0,1,0$ or $(0,-1,0)$

2. Find the angle in degrees between these two vectors: $(0.707, 0.0, 0.707)$ and $(-0.707, 0.0, 0.707)$.

ANSWER 90 degrees

3. What is the length of this vector? $(1, 2, 3)$.

ANSWER square root of 14

4. If the polar coordinates of a point are $(10,45$ degrees), what are the Cartesian coordinates?

ANSWER $(7.07,7.07)$

5. What is the dot product of these two vectors: $(0.866,0.5,0)$ $(0,4,0)$?

ANSWER (2)

6. What is the cosine of the angle and the angle between these two vectors: $(0.866,0.5,0)$ $(0,4,0)$?

ANSWER $(0.5$ and 60 degrees)

7. What is the cross product of these two vectors? $(0.866,0.5,0)$ X $(0,4,0)$?

$(0,0,3.2)$

2 Line Drawing (? points)

1. What is the slope-intersect form of the line equation from (2,1) to (5,3)?

ANSWER $y = 2x/3 - 1/3$

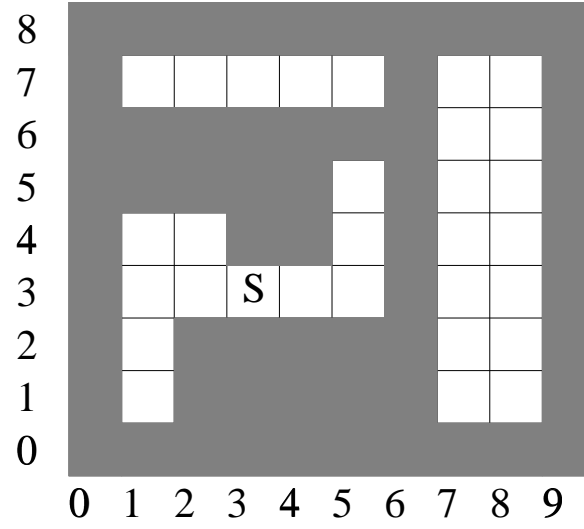
2. What are the parametric line equations for the above line?

ANSWER $x = 2 + 3t; y = 1 + 2t$

3. In Bresenham's algorithm, for lines with integer end points and a slope between 0 and 1, the decision parameter P is initialized to $2*dy - dx$. If the present P is negative, you add $2*dy$ to find the new one; else you add $2*(dy - dx)$. Y is only incremented if P is positive.

For the line above, what pixels are colored in, and what is the value of the decision parameter P at each X integer value?

ANSWER



3 Fill Algorithms (? points)

Using a recursive boundary fill algorithm and a seed of (2,3) for the following picture, how many times will pixel (1,3) be examined to check its color. (The traversal is 4-connected.) Assume the boundary color is dark grey.

ANSWER 3 times

4 3D Line Clipping (? points)

A line runs from $V0 = (-1,-1,0)$ to $V1 = (1,1,1)$.

A 3D window for parallel orthographic projection is defined by

$xwmin = 0; xwmax = 2;$

$ywmin = -1; ywmax = 1;$

$zwmin = 1; zwmax = 0;$

1. What are the Cohen-Sutherland clipping codes for the vertices, using this order:

.....Above.....Below....Right....Left.....Front....Back

V0

V1

ANSWER $V0 = [000100]$ $V1 = [000000]$

2. What is the parametric equation of the line?

ANSWER $x = -1 + 2t; y = -1 + 2t; z = t$

3. What are the line endpoints after it is clipped against the window boundaries in 3D?

ANSWER $(0,0,0.5)$

5 Polygon Clipping (? points)

The window is defined by (0,10) in X and (0,10) in Y. The polygon vertices are (5,5)(15,5)(15,15)(5,15), call them (A,B,C,D) in that order.

Assume that clipping will be against the boundaries in this order: left, right, top, bottom.

The initial list of vertices is L=B-C-D-A.

Show the lists after clipping against each boundary.

ANSWER $L_{left} = A-C-D$; $L_{right} = \text{Int}AB-\text{Int}CD-D-A$; $L_{top} = \text{Int}(\text{Int}AB-\text{Int}CD)-\text{Int}DA-A-\text{Int}AB$;
 $L_{bottom} = L_{top}$;

6 Anti-Aliasing (? points)

Be able to explain how supersampling, area sampling, and filtering can be used for anti-aliasing.

