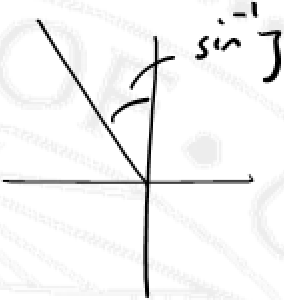
CMPE-242
Applied Feedback Control

Gabriel Hugh Elkaim

Office Hours

25/JAN/2017

#4 LEAD DESIGN



$$G(s) = \frac{10}{s(s+1)(s+10)}$$

$M_p < 16\% \longrightarrow \zeta = 0.5 \longrightarrow 30^\circ$

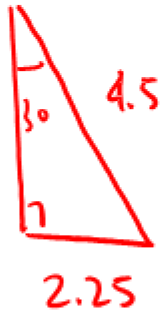
$t_r < 0.4\,\text{sec} \longrightarrow t_r \simeq \frac{1.8}{\omega_n} < 0.4 \longrightarrow \omega_n > 4.5$

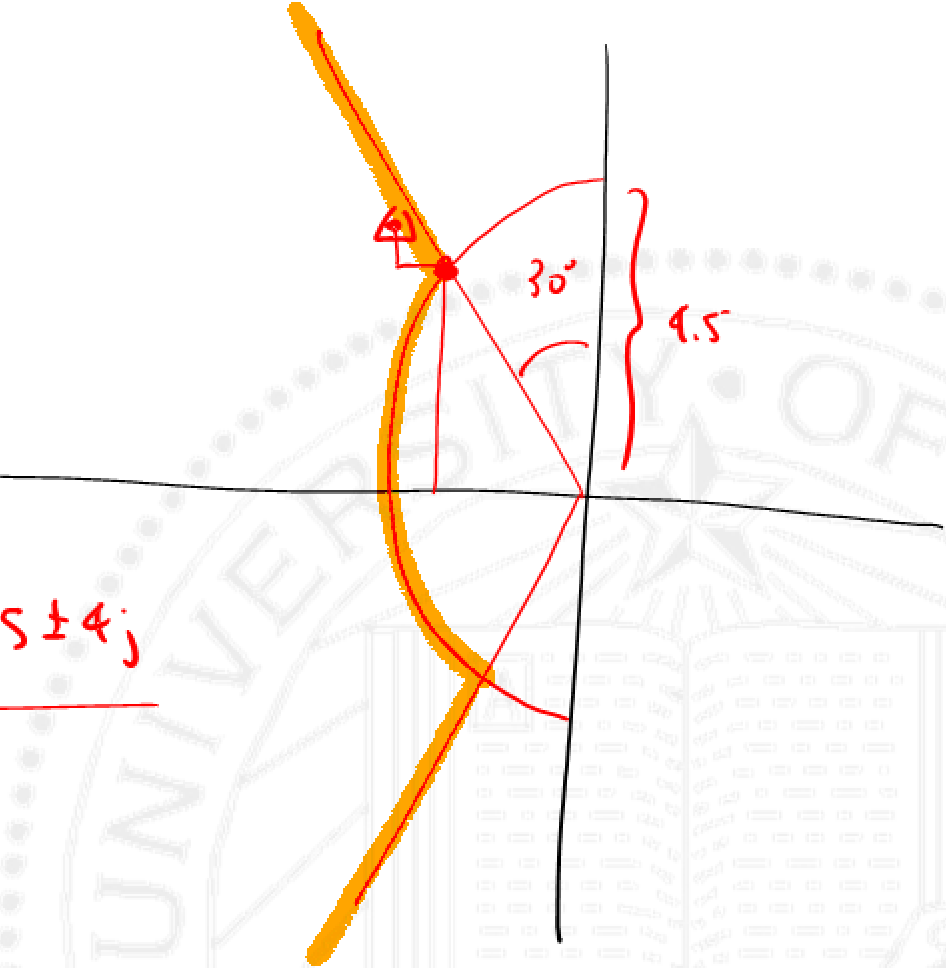
$e_{ss} < 0.02$

$2.25\sqrt{3}$

$\downarrow$

3.9





$\Delta_{des} = -2.5 \pm 4j$

