CMPE-242
Applied Feedback Control

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Office Hours

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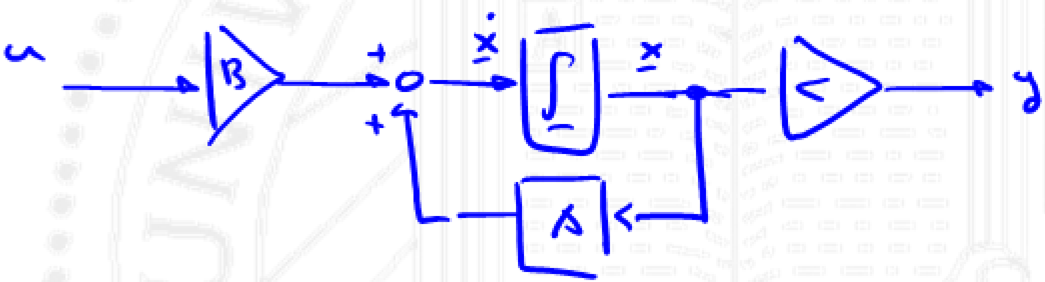
$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A \in \mathbb{R}^{6\times 6}$$

$$B \in \mathbb{R}^{6\times 1}$$

$$C \in \mathbb{R}^{1\times 6}$$

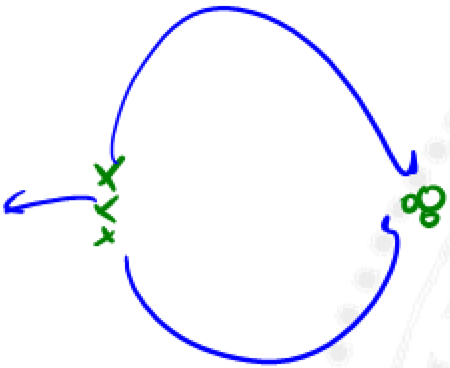


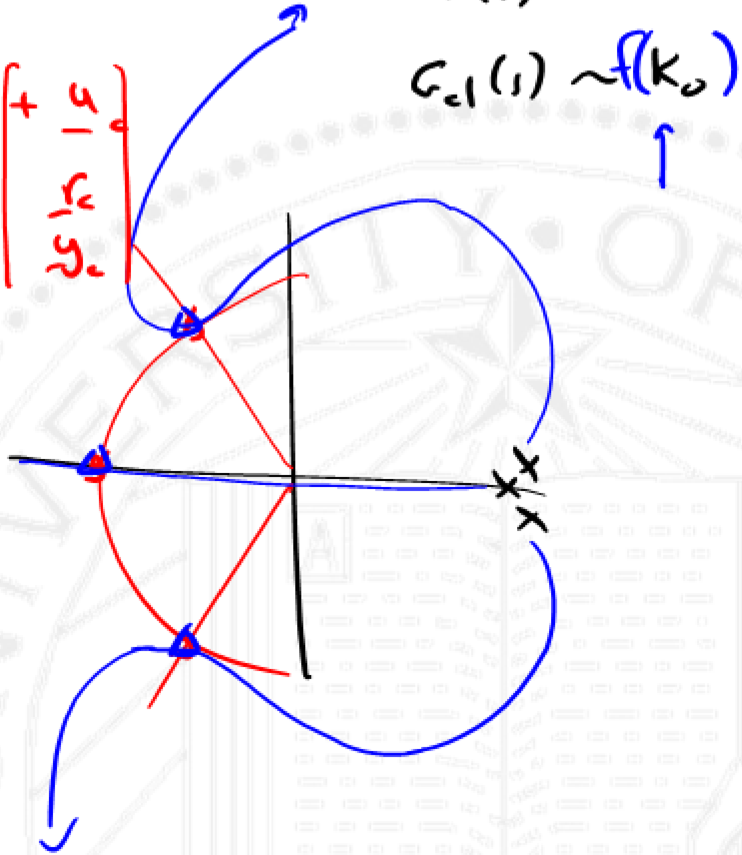
$u = -\underline{K}(\underline{x} - \underline{x}_c)$

