CMPE-242
Applied Feedback Control

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Questions

Symmetric Root Locus

Single input/single output system

$u \rightarrow y$

lqr $\longrightarrow$ symmetric root locus (Bryson's rule)

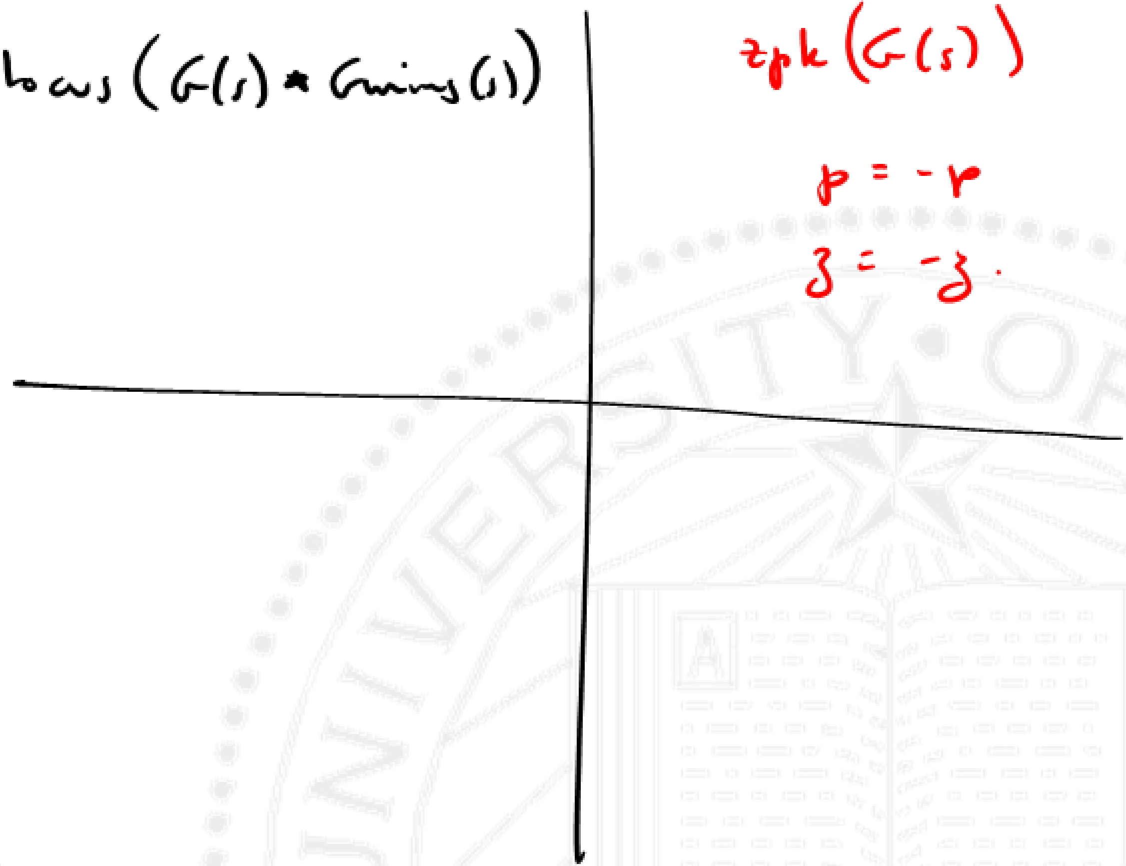
$$1 + \rho G(s) G(-s) = 0.$$

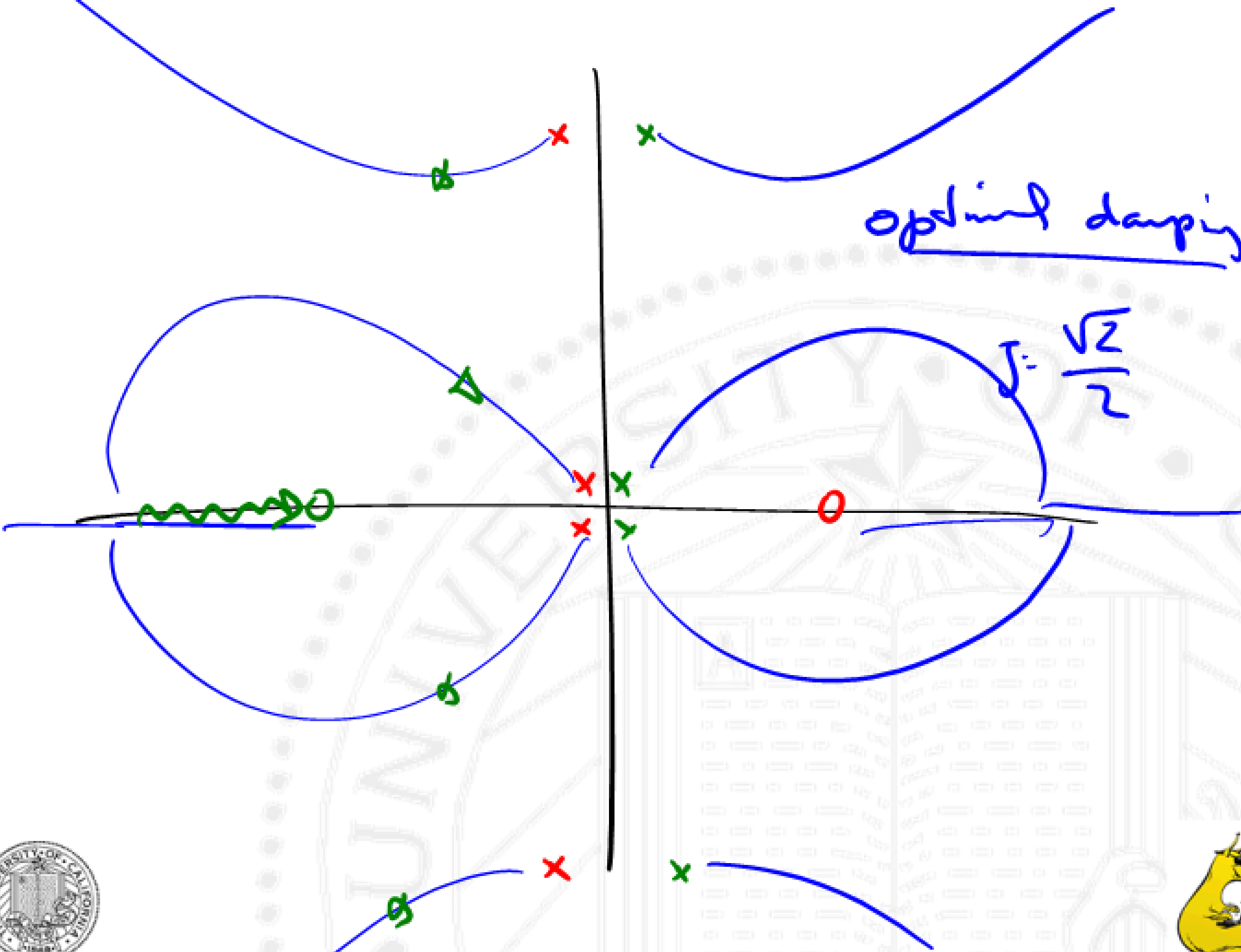
rlocus(G(s)*Gmirr(s))

zpk(G(s))