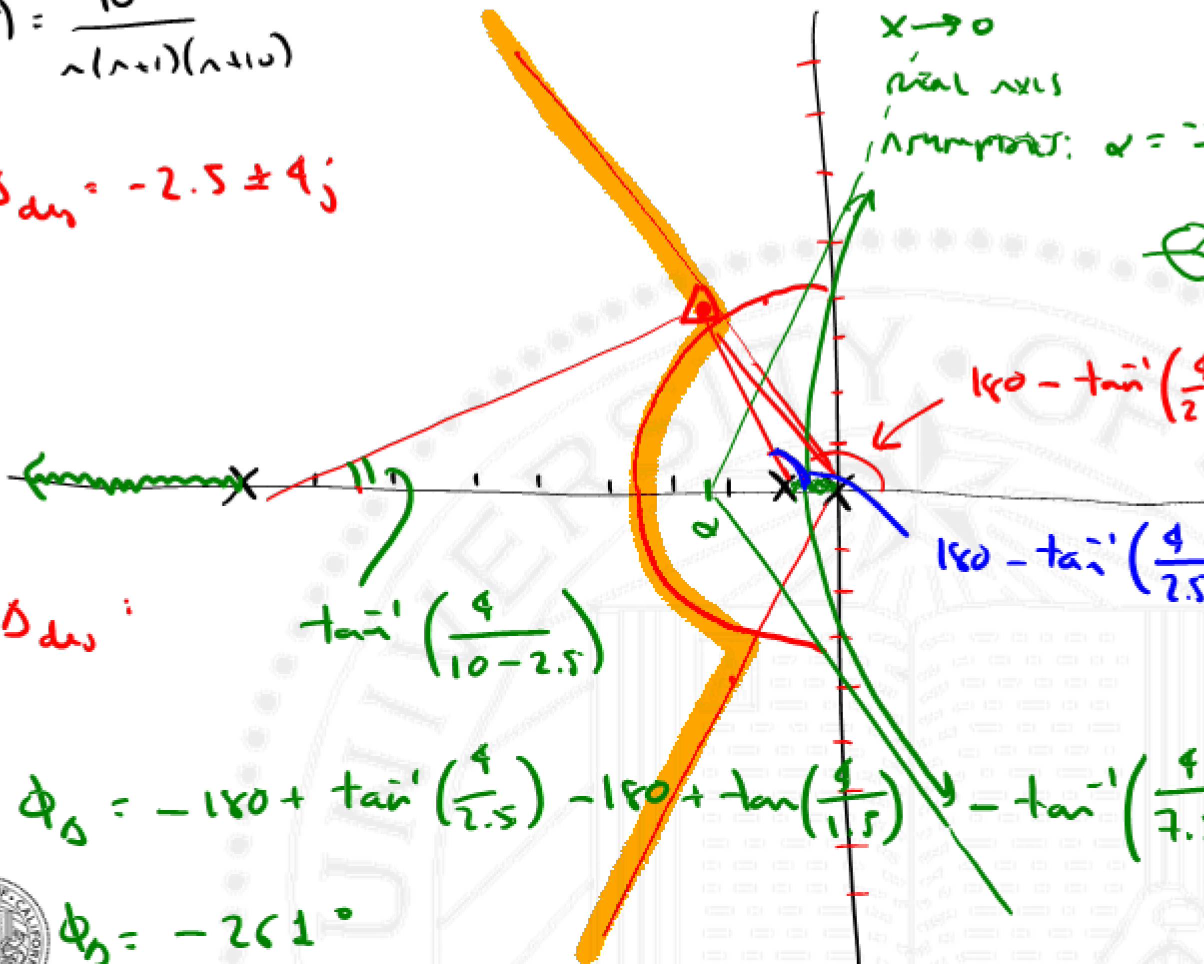
$$G(s) = \frac{10}{s(s+1)(s+10)}$$

$$s_{des} = -2.5 \pm 4j$$

$x \to 0$

real axis

asymptotes: $\alpha = \frac{-11}{3}$



$180 - \tan^{-1}\left(\frac{4}{2.5}\right)$

$180 - \tan^{-1}\left(\frac{4}{2.5-1}\right)$

$\tan^{-1}\left(\frac{4}{10-2.5}\right)$

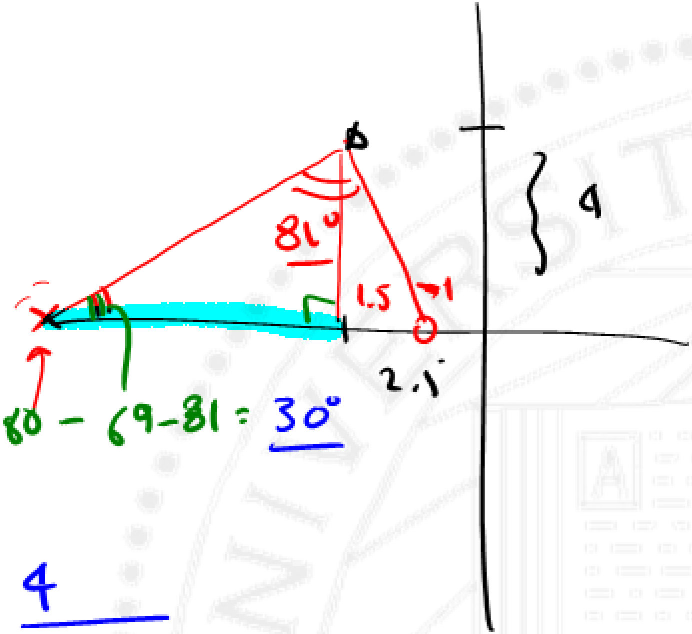
@ s_{des}:

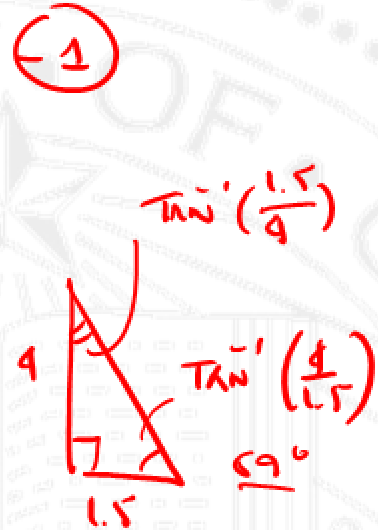
$$\phi_D = -180 + \tan^{-1}\left(\frac{4}{2.5}\right) - 180 + \tan\left(\frac{4}{1.5}\right) - \tan^{-1}\left(\frac{4}{7.5}\right)$$