$-K(\underline{x}) + \begin{bmatrix} \underline{u}_c \\ \underline{r}_c \\ \underline{y}_c \end{bmatrix}$

$G(s)$

$G_{cl}(s) \sim f(K_o)$





$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad \underline{u} = -K(\underline{x}) + \underline{u}_{cmd}$$

$$y = C\underline{x}$$

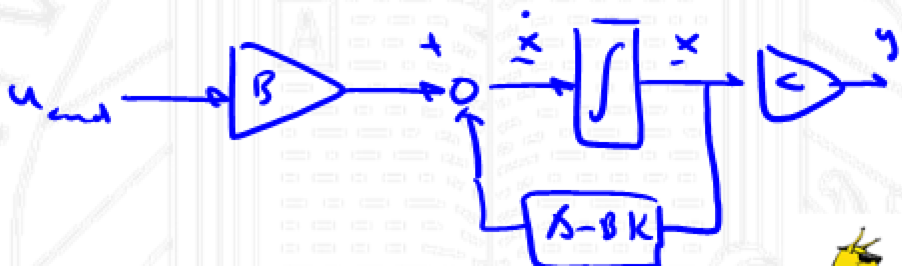
$$\dot{\underline{x}} = A\underline{x} - BK\underline{x} + B\underline{u}_{cmd}$$

$$\underline{y} = C\underline{x}$$

choose $\underline{K}$ place / acker

$$\dot{\underline{x}} = (A - BK)\underline{x} + B\underline{u}_{cmd}$$

$$\underline{y} = C\underline{x}.$$

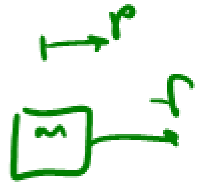


$P_{dm} : \left[6 \; p_{lm} \right]$ ← rank

$X \rightarrow \boxed{s} \rightarrow \dot{X}$

$\underline{x} : \begin{matrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{matrix}$

$$\frac{1}{s^2}$$



$$\begin{bmatrix} \dot{p} \\ p \end{bmatrix}$$

$$u = -Kx = -k_1 \dot{p} - k_2 p$$

$$U(s) = (-k_1 s - k_2) P(s)$$

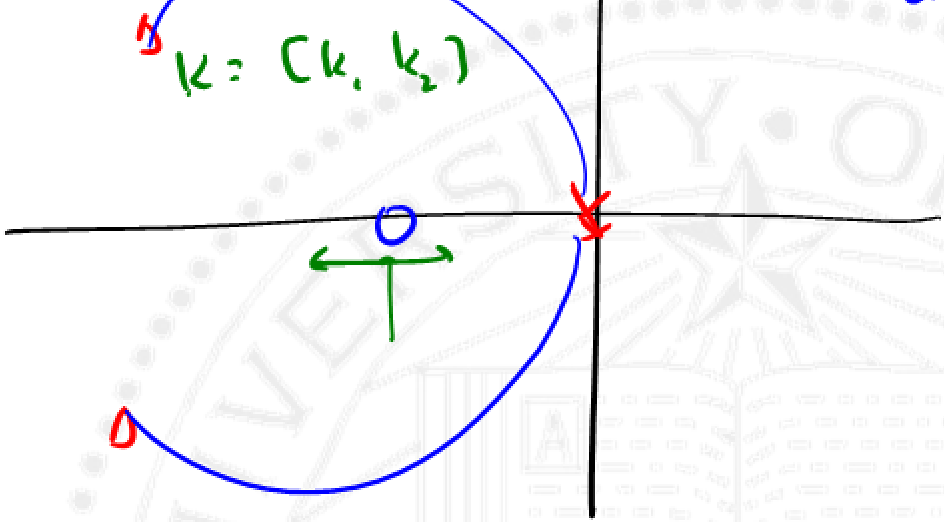
$$\frac{U(s)}{P(s)} = -k_1\left(s + \frac{k_2}{k_1}\right) = \underline{K(s)}$$