7 Polygon Scan Conversion (? points)

You are scan-converting a triangle with vertices $V_0=(3,6)$, $V_1=(5,4)$, $V_2=(3,2)$. Edge A goes from V_0 to V_1 . Edge B goes from V_1 to V_2 . Edge C goes from V_2 to V_3 .

1. Show the data structures at the beginning of the scan. Also what y-bucket is each edge associated with?

Ybucket?	Ymax	X	Delta X
----------	------	---	---------

ANSWER 4 - Edge A - [6,5,-1]

2 - Edge C [6,3,0] - Edge B [4,3,1]

2. What is the active edge list at scan line 5? That is, at the time when that scanline is about to be displayed.

Ybucket?	Ymax	X	Delta X
----------	------	---	---------

ANSWER AEL - Edge C [6,3,0] - Edge A [6,4,-1]

8 Window to Viewport Transformations (? points)

A line is from $V_0 = (50,0)$ to $V_1 = (200,400)$.

The window is defined by

$x_{wmin} = -100$; $x_{wmax} = 200$; $y_{wmin} = -200$; $y_{wmax} = 400$;

The viewport is defined on a 1000 x 1000 screen by

$u_{min} = 500$; $u_{max} = 1000$; $v_{min} = 0$; $v_{max} = 1000$;

Give me the location of the line endpoints in viewport coordinates. Show your reasoning.

ANSWER (750,333)-(1000,1000)

9 2D Geometrical Transformations (? points)

9.1 Line Rotation

A line passes through the points A:(0,0) and B:(40,30). Using point A as a pivot point, what is the transformation matrix to rotate this line to the y-axis?

ANSWER $\cos = 3/5$; $\sin = 4/5$ to rotate into y-axis.
mat $[3/5, -4/5, 4/5, 3/5]$ so B goes to (0,50).

9.2 Point Transformations

If you rotate the point (20,30) by -90 degrees and then translate it by (20,0) and then scale it by (2,1), where will the point be? You must show the vector and matrices used in the correct order, and give the final answer. Do not multiply the matrices by each other to give one matrix.

ANSWER rot (30,-20,1) trans (50,-20,1) scale (100,-20,1);

10 3D Geometrical Transformations (? points)

You are given a line whose vertices are at (1,0,0) to (2,1,0). Rotate a point at (0,1,0) about this line by 180 degrees.

A. Show all the matrix(es) required to do this rotation. You don't have to show identity matrices if there are any. You should NOT multiply them together to get a composite matrix.

B. What is the new location of the vertex (0,1,0)?

ANSWER (others possible)
translate T(-1,0,0). gives a line in XY plane 45 deg angle goes through (1,1,0).
rotate -45 deg around z into x-axis, rotate 180 degrees around x, rotate 45 degrees back translate T(1,0,0).
final location of point is (2,-1,0).
in sequence (0,1,0)-> (-1,1,0)-> (0,1.4,0)-> (0,-1.4,0)
-> (1,-1,0)-> (2,-1,0)

11 Into Plane (? points)

Rotate the line segment from (1,1,2) to (3,3,5) into the XZ-plane. (Don't scale or translate it. Just apply a pure rotation.)

A. Show a rotation matrix that will do this.

B. What are the new locations of the line segment end points?

C. What is the length of this line segment?

- A. rot z -45 degrees (project line into XY plane to find cos and sine)
- B. (1.4,0,2) to (4.2,0,5)
- C. $\sqrt{8 + 9} = \sqrt{17}$. (same as original line)

12 Viewing Transformations (? points)

The eye is on the origin and looking at the point (1,0,1). The "Up" viewing direction is (0,0,1).

A. What is the viewing matrix (call it M_{view}) that will create a "canonical viewing situation" (i.e., create a situation where the eye is at the origin looking down negative z-axis) from the above situation? Note: I mean the matrix that moves the camera to a canonical situation, NOT the projection matrix.

B. What is the coordinate of the point (1,0,1) after this viewing transformation is applied?

ANSWER: UPxZ then, ZxX

```
0 -1 0
-.7 0 .7
-.7 0 -.7
```

pt at (0,0,-1.4).

13 Projection (? points)

13.1 Parallel Orthographic

A line runs from (100,200,-300) to (100,200,-400). No geometrical transformations are applied to it. The viewing situation is canonical. Assume NO clipping is done.

What are the endpoints of this line after projection to 2D using parallel orthographic projection? The projection plane is (0,0,-10).

Answer - (100,200) to (100,200) or (100,200,-10) to (100,200,-10).