$p = -p$

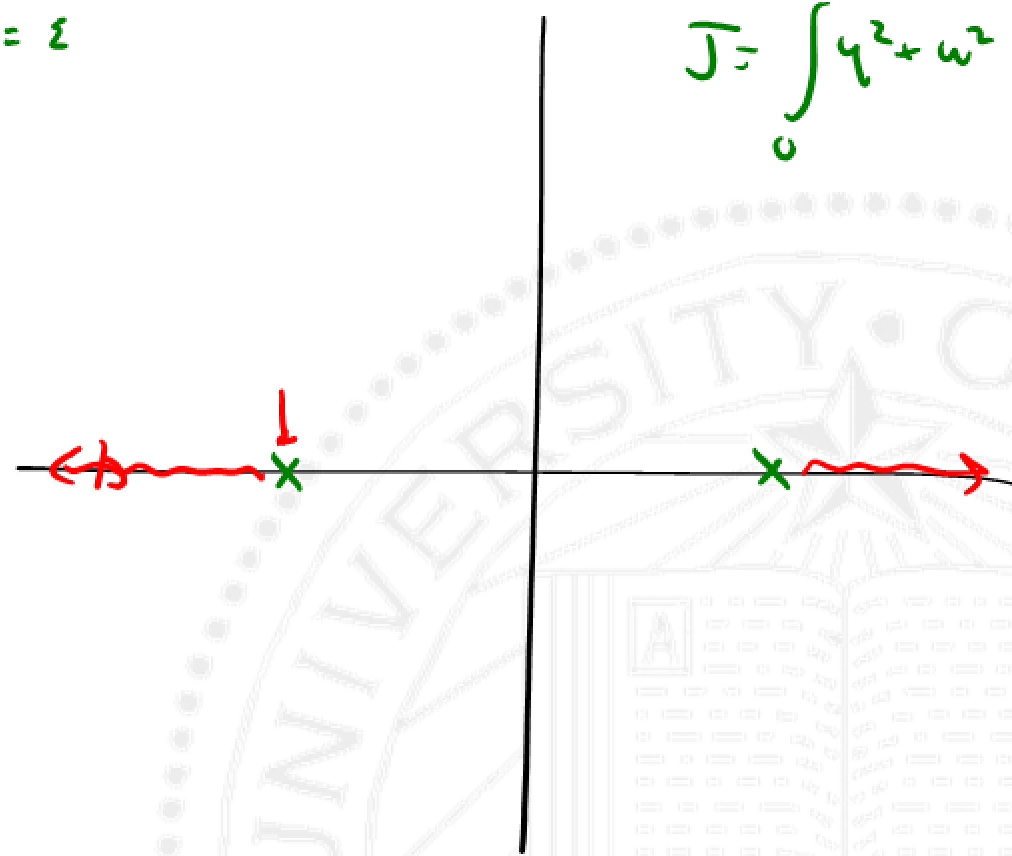
$z = -z$.




