CMPE-242
Applied Feedback Control

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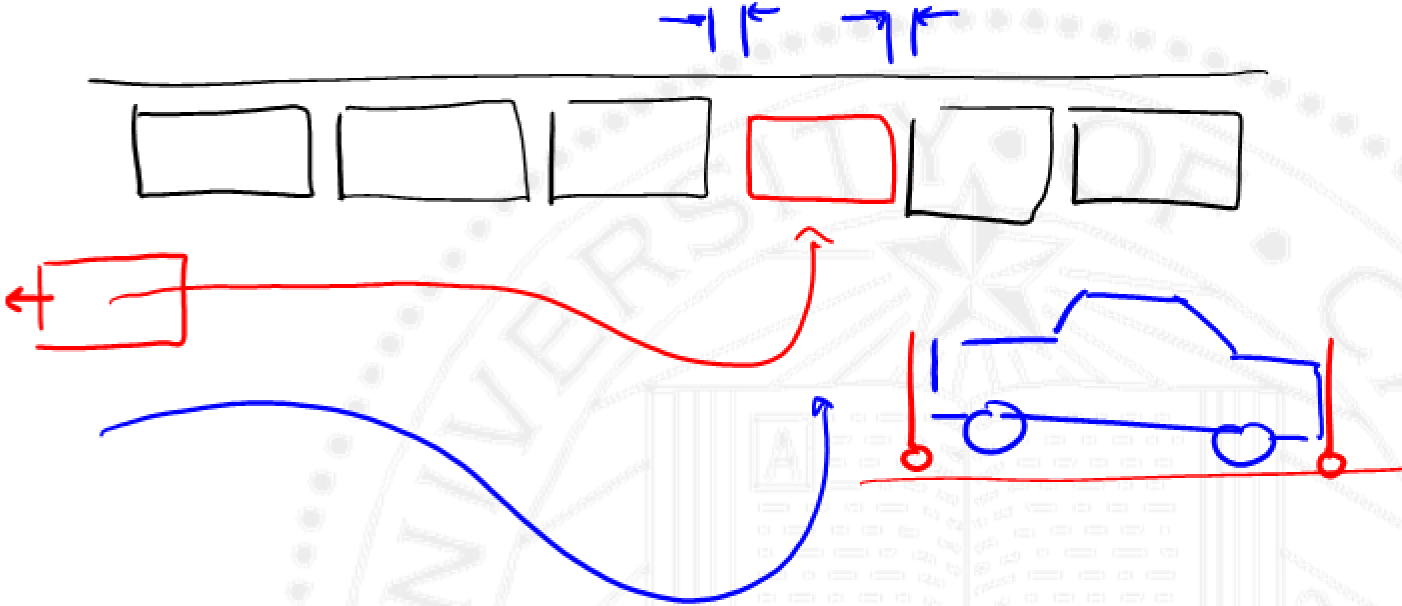
Questions

< 2 cm



J-turn.

Final:

2 parts
- "PAPER" — by hand, no MATLAB
- "MATLAB" — m-file.

> help tf

Course Evals online.

$$G(s) = \frac{1}{s^2} \longrightarrow \frac{y}{u} = \frac{1}{s^2 + 0s + 0}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{A^2}$$

$$\overset{A}{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}} \qquad \overset{B}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \qquad \underline{x} = \begin{bmatrix} \dot{y} \\ y \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \qquad [0]$$

convert to Φ, Γ

$$\Phi = e^{AT} = I + AT + \frac{\cancel{A^2} \xrightarrow{0} T^2}{2!} + \ldots$$

$$\Phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ T & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} = \Phi.$$

$$\Gamma = \left[\int_0^T e^{A\eta}\, d\eta\right] B = \int_0^T e^{\begin{bmatrix}0&0\\1&0\end{bmatrix}\eta}\, d\eta \cdot \begin{bmatrix}1\\0\end{bmatrix} = \int_0^T \begin{bmatrix}1&0\\\eta&1\end{bmatrix} d\eta \cdot \begin{bmatrix}1\\0\end{bmatrix}$$

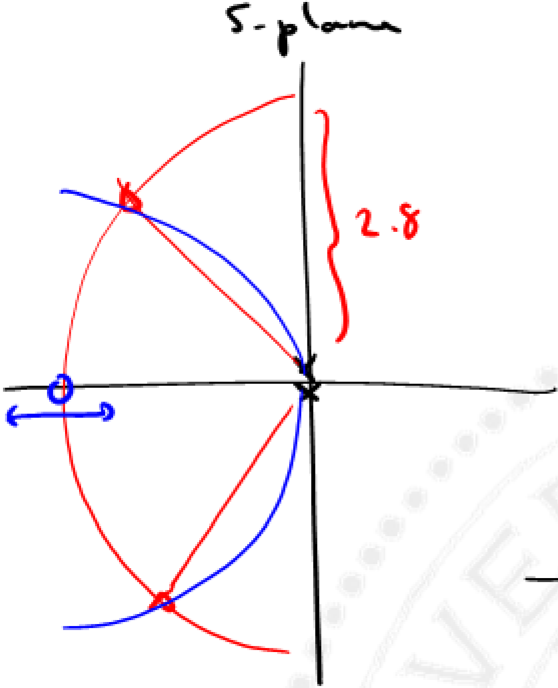
$$= \left.\begin{bmatrix}\eta & 0\\ \frac{\eta^2}{2} & \eta\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix}\right|_0^T = \begin{bmatrix}T & 0\\ \frac{T^2}{2} & T\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix} = \boxed{\begin{bmatrix}T\\ \frac{T^2}{2}\end{bmatrix} = \Gamma}$$