Perspective

Assume a perspective viewing situation such that the projection plane is at $z = -10$. The eye is at (0,0,0).

A. What is the perspective projection matrix such a situation would use?

B. Where does the point (30,10,-20) project to? (Show the point in homogeneous coordinates with a "w" component of 1.0.) Don't forget the homogeneous divide.

Answer:

```
1 0 0 0
0 1 0 0
0 0 1 0
0 0 1/-10 0
```

(15, 5, -10, 1)

14 Order (? points)

The operations below form a sequence that is applied to geometric primitives to make a desired image. However, the operations aren't in the right sequence. Give me the sequence of matrices, geometric primitives (e.g. vectors "V") in the correct order. Also indicate where in the pipeline clipping takes place.

$M_{window-viewport} * M_{geom} * M_{view} * V * M_{proj}$

What is the right order? Give the matrices in the order in which the mathematics would be carried out, using these matrices and vector.

Answer: $M_{vw} * M_p * M_v * M_g * V$

15 Color and Transparency (? points)

15.1 Colors

A. Draw the spectrum for a color with a reddish dominant frequency (wavelength 650) that is 100 energy units in intensity and that is 50% saturated.

Answer:

straight line at 50 energy units with spike at about 650 nanometers to 100 units.

B. Draw a CIE XY diagram. Label white. Draw and label two points representing colors that are complementary. Draw and label a totally saturated green point and a half-saturated green point. Show where a half-saturated green point of half intensity is compared to a half-saturated green point of full intensity.

Answer: Approximately in the middle is daylight; complementary colors are on opposite ends of a line through white. green is on top edge and half-saturated is halfway from there to white. intensity doesn't change position on cie diagram.

C. Given (red,green,blue) triplets

(1.0, .9, .9),

(0.9, .3, .3),

(.1, .1, .1),

Which is most bright? Answer: (1.0, .9, .9).

Which is most pure? Answer: (.9, .3, .3)

D. What is the hue (word in english) of the RGB value (1.0,1.0,0)? Answer: yellow

E. What is the numeric saturation of the RGB value (0.0,1.0,0.0)? Answer: 100% or 1.0

F. What is the value of the RGB value (0,0,0)? Answer: 0.

G. What is the RGB value of the HSV value (0,0,1)? Answer: (1,1,1)

15.2 Transparency

If a red pixel (1, 0, 0, 0.5) that is 50% opaque is in front of a green pixel (0, 1, 0, 0.5) that is 50% opaque, what is the opacity of the composite pixel taking account of both?

Answer :

.75 (.5 from front and .25 (half) of behind gets through).

If you were drawing a picture with a scanline hidden surface algorithm, how could you alter the program to take into account transparency correctly?

Answer :

keep present transparency and interpolate like x.
accumulate depth sorted list at pixels front to back until
opacity is 1 (need alpha buffer)
or do back to front (don't need alpha buffer).

16 Illumination Models (? points)

Assume no distance effect. Triplets refer to (red, green, blue):

Intensity_ambient = (0, 1, 0)

Intensity_light = (1, 1, 1)

ka = (0.1,0,0)

ka = kd = ks = (1, 0, 0)

n = 2

Triplets refers to (x,y,z):

Normal_to_surface = (0, 1, 0)

Light_direction = (.866, .5, 0)

Eye_direction = (0, .7, .7)

a. What is the ambient color of the surface?

answer $(0,0,0) = (1,0,0) * (0,1,0)$

b. What is the diffuse color of the surface?

answer $(N*L)=0.5$. $I_d = 0.5 * (1,1,1) * (1,0,0) = (0.5,0,0)$

c. What is the reflection vector?

answer $(-.866,.5,0)$

d. What is the specular color of the surface?

answer $(V.R)^2 * I * k_s = ((0, .7, .7) * (-.86, .5, 0))^2 * (1, 0, 0) * Z(1, 1, 1)$
 $(.1225, 0, 0)$

e. What is the total color overall of the surface?

