CIS 102: Fall 2001

Homework Assignment # 3

Recurrence Equations

Part A

In the all problems below, give an asymptotic Θ bound for $T(n)$ unless otherwise specified. Assume that $T(n)$ is a constant for $n \leq 2$.

1. (a)

$$T(n) = T(\frac{9n}{10}) + n.$$

   (b)

$$T(n) = 2T(\sqrt{n}) + \log n.$$

2. (a)

$$T(n) = T(\frac{16n}{4}) + n^2.$$

   (b)

$$T(n) = T(\sqrt{n}) + 1.$$

3. Analyze the running time of the following recursive procedure as a function of n. You may assume that each assignment or division takes unit time. An asymptotic "Θ" analysis is fine.

```
Procedure DC(int n)
  if n < 2 then return;
  else
    count := 0;
    for i := 1 to 8 do
      DC(n div 2);
    for i:=1 to n^3 do
      count := count + 1;
```

(Note that this procedure doesn't do anything useful, but the problem is to find out how long it takes before it finishes doing nothing useful.)

4. Find an asymptotic upper bound for the recurrence

$$T(n) = T(n-a) + T(a) + n$$

where $a \geq 1$ is constant, by using recursion tree and/or *iteration* to generate a guess (see the next problem).

5. Complete the *subsitution* method by proving (using strong induction) that the guess generated above is indeed an asymptotic upper bound for the recurrence

$$T(n) = T(n-a) + T(a) + n$$

Part B

1.

$$T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}.$$

2.

$$T(n) = T(n-1) + \log n.$$

3.

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{4}) + T(\frac{n}{8}) + n.$$

Remark: I expect you to understand this material by October 16. There will be a quiz from Part A only. Solutions to Part B will not be posted. You are encouraged to write down the solutions to Part B, acknowledge all the help you get on Part B, write down the date when you did attempt part B, get a signature from a TA or a tutor within a day or two to verify that you did it on the date claimed by you and include it as a part of your portfolio at the end of the quarter.