optimal damping
$\zeta = \frac{\sqrt{2}}{2}$

$\rho = \varepsilon$

$$J = \int_0^\infty y^2 + u^2 \, dt$$



initial $(G(s), x_0)$

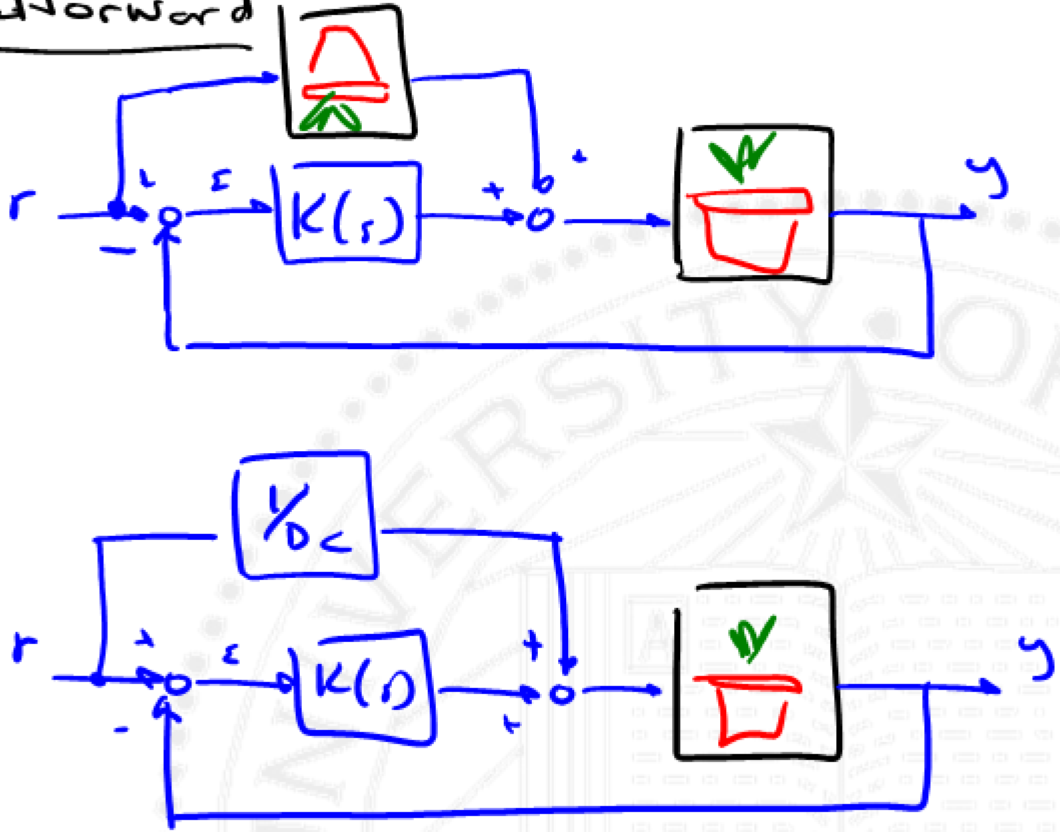
x_2

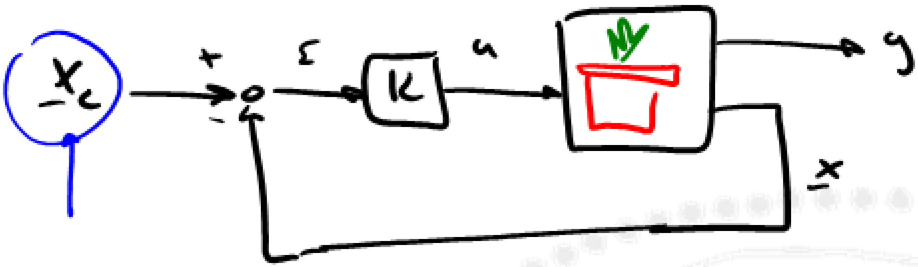
u

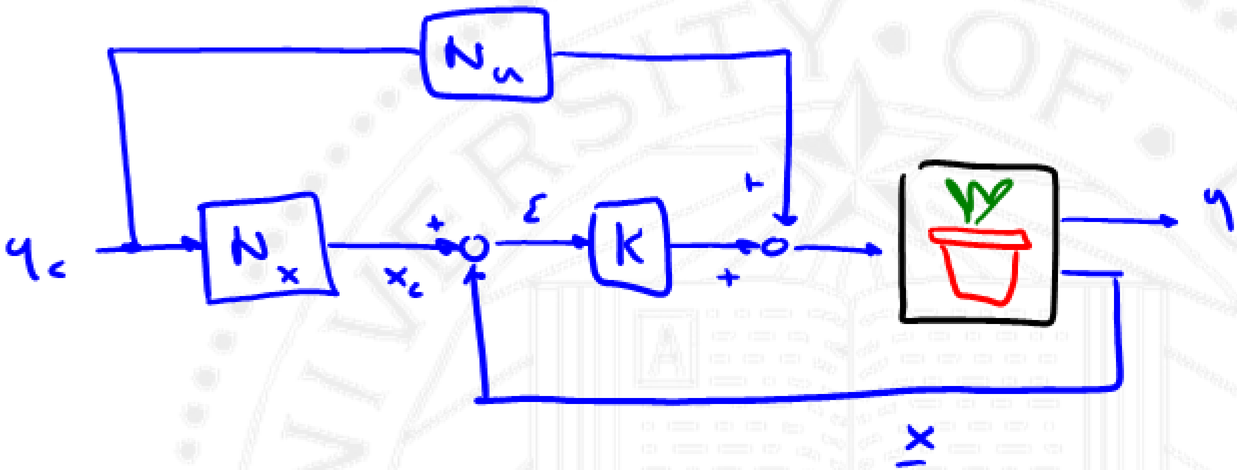
$$C = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & -K & & & \end{bmatrix}$$

$u = -Kx$

Feedforward







N_x converts $y_c \longrightarrow x_c$

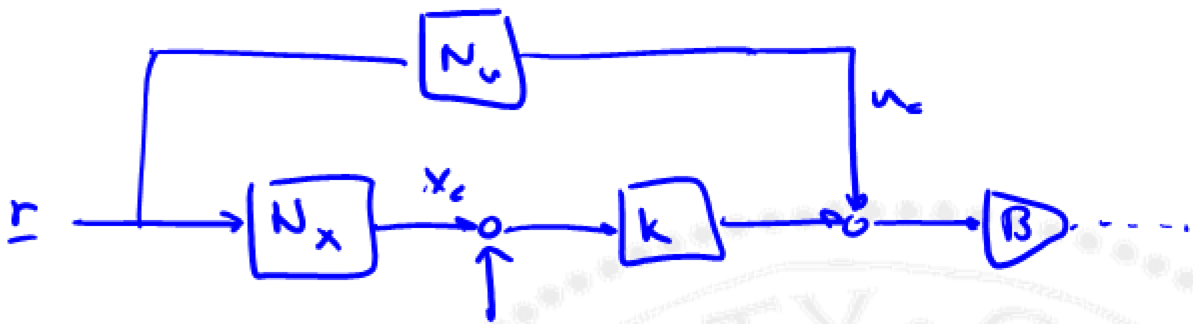
N_u converts $y_c \longrightarrow u_{ss}$.

FPE 7.10

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\underbrace{\begin{bmatrix} \overset{n \times 1}{\dot{x}} \\ \cdots \\ \underset{r \times 1}{y} \end{bmatrix}}_{(n+r) \times 1} = \begin{bmatrix} \overset{n \times n}{A} & \vdots & \overset{n \times m}{B} \\ \cdots & + & \cdots \\ \underset{r \times n}{C} & \vdots & \underset{r \times m}{D} \end{bmatrix} \underbrace{\begin{bmatrix} \overset{n \times 1}{x} \\ \cdots \\ \underset{m \times 1}{u} \end{bmatrix}}_{(n+m) \times 1}$$



$$\overset{0}{\cancel{\dot{x}_{ss}}} = A x_{ss} + B u_{ss} \qquad x_{ss} \triangleq N_x y_{ss}$$

$$y_{ss} = C x_{ss} + D u_{ss} \qquad u_{ss} \triangleq N_u y_{ss}$$

$$\overset{n \times 1}{0} = A N_x y_{ss} + B N_u y_{ss}$$

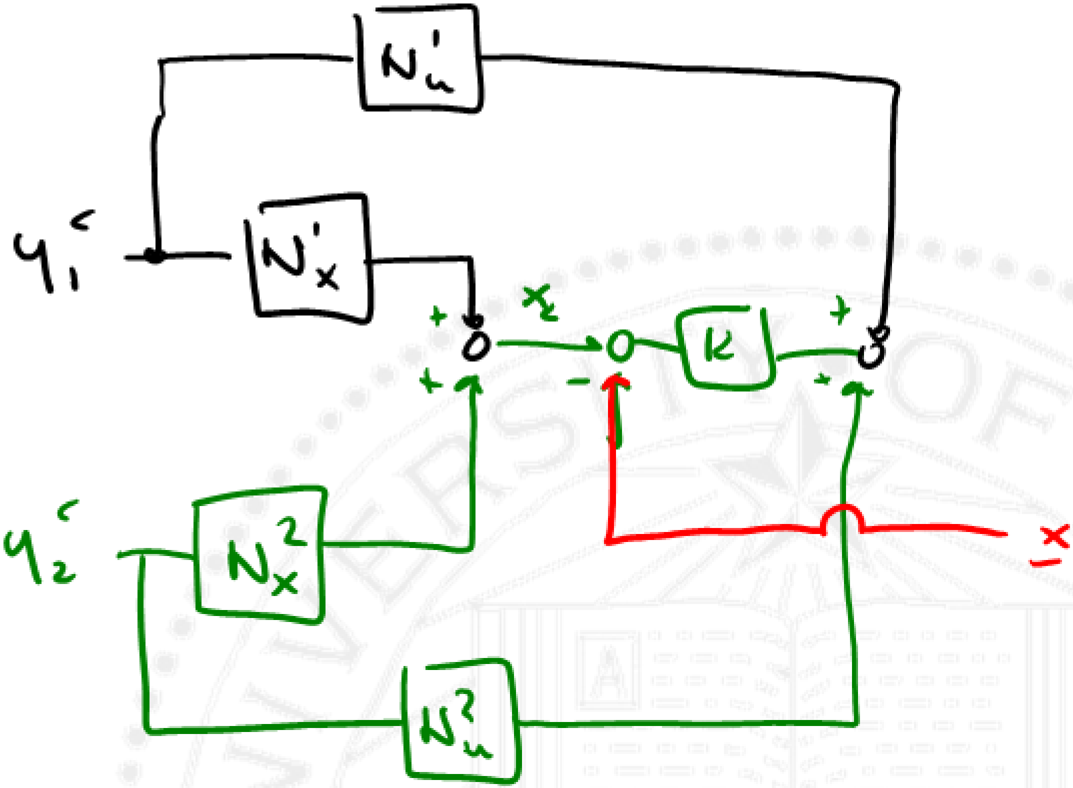
$$y_{ss} = C N_x y_{ss} + D N_u y_{ss}$$

$$\longrightarrow \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A N_x + B N_u \\ C N_x + D N_u \end{bmatrix} y_{ss}$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} N_x \\ \cdots \\ N_u \end{bmatrix} \rightarrow \begin{bmatrix} N_x \\ \cdots \\ N_u \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