$$\dot{\underline{x}} = [A - BK]x + B\underline{u}_{cmd}$$

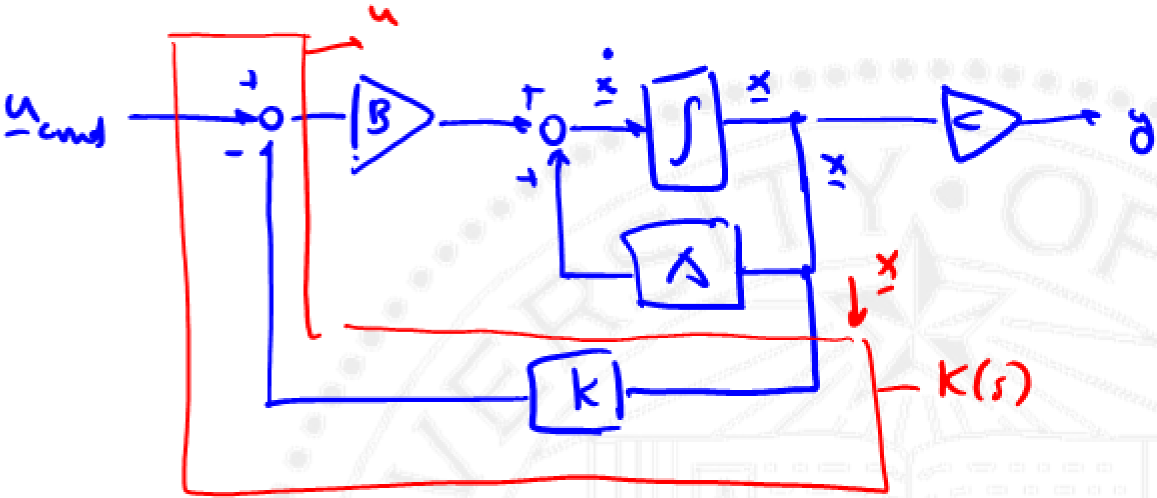
$$u = -K\underline{x} + \underline{u}_{cmd}$$

$$y = C\underline{x}$$

rlocus(G_{cl})



$$K(s) = K\left[sI - A + BK + LC\right]^{-1} L$$



$$K(s) = K\left(sI - A + BK\right)^{-1} I$$

$$G(s) = C[sI - A]^{-1}B$$

$$K(s) = K[sI - (A - BK)]^{-1}B$$

$$K[sI - (A - BK - LC)]^{-1}L$$

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \qquad G(s) = C[sI-A]^{-1}B$$

$$u = -K(\hat{x} + x_c)$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \qquad \hat{y} = C\hat{x}$$

$$u = -K\hat{x} + Kx_c$$

$$\dot{\hat{x}} = (A - BK)\hat{x} + BKx_c + Ly - LC\hat{x}$$

$$\left[\begin{aligned} \dot{\hat{x}} &= (A - BK - LC)\hat{x} + BK\cancel{x_c}^{\,0} + Ly \\ u &= -K(\hat{x} + \cancel{x_c}^{\,0}) \end{aligned}\right] \quad K(s)$$