$$\phi_D = -261^\circ$$

$\phi_\Delta = -260.6 \longrightarrow -180 \longrightarrow 81°$ of phase lead

to put D on locus





$$\ell = \frac{4}{\tan(30°)}$$

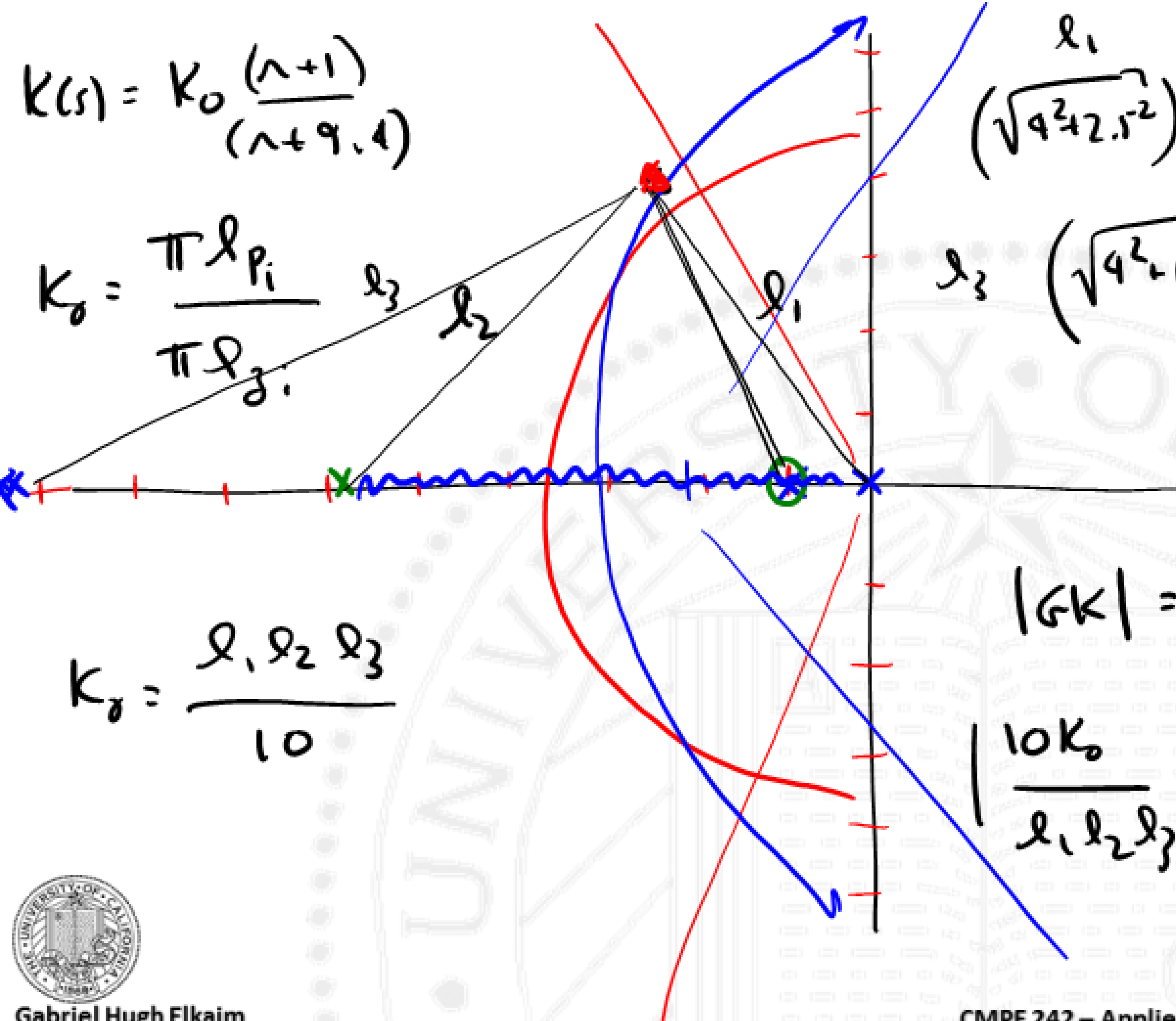
$$D(s) = K_0 \frac{(s+1)}{\left(s + \left(\frac{4}{\tan 30}\right) + 2.5\right)} = K_0 \frac{(s+1)}{(s+9.4)}$$

$$K(s) = K_0 \frac{(s+1)}{(s+9.4)}$$

$$\overset{\ell_1}{\left(\sqrt{4^2+2.5^2}\right)}\overset{\ell_2}{\left(\sqrt{4^2+(6.9)^2}\right)}$$

$$\ell_3 \left(\sqrt{4^2+(10-2.5)^2}\right)$$

$$K_o = \frac{\Pi \ell_{p_i}}{\Pi \ell_{z_i}}$$



$$K_o = \frac{\ell_1 \ell_2 \ell_3}{10}$$

$$|GK| = 1$$

$$\left|\frac{10K_o}{\ell_1 \ell_2 \ell_3}\right| = 1$$

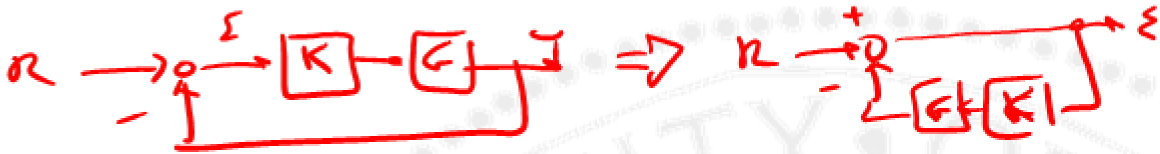
```
Gs = tf([10],conv([1 10 0],[1 10]));
Ds = tf(ko*[1 1],[1 9.4]);

rlocus(Gs);
hold on
sgrid(0.5,4.5);

rlocus(Ds*Gs);
ko = rlocfind(Ds*Gs);

Gcl = feedback(ko*Ds*Gs,1);
step(Gcl);
```

$\varepsilon_{ss} < 0.02$ $\qquad \frac{E}{R} = \frac{1}{1+GK}$



$$\varepsilon_{ss} = \lim_{s\to 0} s\left(\frac{E(s)}{R(s)}\right) \underbrace{R(s)}_{\frac{1}{s^2}} = \lim_{s\to 0} \cancel{s}\left(\frac{E}{R}\right)\frac{1}{s\cancel{s}}$$

$$= \lim_{s\to 0} \frac{1}{1+K_0 D(s) G(s)} \cdot \frac{1}{s}$$

$$\frac{E}{R} = \frac{1}{1 + \frac{K_0 \cancel{(s+1)}}{(s+9.4)} \frac{10}{(s)\cancel{(s+1)}(s+10)}} = \frac{1}{1 + \frac{10K_0}{s(s+9.4)(s+10)}}$$