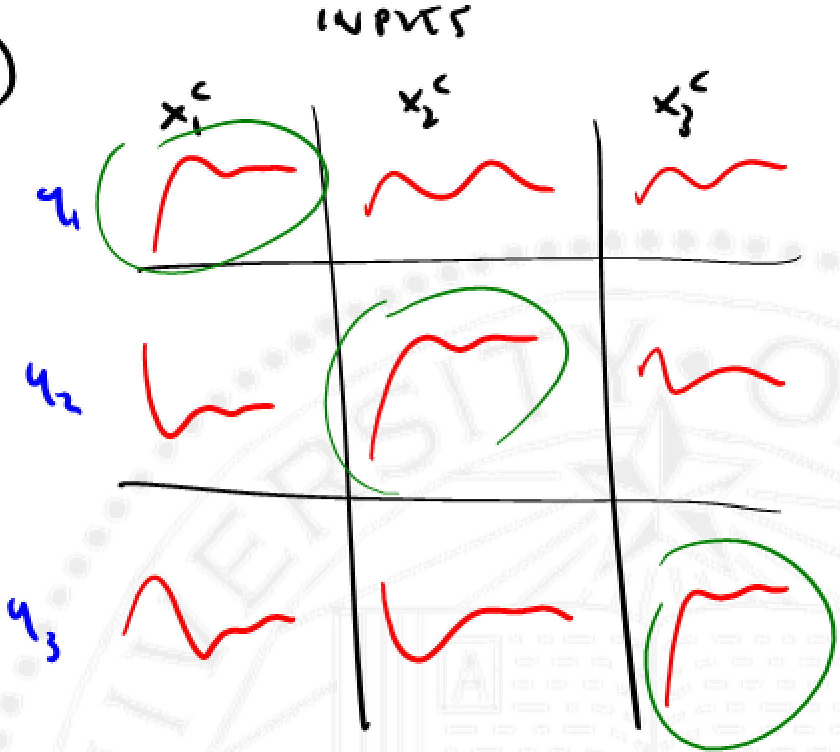
$$y_1^c - \underbrace{[\quad\quad\quad]}_{C_1} \leftarrow N_x^1 \quad N_u^1$$

$$y_2^c = \underbrace{[\quad\quad\quad]}_{C_2} \leftarrow N_x^2 \quad N_u^2$$



N_u^1
y_1^c
N_x^1
x_c
K
y_2^c
N_x^2
N_u^2
$\underline{x}$

step(syscl)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \triangleq M^{-1} \qquad M^{-1} \simeq (M^T M)^{-1} M^T$$

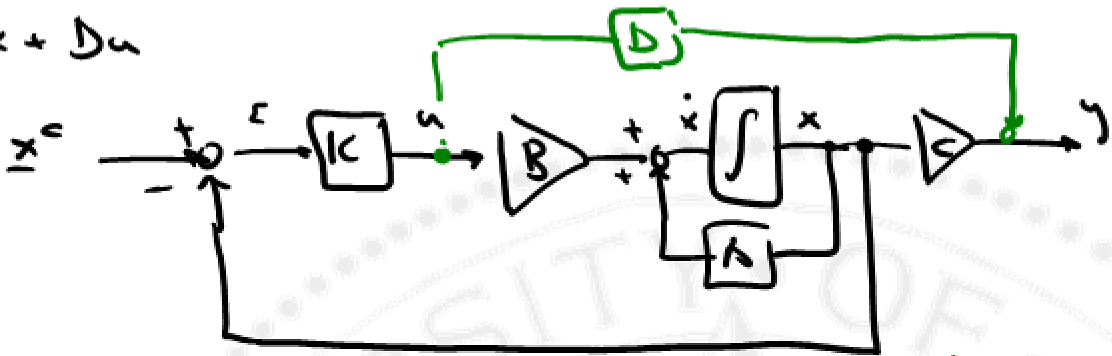
Tychonov Regularization

$$M^{-1} \triangleq (M^T M + \mu I)^{-1} M^T$$

$$\mu > 0.$$

$$\dot{x} = Ax + Bu \qquad u = -K(x - x_c) = K(x_c - x)$$

$$y = Cx + Du$$



$$\dot{x} = Ax + BK(x_c - x) = (A - BK)x + BK\boxed{\underline{x}_c}$$

$$y = Cx + DK(x_c - x) = (C - DK)x + DK\boxed{\underline{x}_c}$$

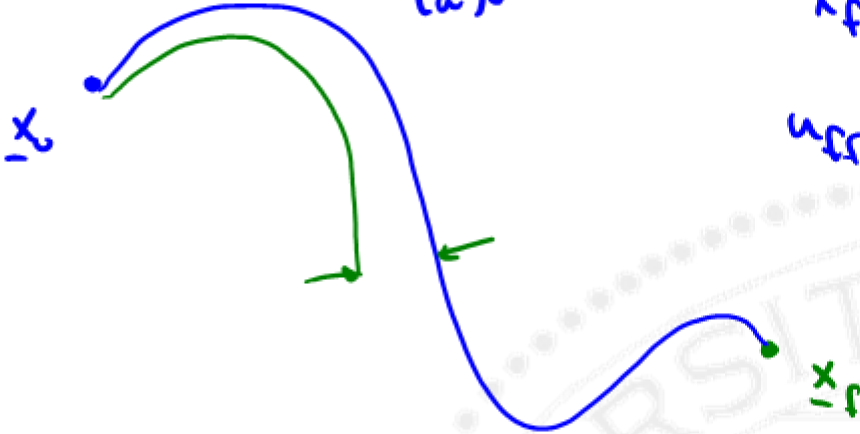
input

$$\left.\begin{aligned}\dot{\underline{x}}_{cl} &= [A - BK]x_{cl} + BK\underline{x}_c \\ y_{cl} &= [C - DK]x_{cl} + DKx_c\end{aligned}\right\} \quad ss(\underbrace{A - BK}_{A}, \underbrace{BK}_{B},$$