$$T_s = 0.1\,\text{sec} \longrightarrow \Phi = \begin{bmatrix}1 & 0\\ .1 & 1\end{bmatrix} \quad \Gamma = \begin{bmatrix}0.1\\ 0.005\end{bmatrix}$$

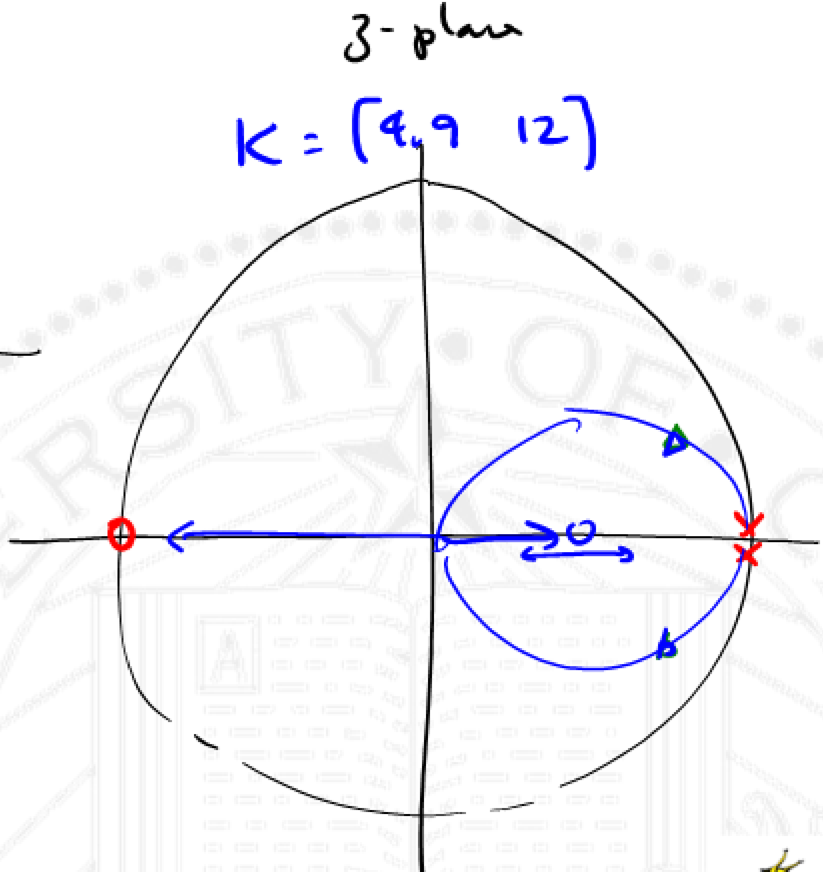
$$\underline{x}_{k+1} = \begin{bmatrix}1 & 0\\ .1 & 1\end{bmatrix}\underline{x}_k + \begin{bmatrix}.1\\ .005\end{bmatrix}\underline{u}_k$$

$$\lambda_{des} = \underline{-2.8 \pm 2.8j}$$

$$z_{des} = e^{s_d T} = \underline{.72 \pm .21j}$$



$K = \text{place}(A, B, P_{des})$



$K = \text{place}(\Phi, \Gamma, z_{des})$

$$J_{qr} = \int_0^\infty y^T Q_y y + u^T R u = \int_0^\infty x^T \underbrace{C^T Q_y C}_{T} x + u^T R u$$

Q_y: $\rho \left[\frac{1}{y_{max}^2} \right]$ R: $[1]$

!! $\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{y_{max}^2} \end{bmatrix}$

dlqr$\left(\Phi, \Gamma, \rho \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{y_{max}^2} \end{bmatrix}, 1\right)$