$$\boxed{K(s) = -K\left[sI - (A - BK - LC)\right]^{-1}L}$$

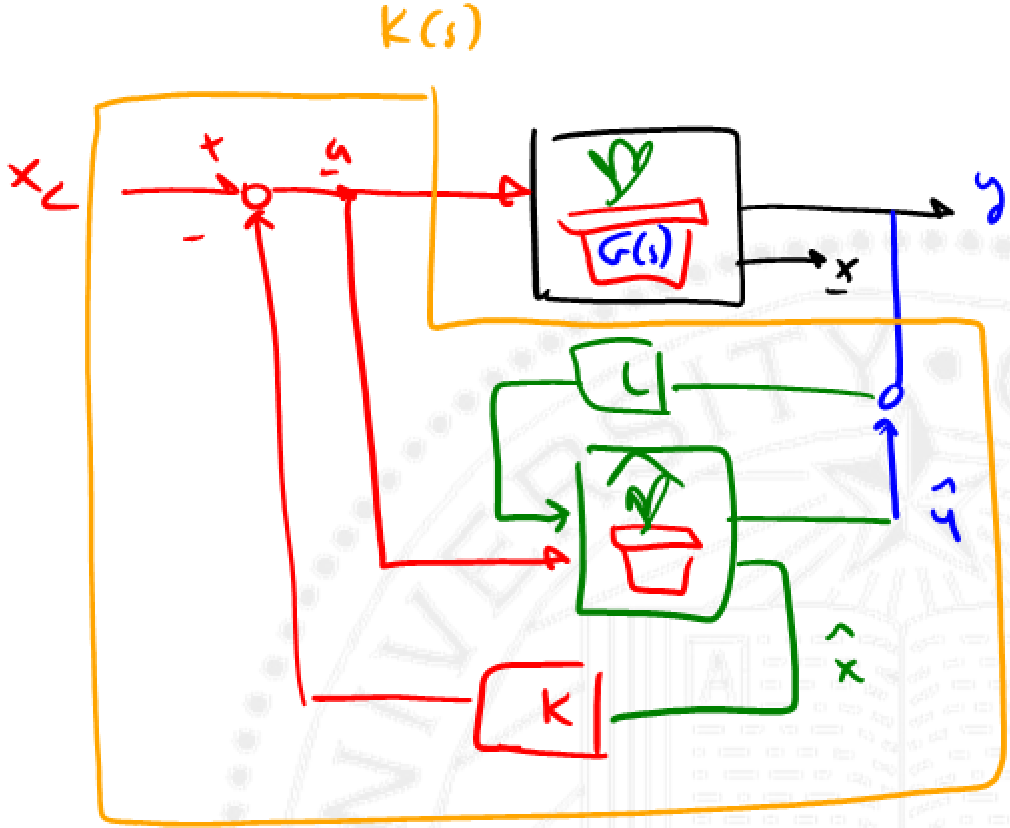
$$\dot{x} = Ax + Bu \qquad u = -K(x - x_c)$$

$$y = Cx$$

$$\dot{x} = (A - BK)\underline{x} + BK\underline{x}_c$$

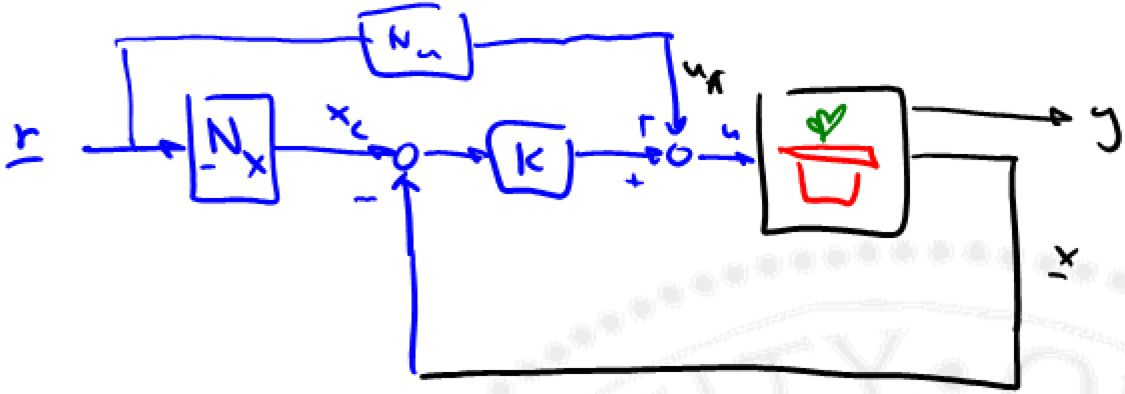
$$u = -K\underline{x}$$

$$\underbrace{\frac{u}{x_c}(s)}_{K(s)} = -\underline{K}\left[sI - (A - BK)\right]^{-1} BK$$