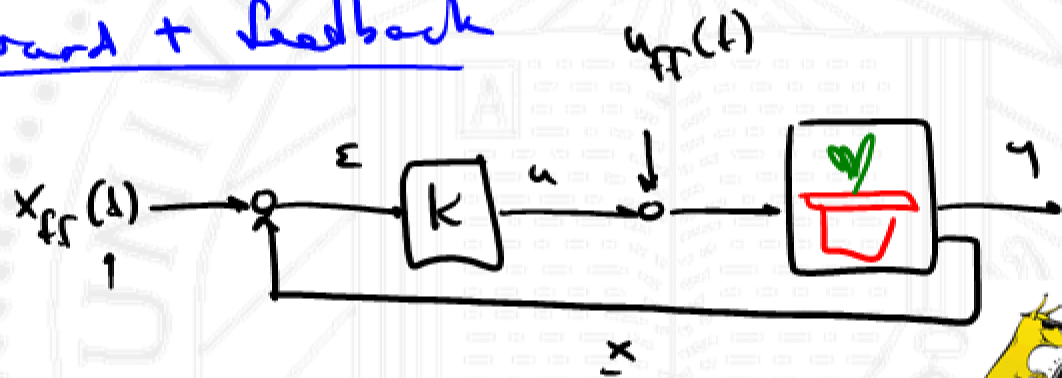
$\begin{bmatrix} x \\ u \end{bmatrix}(t)$

$x_{ff}(t)$ "optimal traj"

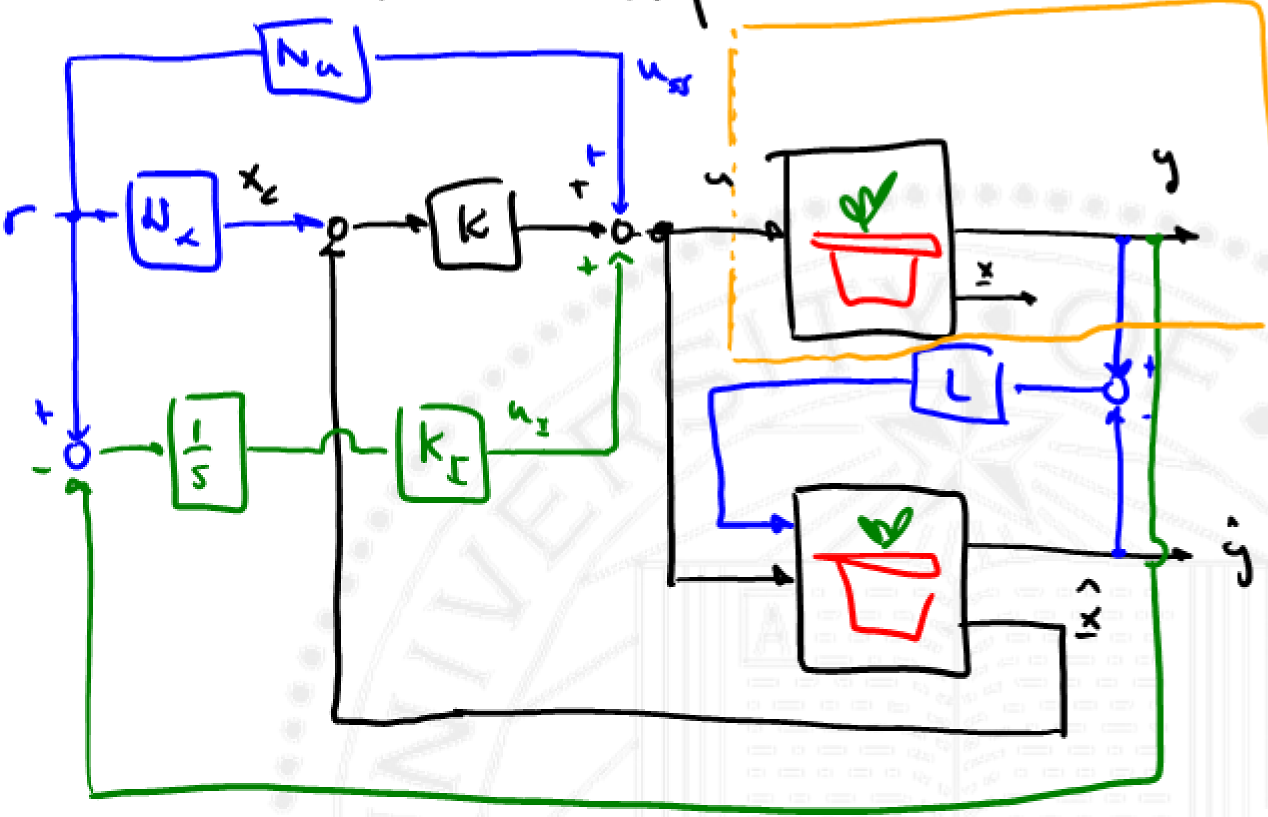
$u_{ff}(t)$



Feedforward + Feedback



"Full Monty"



PUNT
N_u
u_{ss}
r
N_x
x_r
K
+
+
+
u
y
$\underline{x}$
L
+
-
+
-
$\frac{1}{s}$
K_I
u_I
$\hat{y}$
$\hat{\underline{x}}$

$$\dot{x} = Ax + Bu \qquad \dot{\hat{x}} = A\hat{x} + Bu \qquad u = -K(\hat{x} - \cancel{x_c})$$

$$y = Cx + v \qquad \hat{y} = C\hat{x}$$

$$\dot{\hat{x}} = (A - BK)\hat{x} + LCx - LC\hat{x} + Lv$$

$$= (A - BK - LC)\hat{x} + LCx + Lv$$

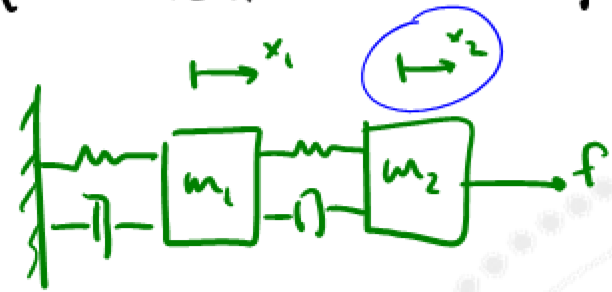
$$\begin{bmatrix} \dot{x} \\ \hline \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A & -BK \\ \hline LC & A-BK-LC \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ L \end{bmatrix} v$$

poles of my controller @ $(A-BK)$

poles of my estimator @ $(A-LC)$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]^{-1} \begin{bmatrix} y \\ 0 \\ 1 \end{bmatrix}$$