$\rho = 256 \sim \frac{1}{y_{max}^2} = \frac{1}{(1/16)^2}$

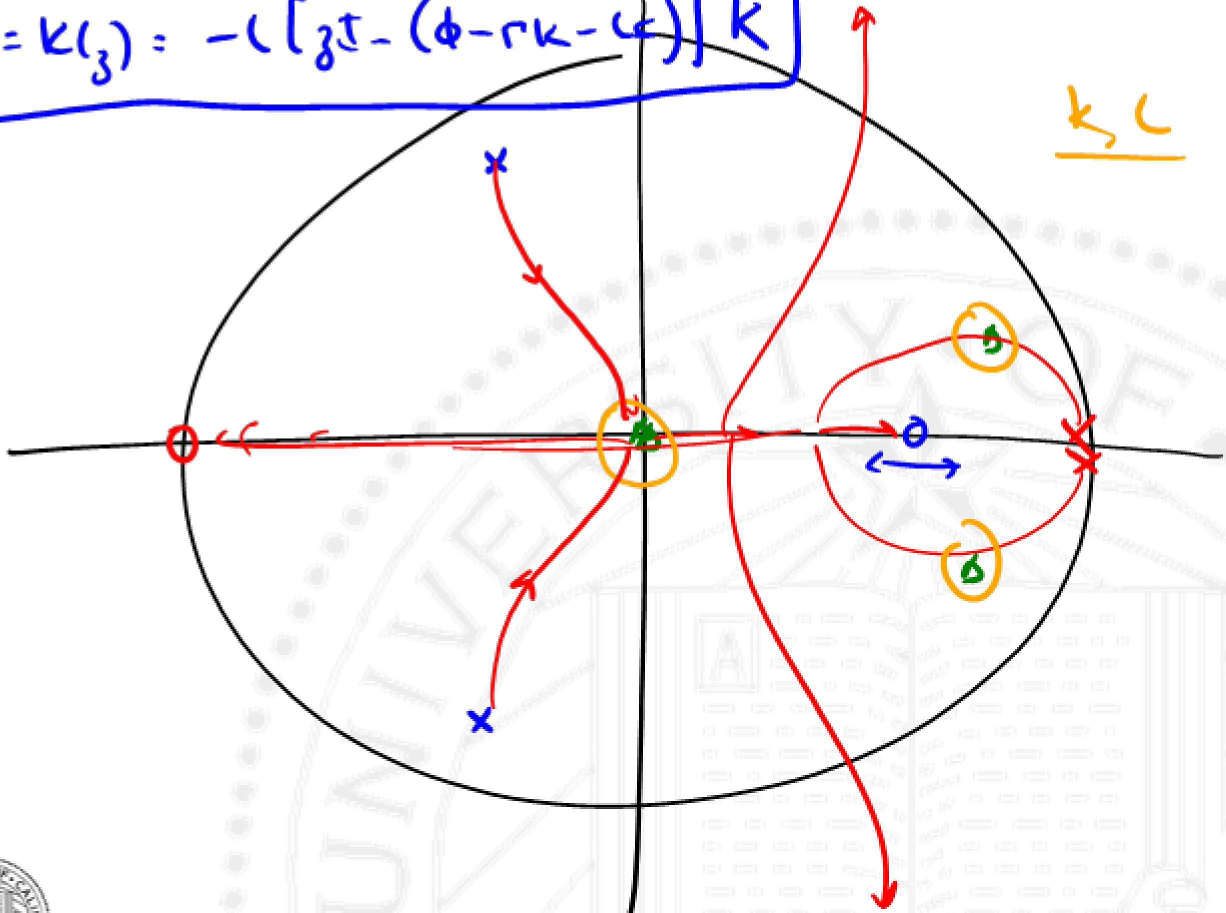
$K = [4.9 \quad 12]$

$Z_D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ "deadbeat" – error goes to ϕ in 2 steps

$L = \begin{bmatrix} 15 \\ 100 \end{bmatrix}$

$$\frac{u}{y} = K(z) = -L\left[zI - (\Phi - \Gamma K - LC)\right]^{-1} K$$

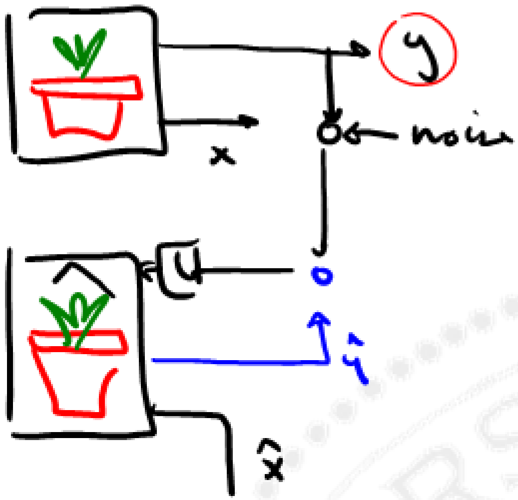
K, L



- Reduced Order Estimator

- Ways to abuse lqr

  - Pincher
  - Implicit Model Following



$$\hat{x} = \begin{bmatrix} \cancel{\hat{y}} \\ \hat{v} \end{bmatrix} \; y$$

$$\underline{X} = \begin{bmatrix} X_A \\ \hline X_B \end{bmatrix}$$

X_A — ALL THE STATE THAT I MEASURE

X_B — EVERYTHING ELSE

$$\begin{bmatrix} \dot{x}_a \\ \hline \dot{x}_b \end{bmatrix} = \left[\begin{array}{c|c} A_{aa} & A_{ab} \\ \hline A_{ba} & A_{bb} \end{array}\right] \begin{bmatrix} x_a \\ \hline x_b \end{bmatrix} + \begin{bmatrix} B_a \\ \hline B_b \end{bmatrix} u$$

$$y = \left[\begin{array}{c:c} I & 0 \end{array}\right] \begin{bmatrix} x_a \\ \hdashline x_b \end{bmatrix}$$