K(s)
x_c
+
-
u
y
G(s)
x
L
$\hat{y}$
$\hat{x}$
K

$$\dot{\begin{bmatrix} \underline{x} \\ \hat{\underline{x}} \end{bmatrix}} = \left[\begin{array}{c|c} A & -BK \\ \hline 0 & A - \cancel{BK} - LC \end{array}\right] \begin{bmatrix} \underline{x} \\ \hat{\underline{x}} \end{bmatrix} + \begin{bmatrix} BK \\ \hline BK \end{bmatrix} \underline{x}_c$$

$$\begin{bmatrix} \underline{x} \\ \hline u \end{bmatrix} = \begin{bmatrix} I \\ \hline -K \end{bmatrix} \begin{bmatrix} \underline{x} \\ \hat{\underline{x}} \end{bmatrix} + \begin{bmatrix} 0 \\ \hline BK \end{bmatrix} \underline{x}_c$$



$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$\begin{bmatrix} N_x \\ \hline N_u \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]^{-1} \begin{bmatrix} 0 \\ \hline I \end{bmatrix}$$

$$u = -K(x - x_c) + u_{ff}$$

$$\uparrow$$

$$u = -K\underline{x} + KN_x r + N_u r = -Kx + [KN_x + N_u]\underline{r}$$

$$\underline{u} = -K\underline{x} + [KN_x + N_u]\underline{r}$$

$$\dot{\underline{x}} = A\underline{x} + B[-K\underline{x} + [KN_x + N_u]\underline{r}]$$

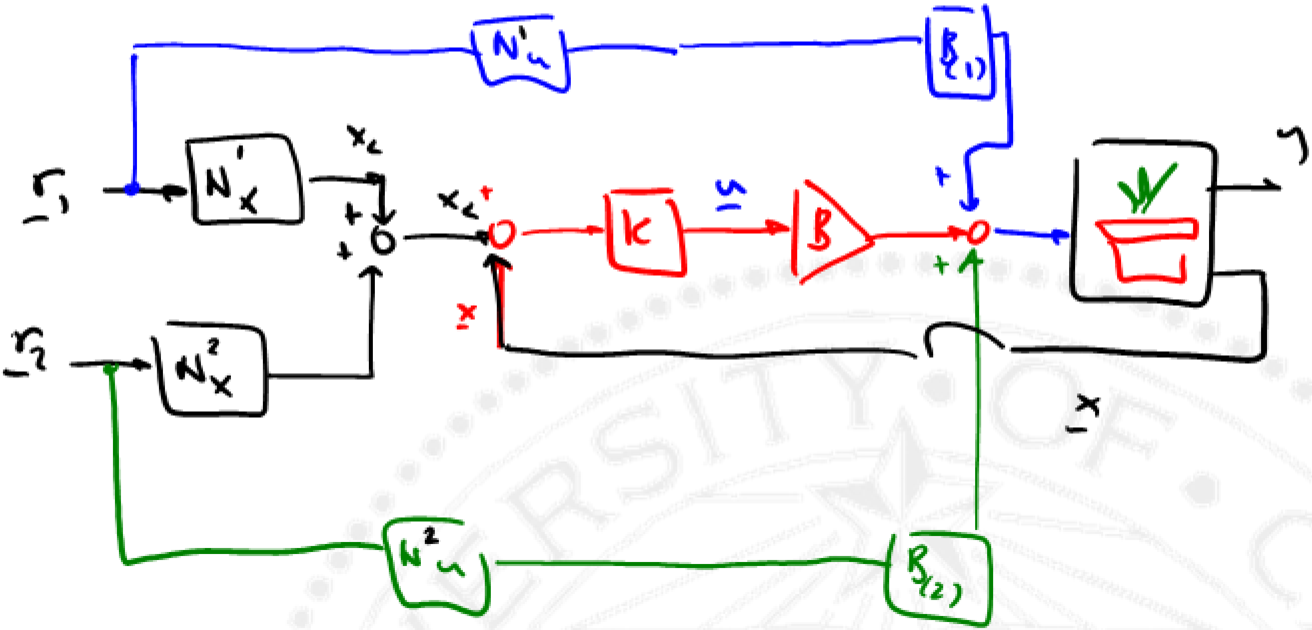
$$y = C\underline{x}$$

$$\dot{\underline{x}} = \overbrace{(A - BK)}^{A_{cl}}\underline{x} + \overbrace{B[KN_x + N_u]}^{B_{cl}}\underline{r}$$