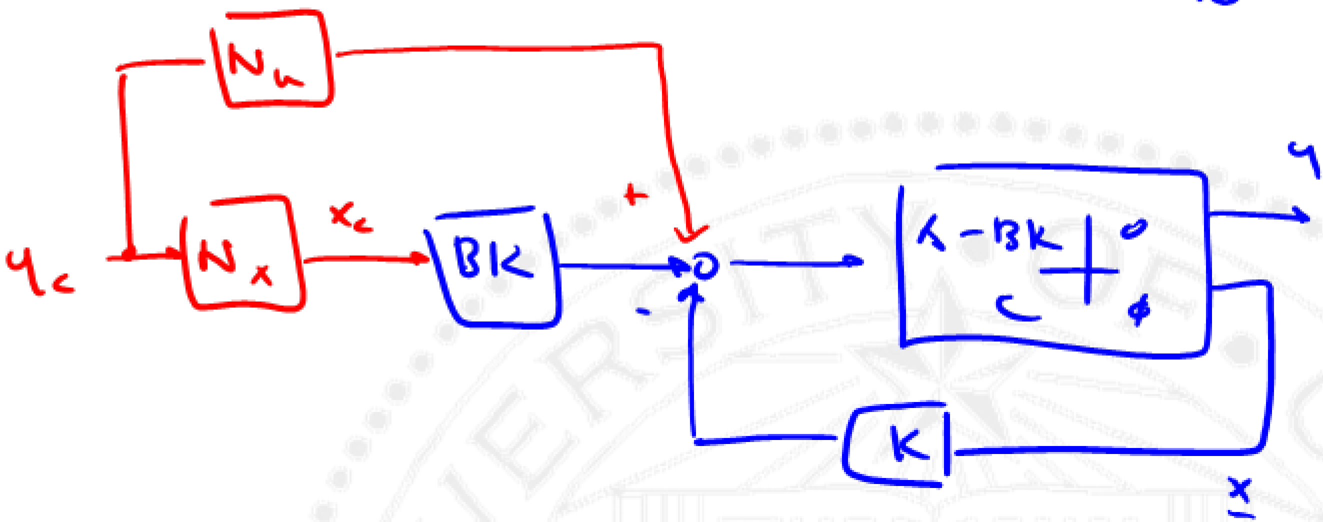
$m_1 = m_2 = k_1 = k_2 = b_1 = b_2 = 1.$

$$\dot{\begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix}} = \begin{bmatrix} -2 & -2 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} f$$

$$y = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underline{x} \qquad D = (0) \qquad \begin{bmatrix} N_x \\ \hline N_u \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]^{-1} \begin{bmatrix} y \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ \hline N_u \end{bmatrix} = \begin{array}{cl} \left[\begin{array}{c} 0 \\ \frac{1}{2} \\ 0 \\ 1 \\ \hline \frac{1}{2} \end{array}\right] & \begin{array}{l} \\ N_x \\ \\ \\ N_u \end{array} \end{array}$$

"State Tracker"

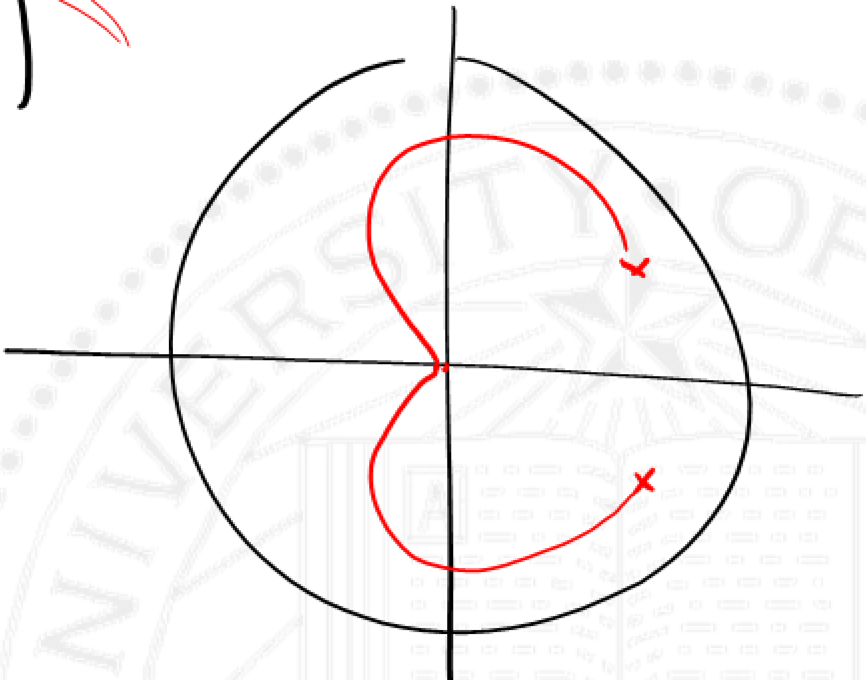


"Control Augmentation"

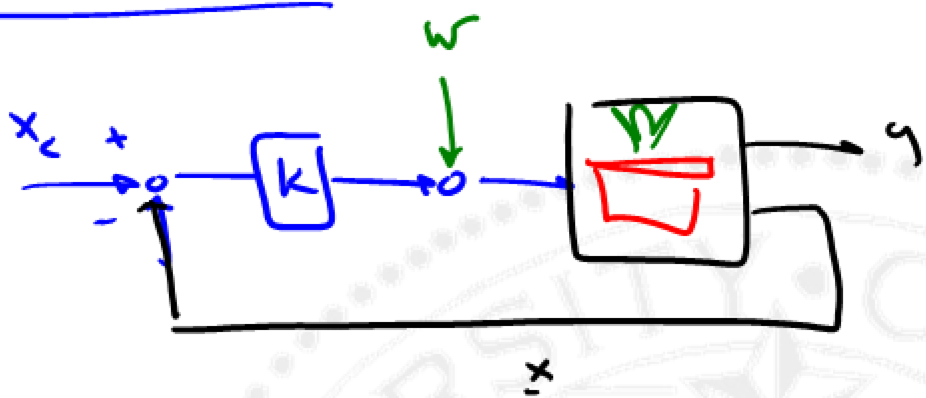
DIGITAL

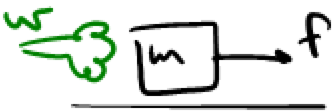
"DEADBEAT"

$z_D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



Bias Estimation





$$\frac{1}{s^2} \longrightarrow A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} y \\ v \end{pmatrix} = \underline{x}$$

$$\dot{y} = v$$

$$\dot{v} = u + w$$

$$\dot{w} = \phi.$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \underline{x}$$

$$\begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$u_{cmd} = u - \hat{w}$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