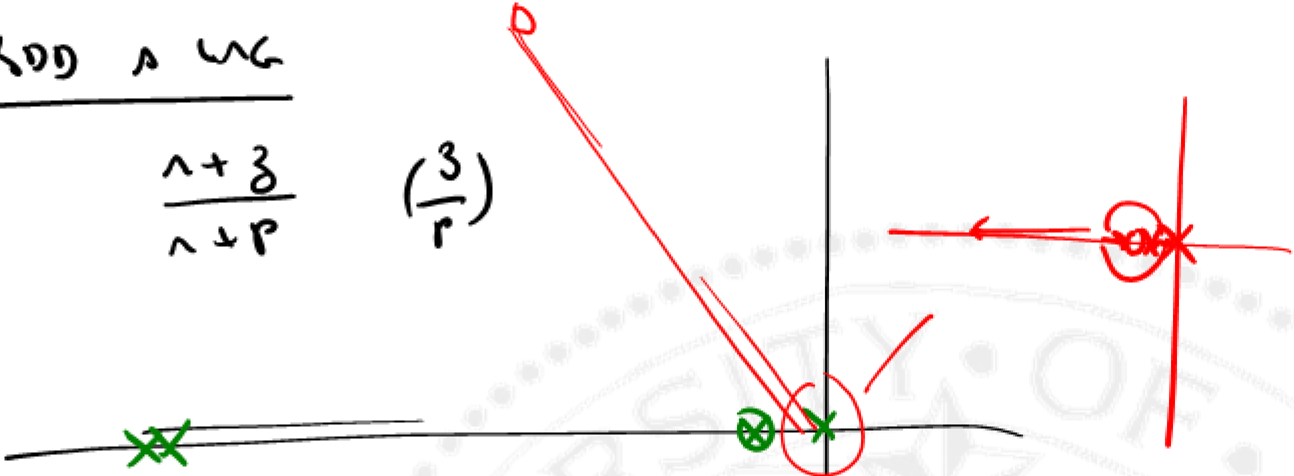
$$\lim_{s\to 0} \frac{\cancel{s}(s+9.4)(s+10)}{s(s+9.4)(s+10)+10K_0}\left(\frac{1}{\cancel{s}}\right) = \frac{(9.4)\cancel{(10)}}{\cancel{10}K_0} < 0.02$$