$I \triangleq$ Identity $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

$$\dot{x}_B = A_{ba} \underbrace{x_a}_{y} + A_{bb} x_b + B_b u$$

$$\dot{\underline{x}}_B = A_{BB} \underline{x}_B + \left[\begin{array}{c:c} A_{ba} & B_a \end{array}\right] \begin{bmatrix} y \\ \hdashline u \end{bmatrix}$$

$$\dot{\hat{\underline{x}}}_b = A_{bb}\hat{\underline{x}}_b + [A_{ba}y + B_b u] + L_r A_{ab}(x_b - \hat{x}_b)$$

$$\tilde{\underline{x}}_b \triangleq \underline{x}_b - \hat{\underline{x}}_b$$

$$\dot{\tilde{\underline{x}}}_b = -A_{bb}\hat{\underline{x}}_b - [\cancel{A_{ba}y + B_b u}] - L_r A_{ab}\tilde{\underline{x}}_b$$
$$+ A_{bb}\underline{x}_b + [\cancel{A_{ba}y + B_b u}]$$

$$\dot{\tilde{\underline{x}}}_b = A_{bb}\tilde{\underline{x}}_b - L_r A_{ab}\tilde{\underline{x}}_b$$

$$\boxed{\dot{\tilde{\underline{x}}}_b = (A_{bb} - L_r A_{ab})\tilde{\underline{x}}_b}$$

$$L_r^T = \text{place}(A_{bb}^T, A_{ab}^T, P_{des})$$

$$\dot{\hat{\underline{x}}}_b = A_{bb}\hat{x}_b + \left[A_{ba} \,\vdots\, B_b\right]\begin{bmatrix} y \\ \hdashline u \end{bmatrix} + L_r\left[\underline{A_{ab}x_b} - A_{ab}\hat{x}_b\right]$$

$$\dot{\underline{x}}_a = A_{aa}x_a + \underline{A_{ab}x_b} + B_a u$$

$$\left(\dot{\underline{y}} - A_{aa}y - B_a u\right) = A_{ab}x_b$$

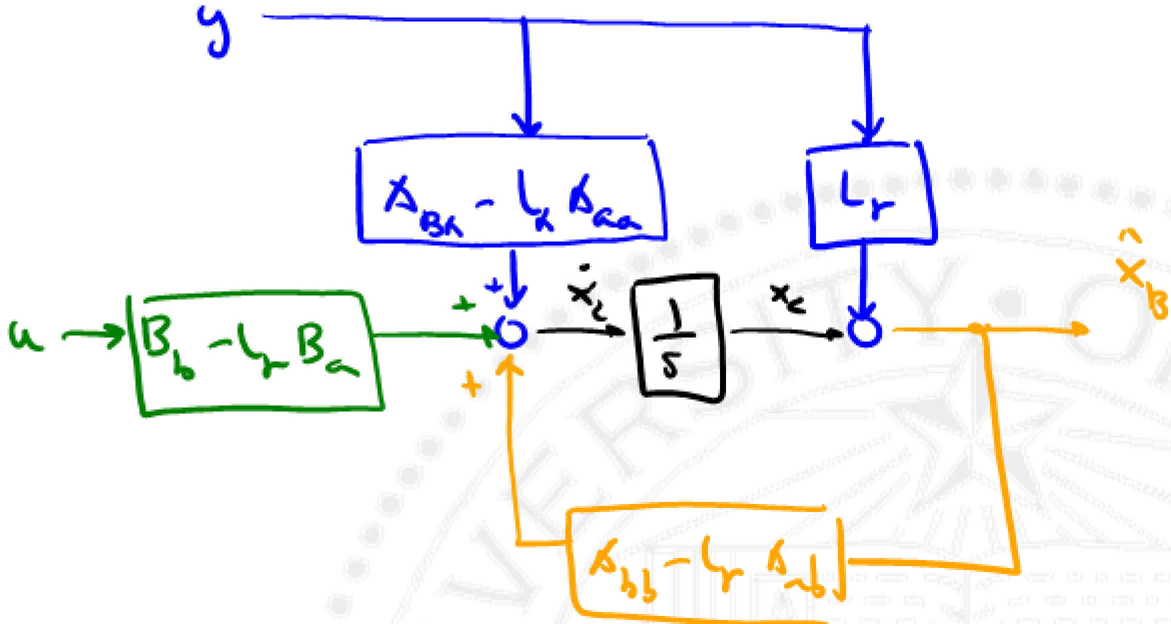
$$\dot{\hat{\underline{x}}}_b = \left[A_{bb} - L_r A_{ab}\right]\hat{x}_b + \left[A_{ba} - L_r A_{aa}\right]y + \left[B_b - L_r B_a\right]u - L_r\dot{y}$$