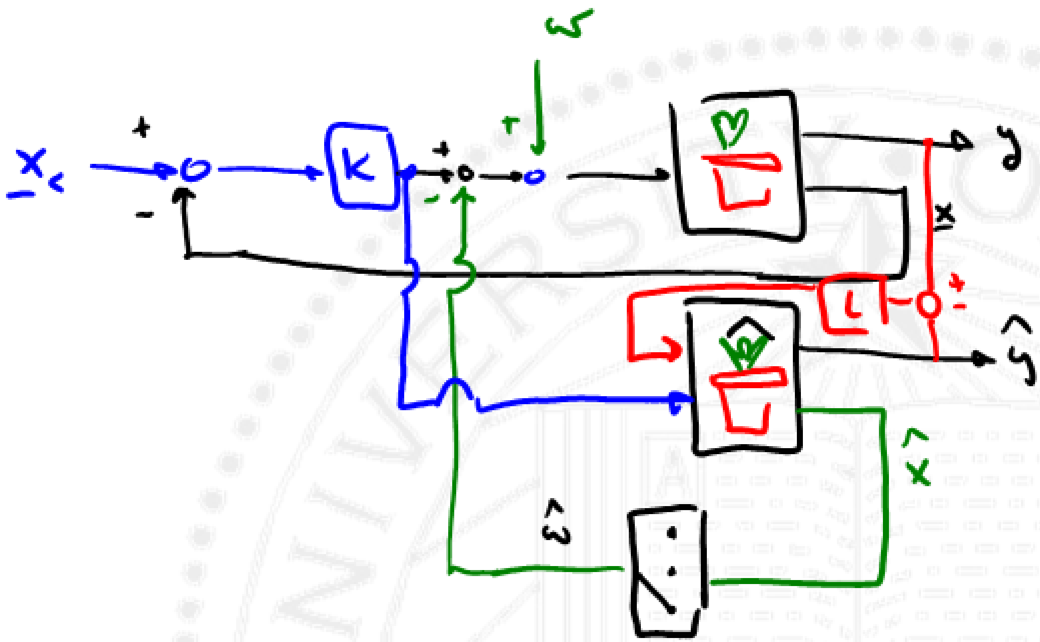
$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Rank}(\mathcal{C}) = 2.$$

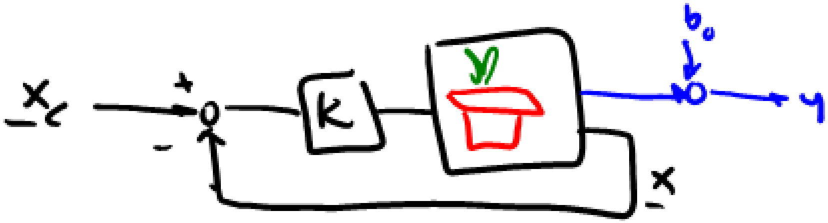
$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Rank}(\mathcal{O}) = 3.$$

can estimate $\hat{w}$ & $\hat{v}$ using only y.

$$u = K(\underline{x}_c - \underline{x}) - \hat{w}$$

← null out any nuisance signal

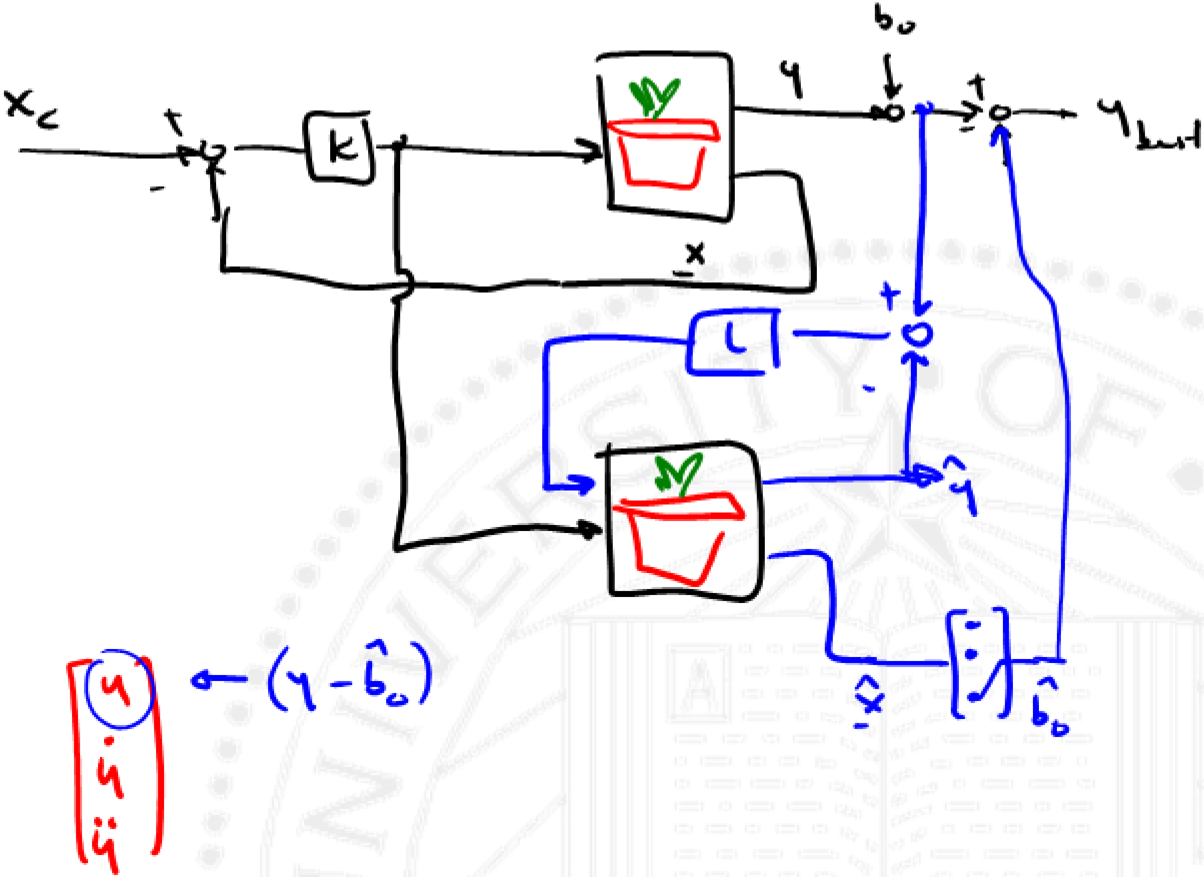




$$\begin{bmatrix} \dot{x} \\ \dot{b}_0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ b_0 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad \dot{b}_0 = \phi$$

$$y = \begin{bmatrix} C & \vdots & 1 \end{bmatrix}\begin{bmatrix} x \\ b_0 \end{bmatrix}$$

$$y = y_{meas} - \hat{b}_0$$


x_c
K
y
b_o
y_{meas}
x
L
$\hat{y}$
$\hat{x}$
$\hat{b}_o$
$(y - \hat{b}_o)$
y
$\dot{y}$
$\ddot{y}$

$$\omega_0 = \frac{1}{f}$$

$$\ddot{w} + \omega_0^2 w = \phi$$

$$\underline{x}_w = \begin{bmatrix} w \\ \dot{w} \end{bmatrix}$$

$$\dot{x}_w = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} x_w$$

$$\begin{bmatrix} \underline{x} \\ \hline x_w \end{bmatrix}^{\bullet} = \begin{bmatrix} A & 0 \\ 0 & A_w \end{bmatrix}$$