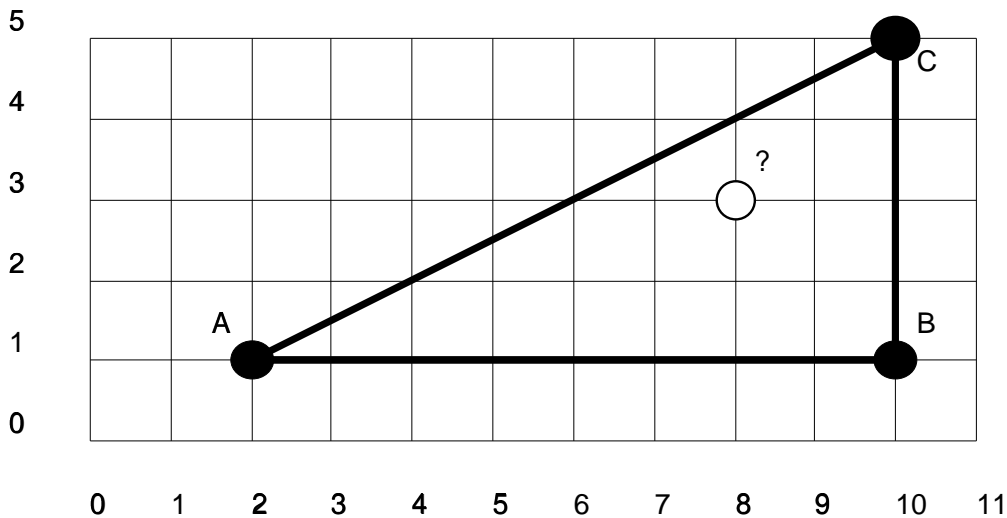
answer $(0,0,0) + (.5,0,0) + (.1225,0,0) = (.6225,0,0)$

17 Gouraud Shading (? points)

Using Gouraud shading with the scanline rendering algorithm, give the color of pixel (scanline 3, pixel 8) in the following diagram (this pixel is shown as an empty circle). By color, assume the light reflected off the surface found using the ambient/diffuse/specular model of illumination.

Pixel A is colored $(0, 0.2, 1.0)$. Pixel B is colored $(0, 1.0, 0.0)$. Pixel C is colored $(1.0, 1.0, 1.0)$.

You must give the equations that you used to calculate your answer, not just the final answer.



Answer: going up the left edge, intensity half way is $(.5, .6, 1.0)$.
 up the right edge, intensity halfway is $(.5, 1., .5)$.
 Midway across scanline intensity is $(.5, .8, .75)$.

Intensity change, left edge, per scanline = $(I_{ytop} - I_{ybottom}) / (ytop - ybottom)$

Intensity change, right edge, per scanline = $(I_{xright} - I_{xleft}) / (xright - xleft)$

18 Dither matrix (? points)

If the dither matrix is

```
-----  
| 0 | 2 |  
-----  
| 3 | 1 |  
-----
```

And the calculated intensities (min 0 max 4) in the image are

```
-----  
| 1 | 1 | 1 | 1 |  
-----  
| 1 | 1 | 1 | 1 |  
-----
```

What are the actual colors (0 = black and 1 = white) in the frame buffer if the image is dithered to black and white? Fill in the frame buffer below with 0 and 1.

```
-----  
|   |   |   |   |  
-----  
|   |   |   |   |  
-----
```

answer - if $fb[] > d[]$ turn on.

```
1 0 1 0  
0 0 0 0
```

19 Texture Mapping (? points)

Imagine a texture that is 100 wide and 25 high that consists of four X's which fill the texture left to right (XXXX). You want to map this onto a triangle with vertices (0,0),(1,0),(0,1). The texture coordinate for vertex (0,0) is (0,0). The texture coordinate for vertex (1,0) is (.25,0.0). The texture coordinate for vertex (0,1) is (0,1.0).

Draw what the texture mapped triangle looks like below.

Answer lower left diagonal of one x.

20 Scanline Algorithms Revisited (? points)

What are the edge data structures for the triangle with vertices $V_0=(3,6,9)$, $V_1=(5,5,8)$, $V_2=(7,7,7)$. Edge A goes from V_0 to V_1 . Edge B goes from V_1 to V_2 . Edge C goes from V_2 to V_0 . Assume you are interpolating grey scale to do Gouraud shading. V_0 and V_2 are colored 1. V_1 is colored 0.4

- What is the z-depth values at pixel on scanline 6 at pixel 5 $(x,y) = (5,6)$.
- What is the intensity at that pixel?

answer

Edge ymax x dx c dc z dz

A 6 5 -2 .4 .6 8 1

B 7 5 1 .4 .3 8 -.5

C 7 3 4 1 0 9 -2

AEL-6: C[or A](7, 3, 4, 1, 0, 9, -2) - B(7, 6, 1, .7, .3, 7.5, 1)

zdepth at (5,6) = 8.00

color is .8

First find the values for x, color, and z for the left edge on scanline 6, then find these values for the right edge. then, interpolate between these two values across the intervening pixels.