$$\underline{x}_c \triangleq \hat{x}_b - L_r y$$

$$\dot{\underline{x}}_c = \dot{\hat{\underline{x}}}_b - L_r\dot{y}$$

$$\dot{\underline{x}}_c = [A_{bb} - L_r A_{ab}]\hat{x}_b + [A_{ba} - L_r A_{aa}]y + [B_b - L_r B_a]u$$

$$\hat{x}_b = \underline{x}_c + L_r y.$$



$$\begin{bmatrix}\dot{x}_a \\ \hline \dot{x}_b\end{bmatrix} = \left[\begin{array}{c|c} A_{aa} & A_{ab} \\ \hline A_{ba} & A_{bb}\end{array}\right]\begin{bmatrix}x_a \\ \hline x_b\end{bmatrix} + \begin{bmatrix}B_a \\ \hline B_b\end{bmatrix} u$$

$$u = -\begin{bmatrix}K_a & \vdots & K_b\end{bmatrix}\begin{bmatrix}y \\ \hline x_b\end{bmatrix}$$

$$\boxed{u = -K_a y - K_b \hat{x}_b}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$C = [1 \quad 0]\,\underline{x}$$

$$\begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$

$$A_{aa} = \phi$$

$$A_{ab} = 1$$

$$A_{ba} = -2$$

$$A_{bb} = 0$$

$x_A = y =$ measurement

x_B — estimate

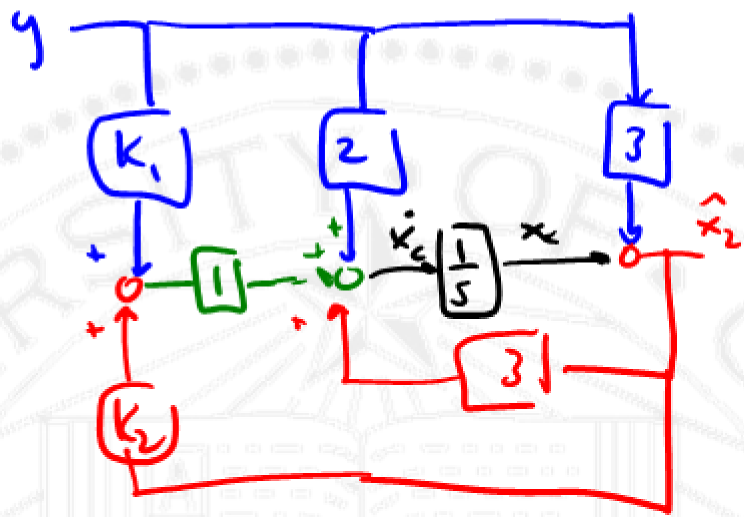
$$\dot{\tilde{x}}_2 = (0 - L_r)\,\tilde{x}_2$$