$$u = -k(\underline{x} - \underline{x}_c) - \hat{w}$$

GO DIGITAL

$$\dot{x} = Ax + Bu \longrightarrow \mathcal{L}\{\} - sX = AX + BU$$

$$\frac{X}{U} = \underbrace{(sI - A)^{-1}}_{\Delta_{OL}} B$$

$$x_{k+1} = \Phi x_k + \Gamma u_k \longrightarrow Z\{\} - zX = \Phi X + \Gamma U$$

$$\frac{X}{U} = \underbrace{(zI - \Phi)^{-1}}_{\Delta_{OL}} \Gamma$$

Scalar Case

$$\dot{x} = ax + bu \qquad y = 1x$$

$$\frac{x}{u} = \frac{b}{s-a} \qquad h(t) = be^{at} \qquad \text{1st order system}$$

$$\boxed{x(t) = e^{a(t-t_0)}\, x(t_0) \leftarrow \text{I.C. Response}}$$

$$\text{forced: } \int_0^t h(t-\tau)u(\tau)\,d\tau = \int_0^t e^{a(t-\tau)}u(\tau)\,d\tau$$

$$e^{at} \cong 1 + at + \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} + \ldots$$

$$e^{At} \cong I + At + \frac{A^2t^2}{2!} + \ldots$$

$$t_0 = kT$$

$$t_0 + t = kT + T = (k+1)T$$

across right time step.

$$x(kT+T) = e^{AT} x(kT) + \int_{kT}^{kT+T} e^{A((k+1)T-\tau)} B u(\tau)\, d\tau$$

$$x_{k+1} = \underbrace{e^{AT}}_{\Phi} x_k + \underbrace{\left[\int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)}\, d\tau\right] B}_{\Gamma} u_k$$

$$\eta = (k+1)T - \tau$$

$$d\eta = -d\tau$$

$$\tau = kT \rightarrow \eta = T$$

$$\tau = (k+1)T \rightarrow \eta = 0$$

$$\int_T^0 e^{A\eta}(-d\eta) = \int_0^T e^{A\eta}\, d\eta$$

$$x_{k+1} = \underbrace{e^{AT}}_{\Phi} x_k + \underbrace{\left[\int_0^T e^{A\eta}\, d\eta\right] B}_{\Gamma} u_k$$

$\Phi = \text{expm}(A*T)$

c2d(sys, Ts, 'zoh')

$u = -Kx$ or $-K\hat{x}$

$\dot{x} = (A - BK)x_k$

$sIx = (A - BK)x$

$x = \underbrace{[sI - (A - BK)]^{-1}}_{\Delta_{cl}(s) = \phi}$

$u_k = -Kx_k$ or $-K\hat{x}_k$

$x_{k+1} = (\Phi + \Gamma K)x_k$

$zIx = (\Phi - \Gamma K)x$

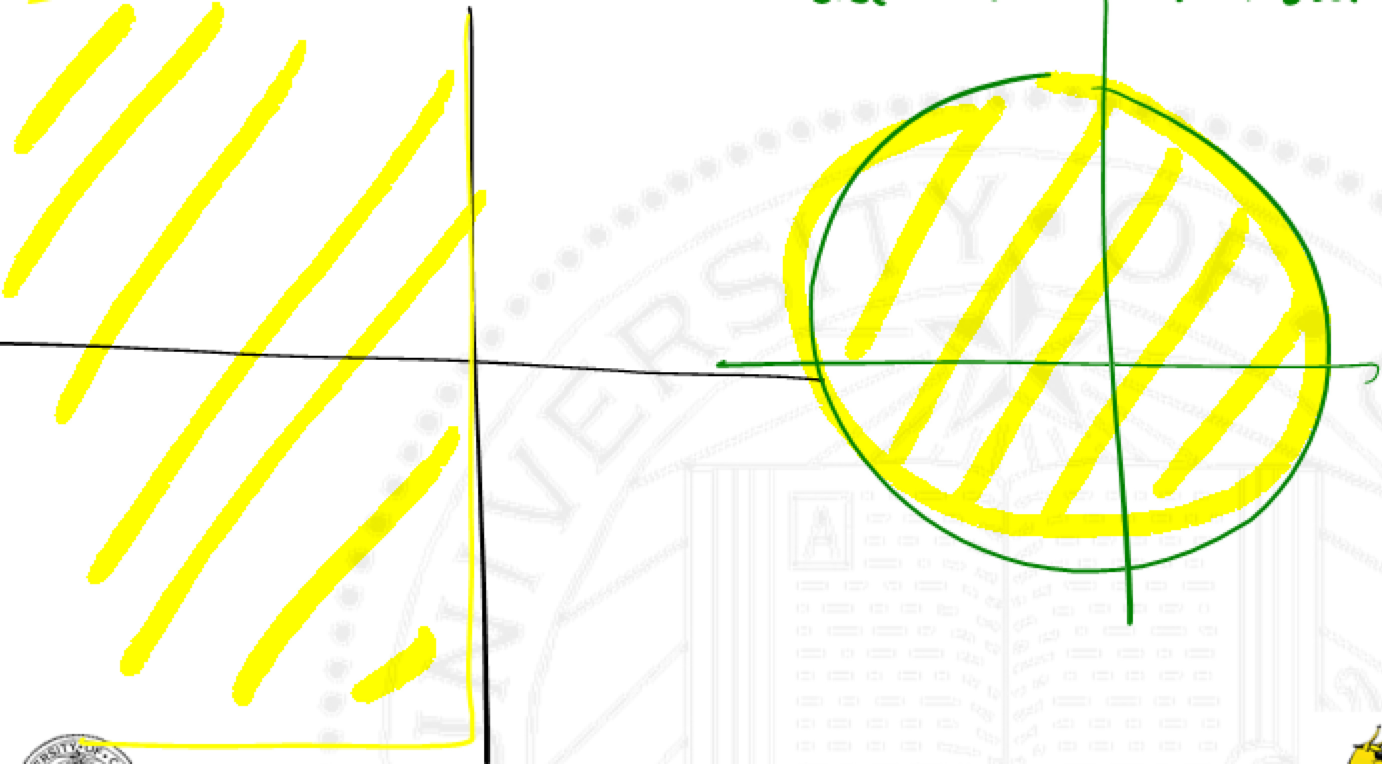
$x = \underbrace{[zI - (\Phi - \Gamma K)]^{-1}}_{\Delta_{cl}(z) = \phi}$

s-plane

$s_{des} \Rightarrow$ place(A, B, P_{des})

z-plane

$z_{des} \Rightarrow$ place(Φ, Γ, z_{des})



LQ:

$$J = \int_0^\infty (x^T Q x + u^T R u)\,dt \longrightarrow \mathrm{lqr}(A, B, Q, R)$$

$\neq$

$$J = \sum_0^\infty x_k^T Q x_k + u_k^T R u_k \longrightarrow \mathrm{dlqr}(\Phi, \Gamma, Q, R)$$

$$\dot{x} = Ax + Bu \qquad x_{k+1} = \Phi x_k + \Gamma u_k$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \qquad \hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k)$$