21 Parametric Curves and Surfaces (? points)

21.1 Curve Comparisons

Give one case where a hermite formulation would be superior to a bezier, and one case where a b-spline would be preferable to a bezier.

control tangents independently, interpolates

b-spline gives c2 continuity.

Are the two bezier segments C_1 continuous? SHOW why or why not, using equations and figures.

if lines are continuous, subtract vectors should be equal.

i.e. $V_3 - V_2$ of first curve = $V_1 - V_0$ of second curve.

21.2 Curves

The control points for a 2D Bezier curve are (P0 to P3) (0,0),(1,10),(2,-10),(3,0). Draw the curve that interpolates P0 to P3 for parameter values $t=0$ to $t=1$.

Calculate the location of point $t=0.5$. You must show how you calculate this mathematically to get full credit.

Answer $P_t(t) = T M G = (.125, .25, .5, 1) * M_{\text{bezier}} * (P_{tx}, P_{ty}) = (1.5, 0)$

21.3 Continuity

For the Bezier curve below, are the two segments C1 continuous.

If not, what point would you have to move and to where, in order to achieve C1 continuity?

21.4 Surfaces

Give the matrices (with numeric values) for calculating any point x, y , and z for a parametric cubic bezier patch which is planar and is perpendicular to the z -axis. One corner of the patch is point (1,1,1) and the opposite corner is (4,4,1) and the control points are equally space.

```
%answer for x
%1 2 3 4
%1 2 3 4
%1 2 3 4
%1 2 3 4
%answer for y
%1 1 1 1
%2 2 2 2
%3 3 3 3
%4 4 4 4
%answer for z
% all ones
```

to do the surface, $P_t(s,t) = T * M_{\text{bezier}} * G_{\text{matrix}} * M_{\text{bezier_transpose}} * S_t$

21.5 Characteristics

What curves that we discussed in class are interpolating? answer : hermite, catmull-rom (cardinal)

What curves that we discussed in class have C2 continuity.

answer b-splines, nurbs

What curves that we discussed in class have the convex hull property.

answer bezier, b-spline, nurb

Draw the convex hull for the control points shown below.

22 Other Modeling Approaches (points)

22.1 Techniques

Give 3 ways to create an image of pebbles using 3D computer graphics. E.g., one way would be to create a lot of polygon primitives, one for each pebble, and render it. (You can't use that one!) Explain briefly.

answer bump map, texture map, displacement map, fractal. (also need to explain)

22.2 Quadric Surfaces

$$f(x, y, z) = 3x^2 - 4y^2 + z^2 + 2x - y + 10$$

is a quadric surface. Are the points (1,1,1) and (0,0,0) on the surface? If not, are they on the same side of the surface?

answer 1,1,1 is 11; 0,0,0 is 10, so both on same side not on. just plug in the (x,y,z) value in the equation. if the point is on the surface, the value of f(x,y,z) will be 0. if it is on the positive side, it will be positive. if it is on the negative side, it will be negative.

22.3 Implicit Surfaces

The two points below at a distance of 1.5 apart each radiate an implicit field that increases with distance from the point. The value at each point in the field is the sum of the values from the contributing points. Draw the isosurface where the field value is 3.

answer: an ellipse. generally the implicit function drops off with distance, but in this simple case, you just add the actual distance from any point to the two points of interest to get the function value. if the function value (this sum) is three, you are on the curve. this turns out to be an ellipse, given two points.

23 Hierarchies (? points)

Assume a 2D three-segment arm where the first segment is length 3, the second length 2, and the third length 1. Initially, all are lined up left to right along the X-axis. A rotation of -90 is applied between the second and third segments

How can you calculate the world space locations of points in local segment spaces? Show the matrices.

```
Pt world = T_seg1_default * T_seg1_state * Tseg2_default
          *Tseg2_state * Tseg3_default * Tseg3_state * Pt_seg3
Pt(seg3) = (1,0,1) (homogeneous coordinates;
Tseg3_state = rotate -90 degrees;
```

```
Tseg3_default = translate (2,0);
Tseg3_default = translate (3,0);
all other matrices are identity matrices.
```

Where is the end of the third segment in world space (locally is 1,0,0)? (Even if you don't know the matrices, you can probably figure it out.)