$$\frac{9.4}{K_0} < 0.02 \quad \therefore \quad \boxed{K_0 > \frac{9.4}{0.02}}$$

$$\boxed{\frac{9.4}{K_0} = e_{ss} \sim 0.7}$$

Add a lag

$$\frac{s+z}{s+p} \qquad \left(\frac{z}{p}\right)$$

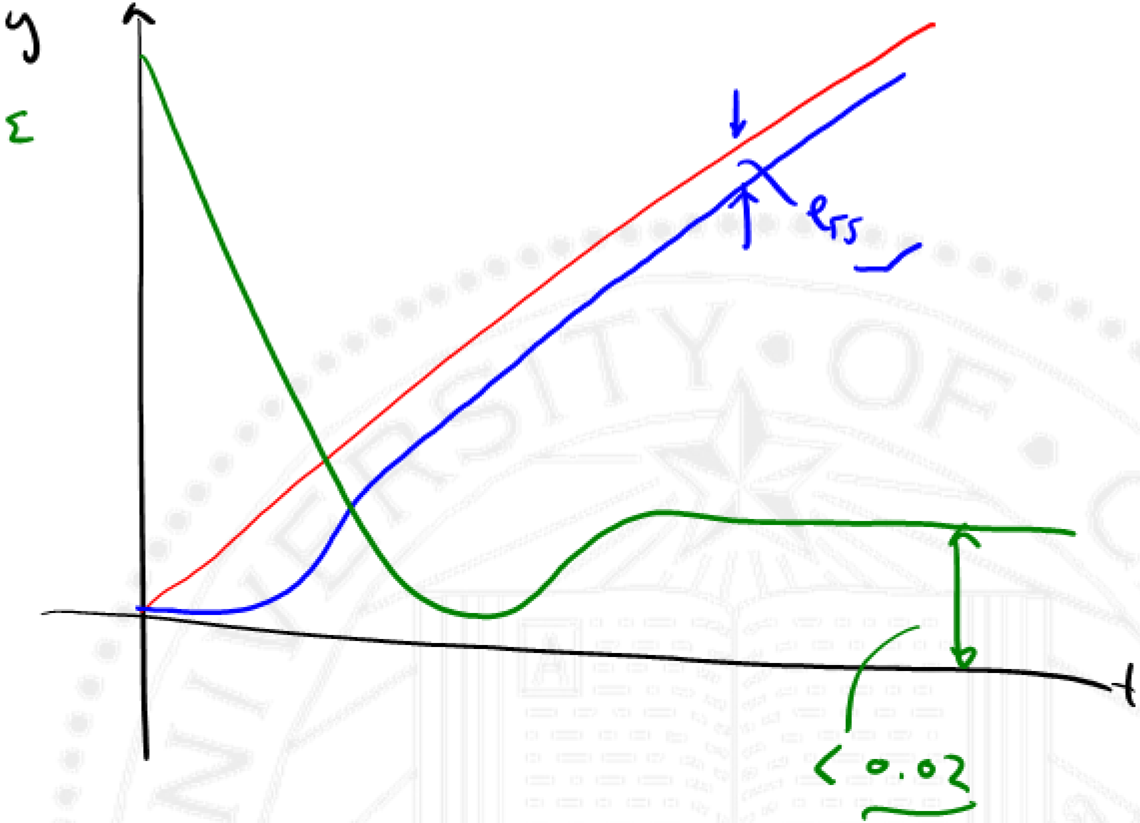


$$\left(\frac{z}{p}\right) > \frac{(9.4/K_0)}{0.02} \rightarrow \underline{30}$$

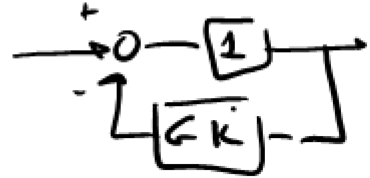
$$z = 0.001$$

$$p = 30(0.001)$$

$$\boxed{D(s) = \frac{K_0(s+1)(s+0.03)}{(s+9.4)(s+0.001)}}$$


y
ε
e_{ss}
t
< 0.02

$$\frac{E}{R} = \frac{1}{1+GK}$$




```
Err = feedback(1,Ds*Gs);
```

#2:

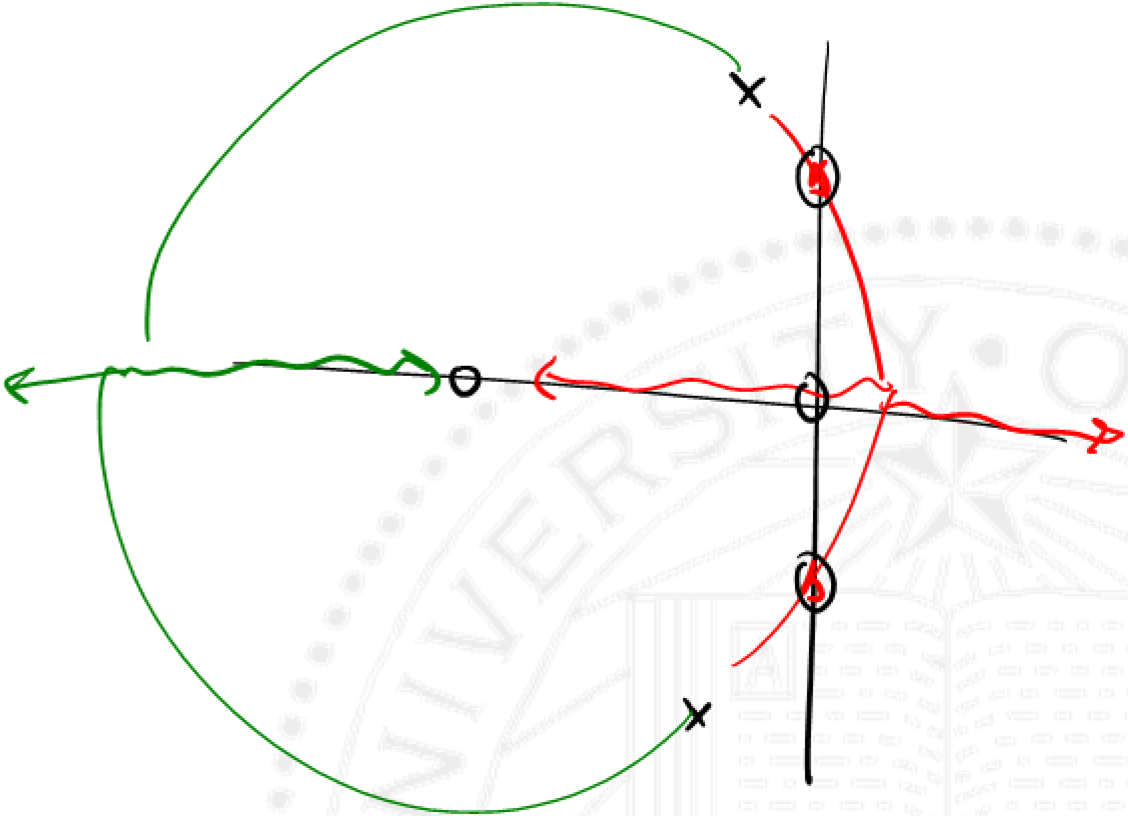
$$G(s) = \frac{1}{s^2+(1+\alpha)s+(1+\alpha)}$$

$$\alpha \frac{N}{D} = -1$$

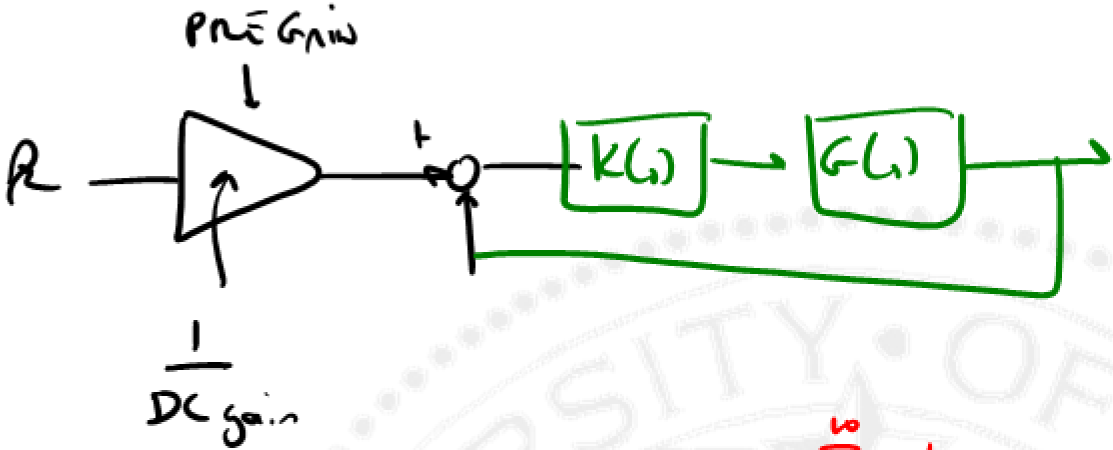
$$\frac{1}{(s^2+s+1)+\alpha(s+1)} = \phi$$

$$(s^2+s+1) + \alpha(s+1) = \phi$$

$$1 + \frac{\alpha(s+1)}{s^2+s+1} = \phi \longrightarrow \boxed{\alpha \frac{s+1}{(s^2+s+1)} = -1}$$