$$\tilde{x}_2(t) = e^{-L_r t}\,\tilde{x}_2(0)$$

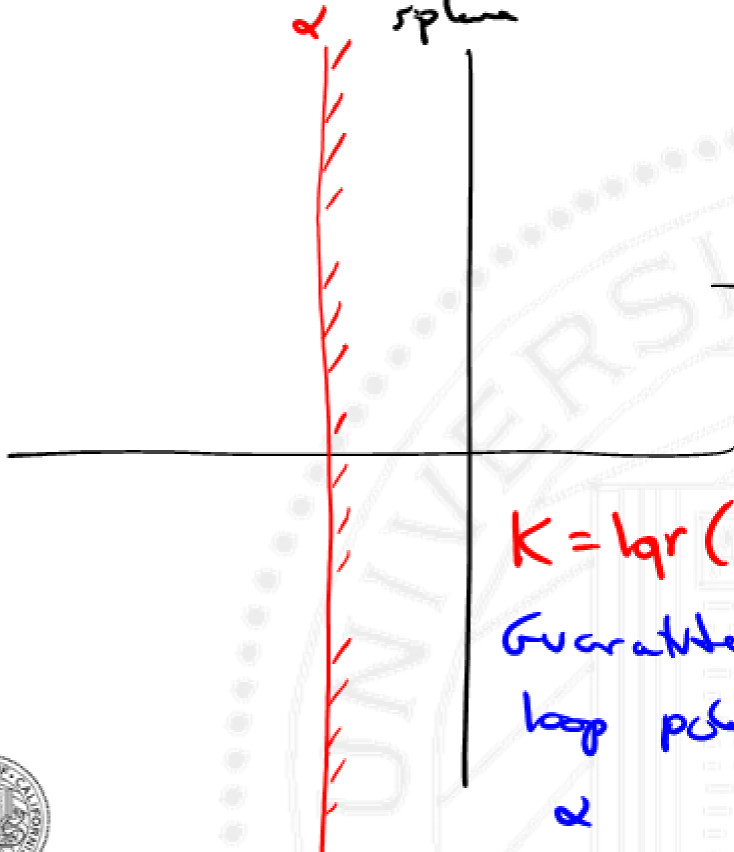
$$\dot{x}_c = -3\hat{x}_2 + 2y + u$$

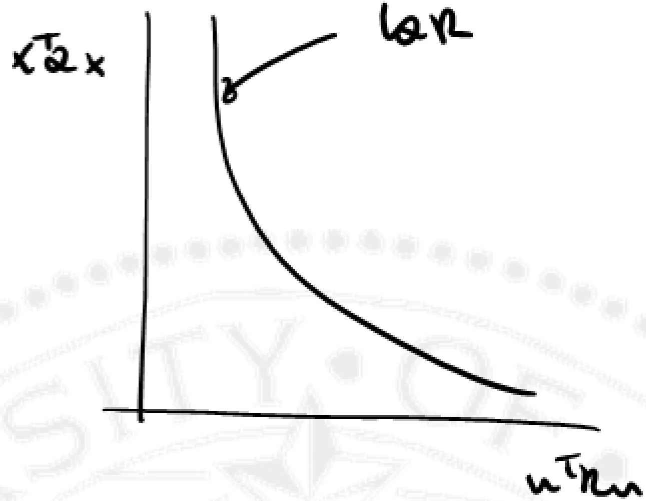
$$\hat{x}_2 = x_c + 3y$$

$$\underline{u = -k_1 y - k_2 \hat{x}_2}$$



Alpha LQR





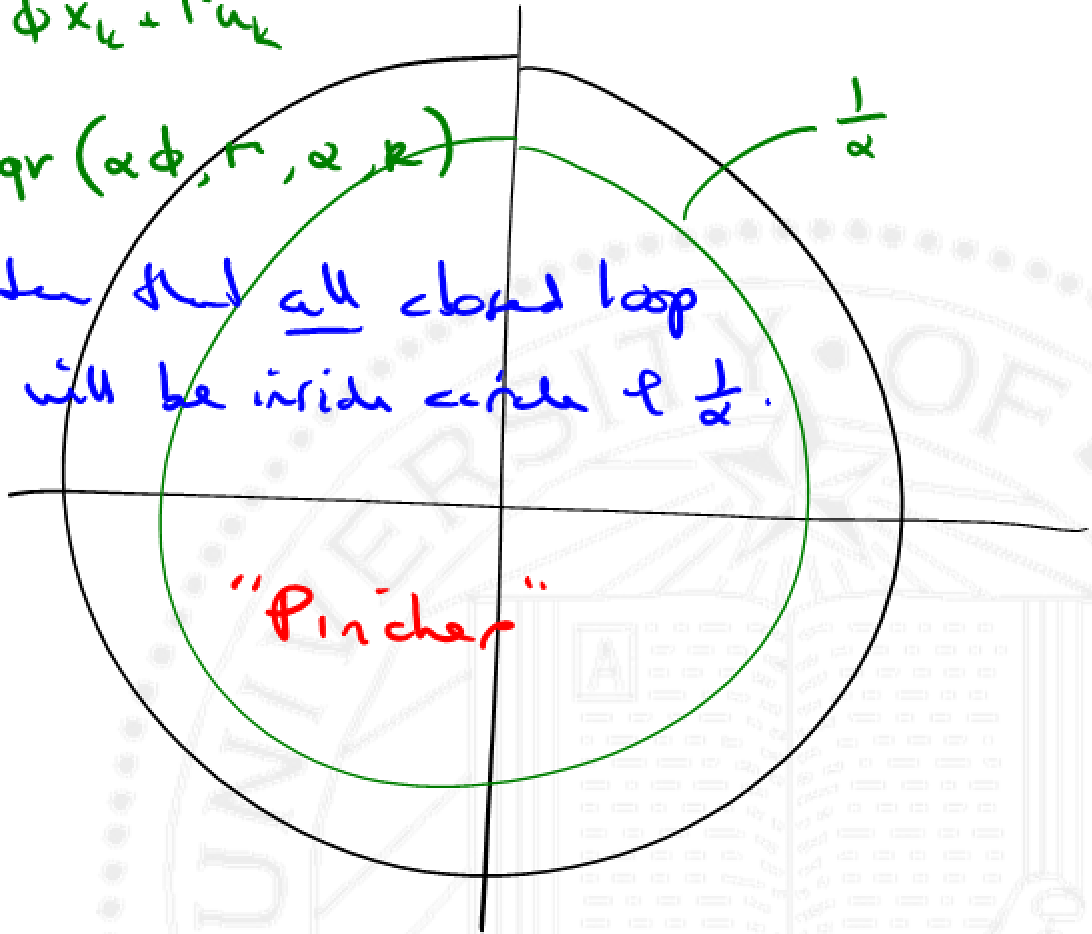
$K = \mathrm{lqr}(A + \alpha I, B, Q, R)$