Answer (5,-1)

24 Hidden Surface Algorithms (? points)

24.1 General Methods

Explain briefly how each of these works: Warnock Area Subdivision, Weiler-Atherton Cookie-Cutter, Ray-casting, Z-buffer, Octree, Floating Horizon, Watkin's Scanline, Painter's Algorithms, Ray-tracing.

When are the pros and cons of these different algorithms.

24.2 Efficiency

For the below scene, which algorithm would probably be most efficient and why? A painter's, zbuffer, ray-tracing, or area-subdivision? Explain your reasoning.

For example:

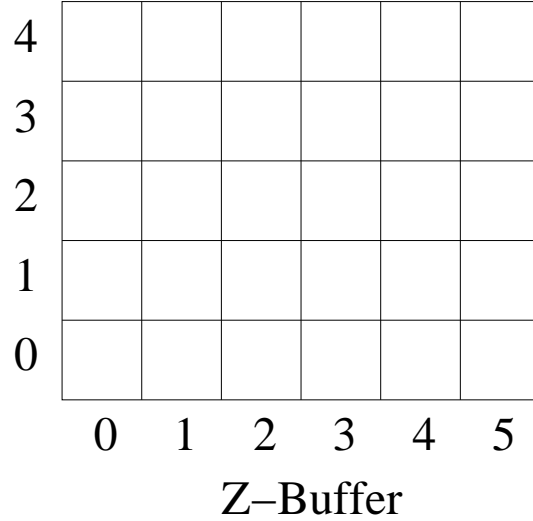
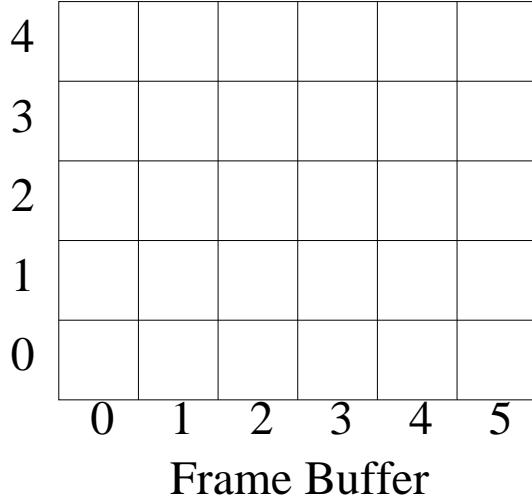
```
if it is large square polygons, area-subdivision.
if it is large oblique polygons, cookie-cutter.
if it is lots of tiny polygons, z-buffer.
```

24.3 Z-Buffers

Red polygon (R) has vertices (1,1,0) to (3,1,0) to (3,3,-4) to (1,3,-4). Cyan polygon (C) has vertices(2,2,-1) to (3,2,-1) to (3,3,-1) to (2,3,-1). The frame buffer is initially set to Black (B) and the Z-buffer to a large value (i).

Show the values in the frame buffer and z-buffer after these two polygons are rendered.

```
%answer
%B B B B B B
%B R C C B B
%B R C C B B
%B R R R B B
%B B B B B B
%answer z
%i i i i i i
```



```
%i 4 1 1 i i
%i 2 1 1 i i
%i 0 0 0 i i
%i i i i i i
```

There is some possible confusion here between left- and right-handed coordinate systems. the hidden surface algorithm is applied after the viewing transformation transforms everything so that the viewer is looking down the negative Z-axis. so in this case, visible objects are normally at negative z-values.

however, traditionally, z-buffers actually hold distance values (which amount to using a left-handed coordinate system), so if the point is at value (1,1,-5), and was visible, then a 5 should be put in the z-buffer, assuming the eye is at the origin. in that case, an object at z = -5 would be closer than an object at z = -10.

25 Ray Tracing (? points)

Show the path of the traced specular ray from the eye to the light assume object B is a mirror. Show the path of a transmitted ray with refraction. Show the path of a shadow ray. Is the point in shadow?

26 Plane Equation (? points)

What is the plane equation for this triangle? $(0, 0, 0)$ to $(1, 1, 1)$ to $(1, 0, 0)$

answer

$(0, 1, -1, 0)$

between scan lines $-b/c = 1$

between x pixels $-a/c = 0$

If you were scan converting this triangle, what would the difference in z value at a pixel that was directly above $(y+1)$ a given pixel?

What would be the difference in z value at a pixel directly to the right of $(x+1)$ a given pixel? $z = (-ax - by - d)/c$. at the pixel to the right, $z' = (-a(x+1) - by - d)/c$. the difference is $z' - z$. similarly for $y+1$.