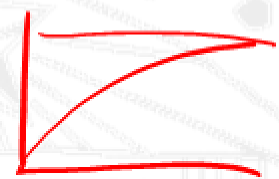


$$\Delta_d(s)\Big|_{s=j\omega} = 0 + 0j \quad — \omega, \alpha$$

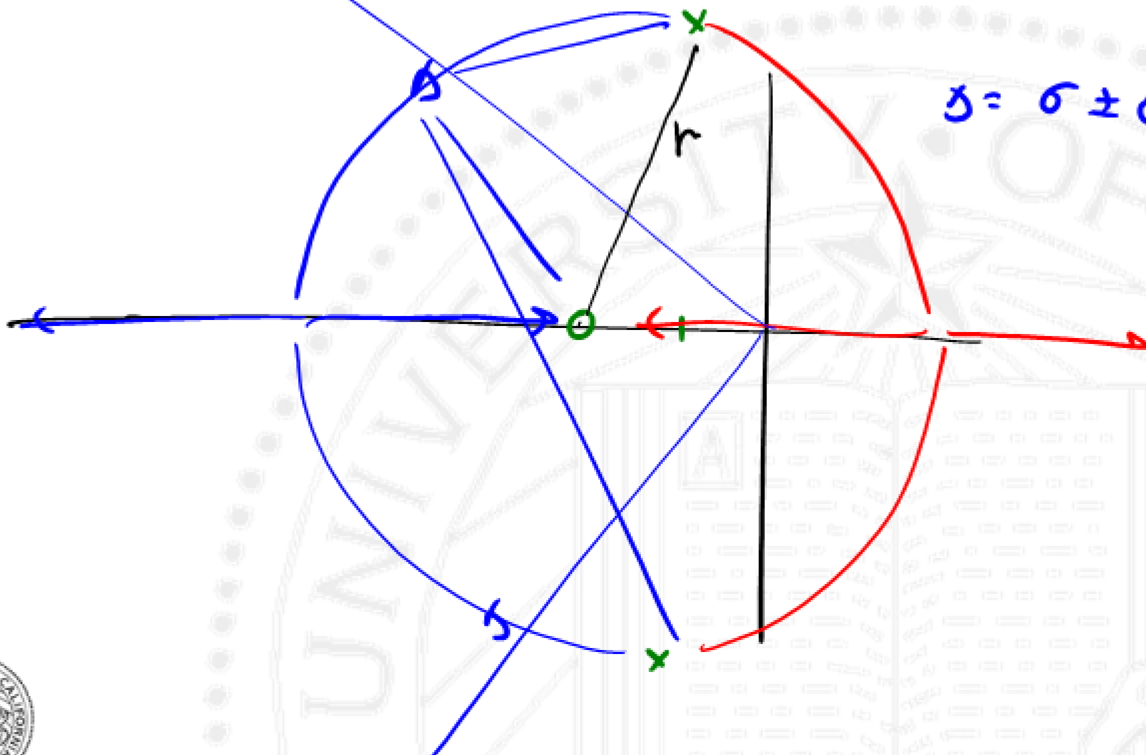


$$\frac{K_0}{(s+10)}$$

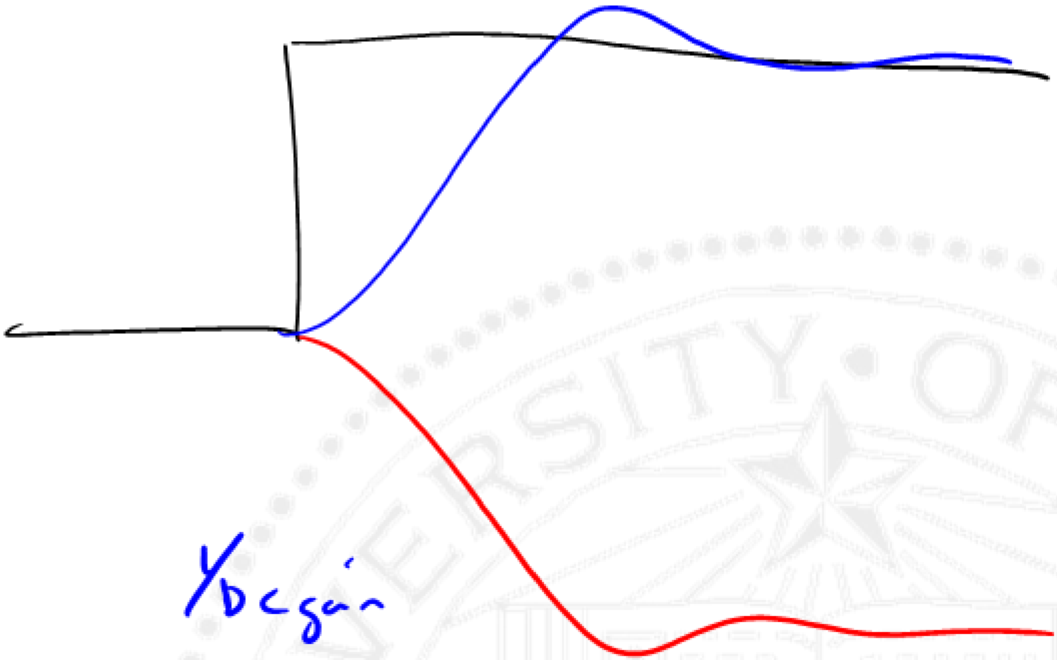
$$\frac{10}{s+10}$$



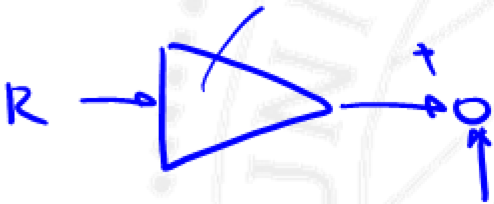
$$G(s) = \frac{4-2s}{(s^2+s+4)} = -\frac{2(s+2)}{(s^2+s+4)}$$