Guarantees that all closed loop poles are to the left of α

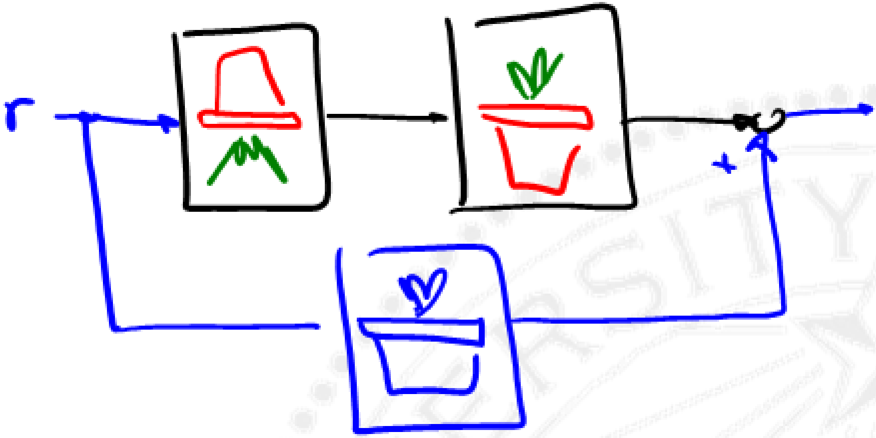
$x_{k+1} = \phi x_k + \Gamma u_k$

$K = \text{dlqr}(\alpha\phi, \Gamma, Q, R)$

Guarantee that all closed loop poles will be inside circle of $\frac{1}{\alpha}$.



Implicit Model Following



"Desired Plant"

$$J = \int_0^\infty \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

$\dot{x} = Ax + Bu$

$y = Cx$

$u = -Kx$

$\dot{z} = A_m z$ ← model

$z \simeq y$

$\dot{y} = C\dot{x} = C[Ax + Bu]$

$$J = \int_0^\infty \{[\dot{y} - \dot{z}]^T Q [\dot{y} - \dot{z}] + u^T R u\} \, dt$$

$$\dot{z} = A_m z \overset{\sim}{=} A_m y = \underline{A_m C x}.$$

$$\dot{y} = CAx + CBu$$

$$\dot{z} = A_m z \approx A_m y = A_m C x$$

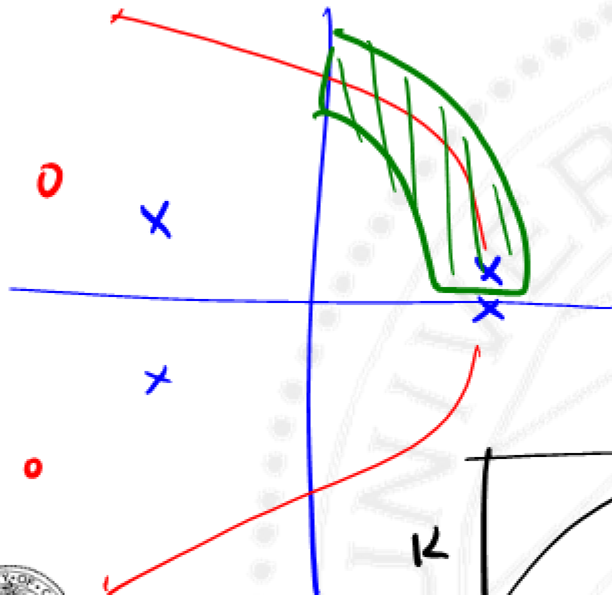
$$(\dot{y} - \dot{z}) = [CA - A_m C]x + CBu$$

$$J = \int_0^\infty (\dot{y} - \dot{z})^T Q (\dot{y} - \dot{z}) + u^T R u$$

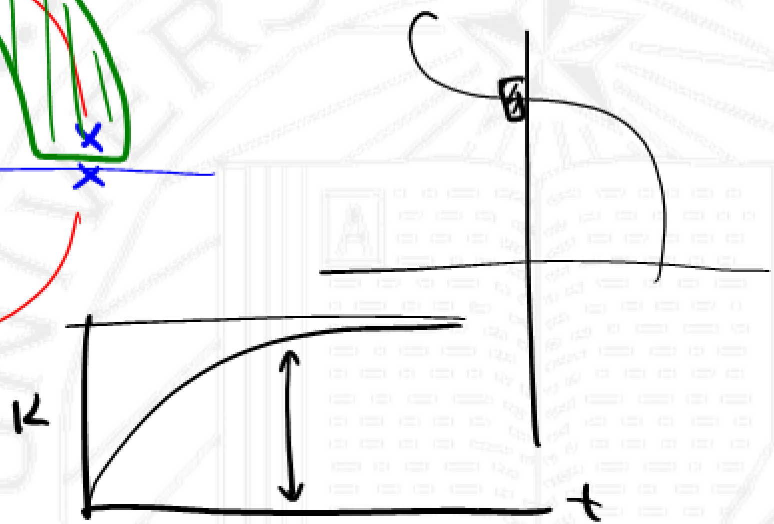
$$J = \int_0^\infty \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} \underbrace{(CA - A_m C)^T Q (CA - A_m C)}_{Q} & \overbrace{[CA - A_m C]^T Q [CB]}^{N} \\ \underbrace{(CB)^T Q (CA - A_m C)}_{N^T} & \underbrace{R + (CB)^T Q (CB)}_{R} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