$$y = \underbrace{C}_{C_{cl}}\underline{x}$$

$$\frac{y}{r} = G_{cl}(s) = C[sI - (A - BK)]^{-1} B(KN_x + N_u)$$

$$G_{cl} = ss(A - BK, B*(KNx + Nu), C, \phi)$$



$$\begin{bmatrix} N_x^1 \\ \hline N_u^1 \end{bmatrix} = \left[\begin{array}{c|c} A & B_{(1)} \\ \hline C^{(1)} & 0 \end{array}\right]\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} N_x^2 \\ \hline N_u^2 \end{bmatrix} = \left[\begin{array}{c|c} A & B_{(2)} \\ \hline C^{(2)} & 0 \end{array}\right]\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$u = -K(\underline{x} - x_c) + \underline{u}_{FF}$$

$$u = \underline{-K\underline{x} + KN_x^1 r_1 + KN_x^2 r_2} + \underbrace{N_u^1 B_{(1)} \underline{r}_1 + N_u^2 B_{(2)} r_2}_{\text{2nd ``B''}}$$

$$\dot{x} = Ax + Bu$$

$$\underline{\dot{x}} = Ax - BK\underline{x} + BKN_x^1 r_1 + BKN_x^2 r_2 + \underline{N_u^1 B_{(1)}} r_1 + \underline{N_u^2 B_{(2)}} r_2$$

$$\underline{\dot{x}} = \underbrace{(A - BK)}_{A_{cl}}\underline{x} + \begin{bmatrix} BKN_x^1 & BKN_x^2 \end{bmatrix}\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$\underbrace{BK\left[N_x^1 \,\vdots\, N_x^2\right]\underline{r} + \left[N_u^1 B_{(1)} \,\vdots\, N_u^2 B_{(2)}\right]\underline{r}}_{B_{cl}}$$

$n \times 2$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$2 \times n$

$y_1 \rightarrow u_1$

$y_2 \rightarrow u_2$

$B_{(1)} \quad B_{(2)}$

$$\dot{x} = Ax + Bu$$

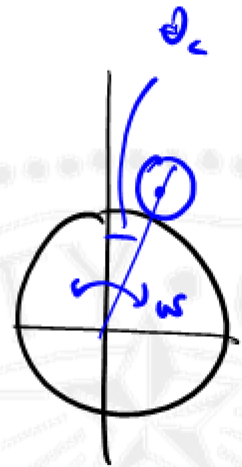
$$y_1 = C^{(1)} x$$

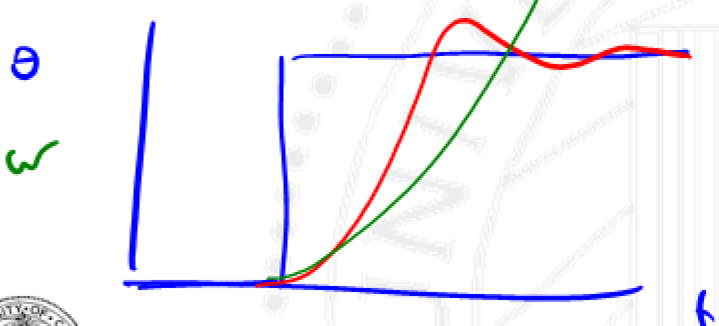
$$y_2 = C^{(2)} x$$

$\frac{y}{r}$ — stable

$\frac{x}{r}$ — stable

unstable





sig (1)

damp (1)