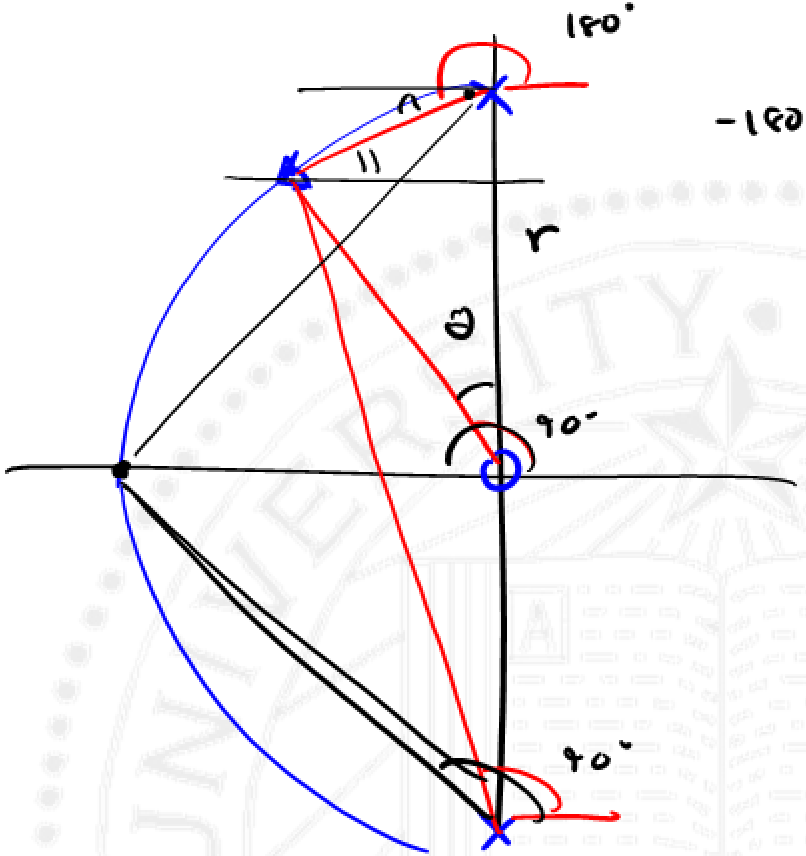
$s = \sigma \pm \sigma j$



1/DC gain



ϕ_A :