WARNINGS

UNSTABLE CONTROLLERS / SYSTEM



STARTUP PROBLEMS



Non-Linear Systems

$$\dot{x} = Ax + Bu \longrightarrow \dot{x} = f(x,u)$$

$$\underline{\dot{x}} = \underbrace{\frac{\partial f}{\partial x}\bigg|_{x=x_0,u_0}}_{A}(x-x_0) + \underbrace{\frac{\partial f}{\partial u}\bigg|_{x_0,u_0}}_{B}(u-u_0)$$

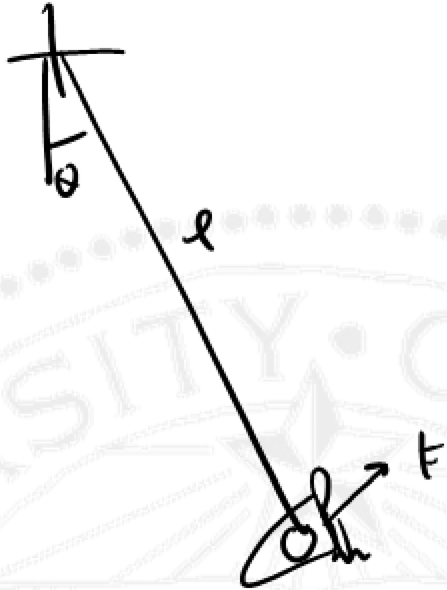
$$u = -f_{nl}(x,u) + K\underline{x}$$

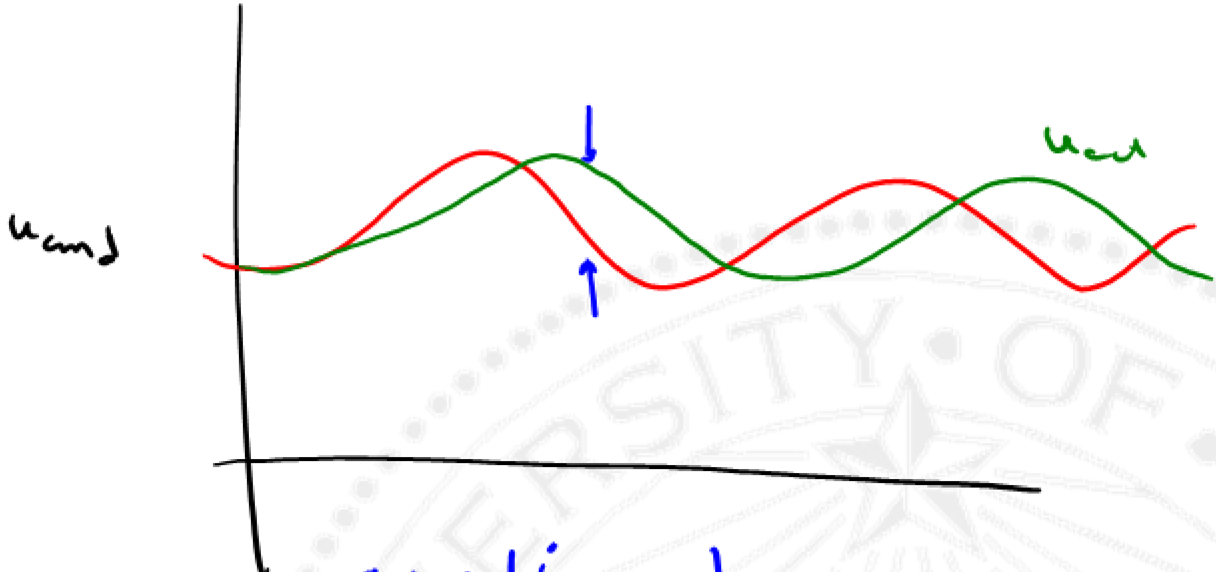
$$m\ell^2\ddot{\theta} + mg\ell\sin\theta = F$$

$$\ddot{\theta} + \frac{g}{\ell}\sin\theta = F$$

$$u = F - \frac{g}{\ell}\sin\theta$$

$$\ddot{\theta} = u$$





$$|\dot{u}| < |\dot{u}_{max}|$$

$$|\ddot{u}| < |u_{acc\,max}|$$

$$\dot{\underline{x}} = \left[\begin{array}{c|c} \lambda & \\ \hline & \end{array} \right] \begin{bmatrix} \vdots \\ - \\ \dot{u}_{ACT} \end{bmatrix} + \begin{bmatrix} \\ \\ \end{bmatrix} \dot{\underline{u}}_{cmd}$$

$\dot{u}_{act}$

$$u^{+}_{cmd} = u^{-}_{cmd} + \dot{u}_{cmd} \Delta T$$

$$u^{+}_{cmd} = u^{-}_{cmd} + \dot{u}_{cmd} \Delta T + \ddot{u}_{cmd} \frac{\Delta T^{2}}{2}$$