

## Written Assignment 2

40 pts, due at the start of lecture Tuesday, Oct. 27.

1. (5 pts) Prove formally that  $10\sqrt{n}$  is in  $O(n)$  using the  $c$  and  $n_0$  definitions in the book.
2. (7 pts) Rank the following functions by order of growth, list the fastest growing functions at the top, the slowest growing ones at the bottom. Functions that grow at the same asymptotic rate (i.e. are  $\Theta$  of each other) should be listed on the same line (see instructions for problem 2-3 on page 38, no proofs required for this problem). Recall that  $\lg$  means the base 2 logarithm.

$n!$	$n^4$	$4^n$
$\lg n$	$2^{2n}$	$4^{\lg n}$
$2^{(2^n)}$	$2^{\lg n}$	$n$
$\sqrt{n}$	$n^{\lg 3}$	$(\lg n)^2$
$\log_{10} n$	$n^{\lg 4}$	$n^n$

3. (6 pts) Prove that (for any positive  $g(n)$ ) no function is in both  $o(g(n))$  and  $\Theta(g(n))$ . Use the  $c$  and  $n_0$  style definitions of  $o$  and  $\Omega$  given in the book. (Hint: consider proof by contradiction.)
4. (6 pts) Use the master theorem to obtain asymptotic expressions for the following recurrences. Assume that  $T(n)$  is positive for  $n = 1$  and  $n = 2$ .
  - $T(n) = 3T(n/2) + n^2$ .
  - $T(n) = 3T(n/2) + n \lg n$ .
  - $T(n) = 16T(n/4) + n^2$ .
  - $T(n) = T(9n/10) + n$ .
  - $T(n) = 2T(n/4) + n$ .
5. (7 pts) Formally prove by induction that  $n^2 \leq 2^n$  for all  $n \geq 4$ . Pay particular attention to the structure and style of your proof.
6. (9 pts) Problem 5.5-4 on page 96. Use the definition of Binary tree on page 94, and consider cases based on how many children the root has. Pay particular attention to the structure and style of your proof.

Recommended problems, not to be turned in

1. Problem 3.1-1 on page 45 of the text.
2. Problem 4.3-2 on page 64 of the text.
3. Prove theorem 2.1 on page 27 of the text. You may want to break this into two parts: showing first that if  $f(n)$  is  $\Theta(g(n))$  then  $f(n)$  is  $O(g(n))$  and also  $f(n)$  is  $\Omega(g(n))$ ; and later showing that if both  $f(n)$  is  $O(g(n))$  and  $f(n)$  is  $\Omega(g(n))$  then  $f(n)$  is  $\Theta(g(n))$ .
4. Problem 2-4 on page 39.
5. (This is hard) Give a closed form solution for  $\sum_{i=0}^n i2^i$ .
6. (Also hard) Find two positive functions  $f(n)$  and  $g(n)$  such that both  $f(n) \notin O(g(n))$  and  $g(n) \notin O(f(n))$ .