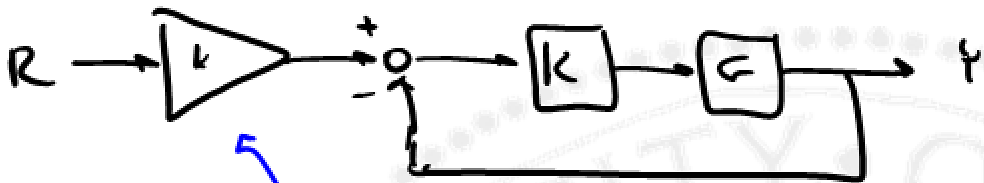
$-180 - 90 + 90 = ?$

-180

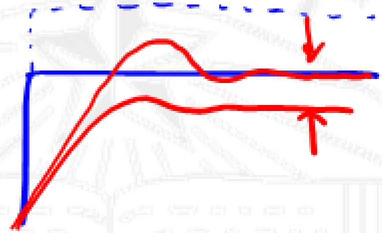
$-180 - 135$

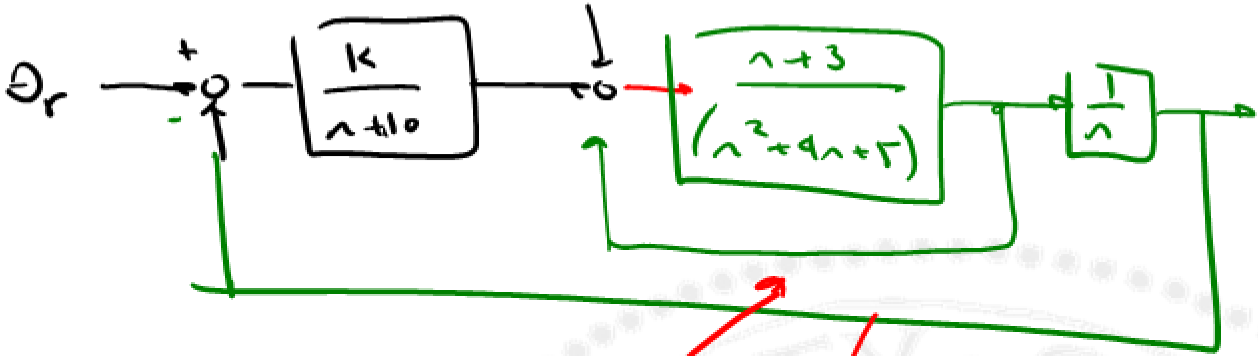
$+135 = ? \; -180$

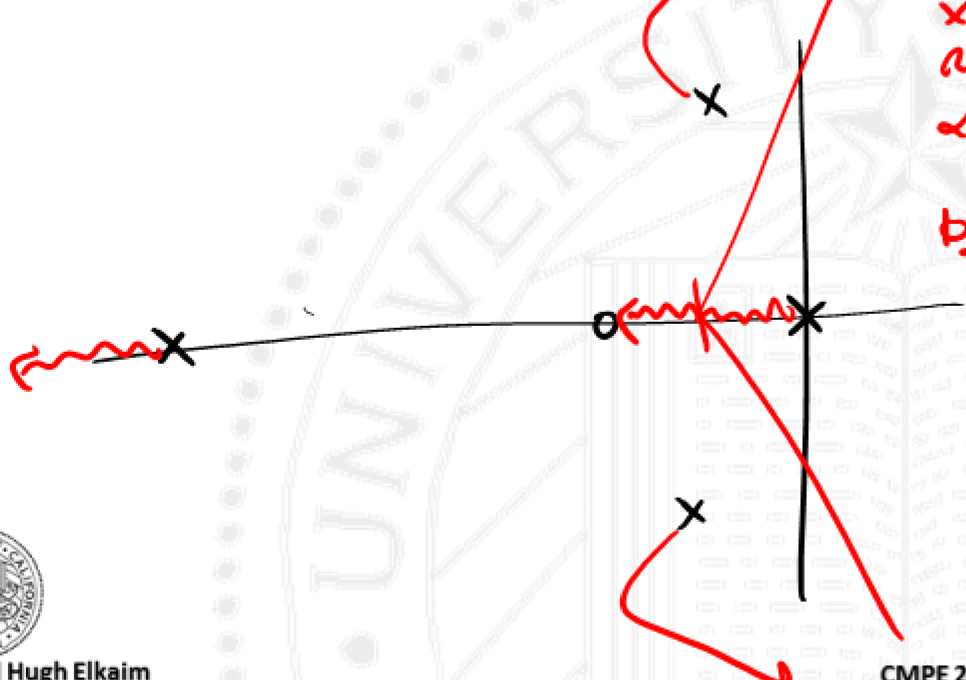
PRE-GAIN



$$\frac{Y}{R} = \frac{GK}{1+GK} \left(\frac{1}{\text{DC gain}} \right)$$





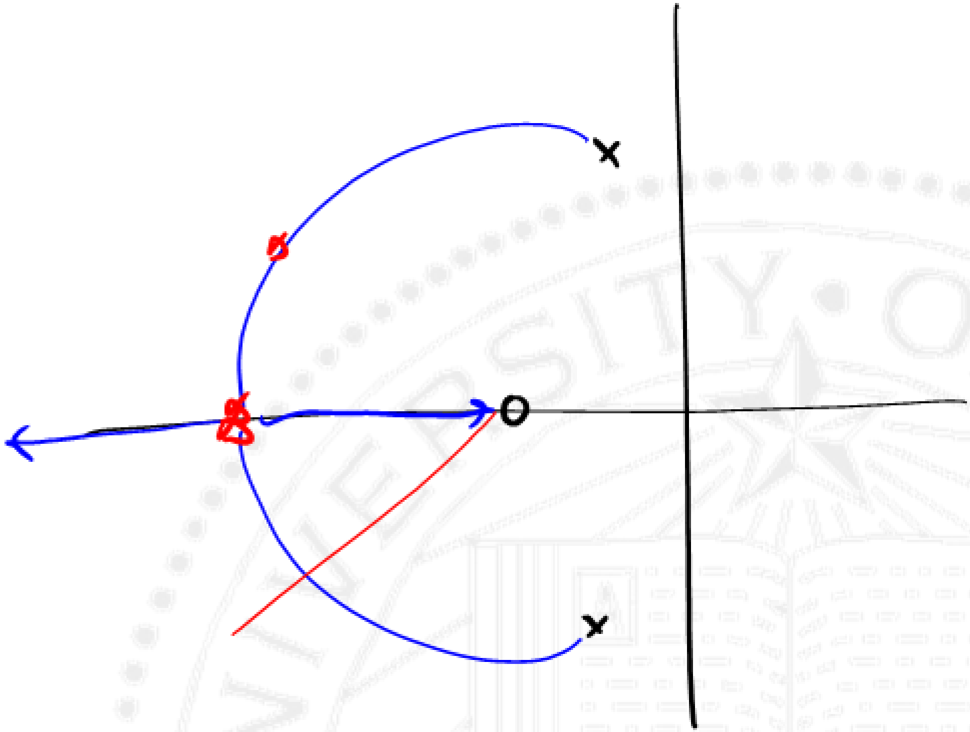


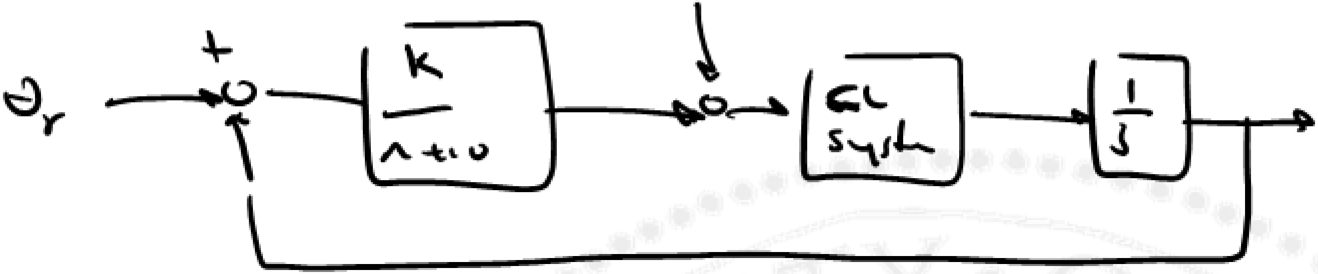
x → 0

real axis

$\alpha = \frac{\sum p - \sum z}{3}$

b_1





Successive Loop Closure