CMPE-242
Applied Feedback Control

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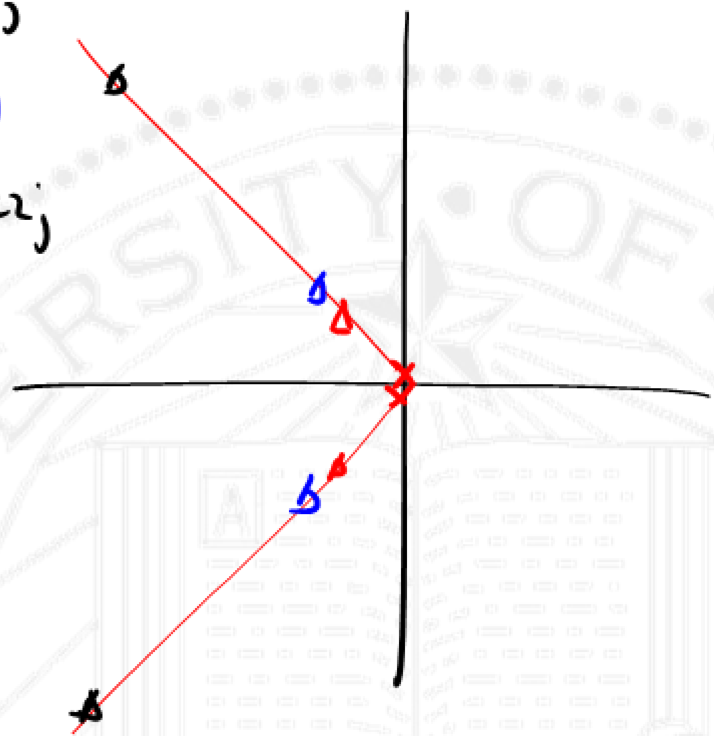
$$G(s) = \frac{1}{s^2} \qquad A: \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C: \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D: \begin{bmatrix} 0 \end{bmatrix}$$

$$\rho y^T Q y = \rho x^T \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underbrace{Q_y}_{1/y_{max}^2} \begin{bmatrix} 1 & 0 \end{bmatrix} x + u^T \underbrace{R}_{1} u$$

$$J = \int_0^\infty \Big[\overbrace{\frac{\rho}{y_{max}^2} \Big(x^T \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{Q} x \Big) + u^T u}^{\bar{J}} \Big] dt$$

$$K = \text{lqr}(A, B, Q, 1) \text{ — } \text{eig}(A - BK)$$

ρ	K		$eig(A-BK)$
1	$\sqrt{2}$	1	$-.7 \pm .7j$
4	2	2	$-1 \pm j$
100	4.7	10	$-22 \pm 22j$



STATE SPACE

CONTROL — PLACE, LQR

ESTIMATION — PLACE, LQE

Kalman Filter

$$\dot{x} = Ax + Bu + \Gamma w$$ ← process noise

$$w \sim N(0, R_w)$$

$$y = Cx + v$$ ← measurement noise

$$v \sim N(0, R_v)$$

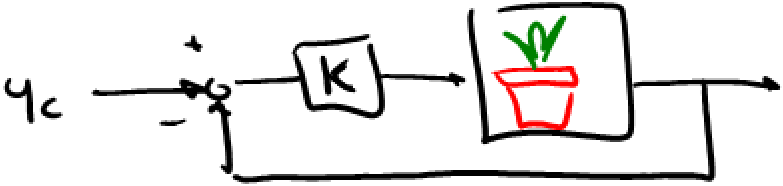
$R_w \gg R_v$ — believe measurements, L ← big

$R_w \ll R_v$ — believe model, L ← small

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$L = \mathrm{lqe}(A, \Gamma, C, R_w, R_v)$$ — "steady state Kalman"

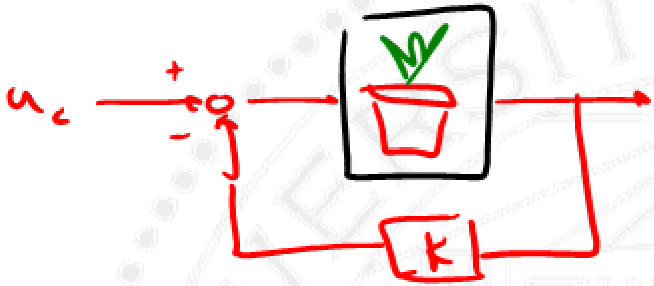
$$L^T = \mathrm{lqr}(A^T, C^T, \underbrace{\Gamma R_w \Gamma^T}_{Q}, \underbrace{R_v}_{R})$$



output follows input

Full autopilot

"AIRBUS"



Augment the stability for the pilot

Stability Augmentation

"SAS"

"Boeing" Model

$\mathrm{eig}(A - BK)$

DIGITAL STATE SPACE

$\dot{\underline{x}} = A\underline{x} + B\underline{u}$

$\underline{y} = C\underline{x} + D\underline{u}$

$\longrightarrow$

$\underline{x}_{k+1} = \Phi \underline{x}_k + \Gamma \underline{u}_k$

$\underline{y}_k = H\underline{x}_k + D\underline{u}_k$

$\Phi = e^{A\Delta T}$ ← matrix expansion:

$e^{At} = I + At + \frac{A^2t^2}{2!} + \dots$

$\Gamma = \left[\int_0^{\Delta T} e^{A\tau} d\tau\right] B$

c2d(sys, 'zoh')

$$\text{lqr} \leftarrow J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

$$\text{dlqr} \leftarrow J = \sum_0^\infty \underline{x}_k^T Q \underline{x}_k + u_k^T R u_k$$

lqr – overloaded

$$K = \text{lqr}(A, B, Q, R) \qquad K = \text{lqr}(\Phi, \Gamma, Q, R)$$

commanded state

$$\underline{u}_k = -K(\hat{\underline{x}}_k - \underline{x}^c_k)$$

$$\underline{x}_{k+1} = \Phi \underline{x}_k + \Gamma \underline{u}_k \qquad u_k = -K(\hat{\underline{x}}_k - \underline{x}_k^c)$$

$$\underline{y}_k = H \underline{x}_k + D \underline{u}_k$$

$$\hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k) = [\Phi - LH]\hat{x}_k + [\Gamma \;\; L]\begin{bmatrix} u_k \\ y_k \end{bmatrix}$$

$$\hat{y}_k = H \hat{x}_k + D u_k$$

initialize $\hat{x}_k, u_k$

measure $\underline{y}_k$

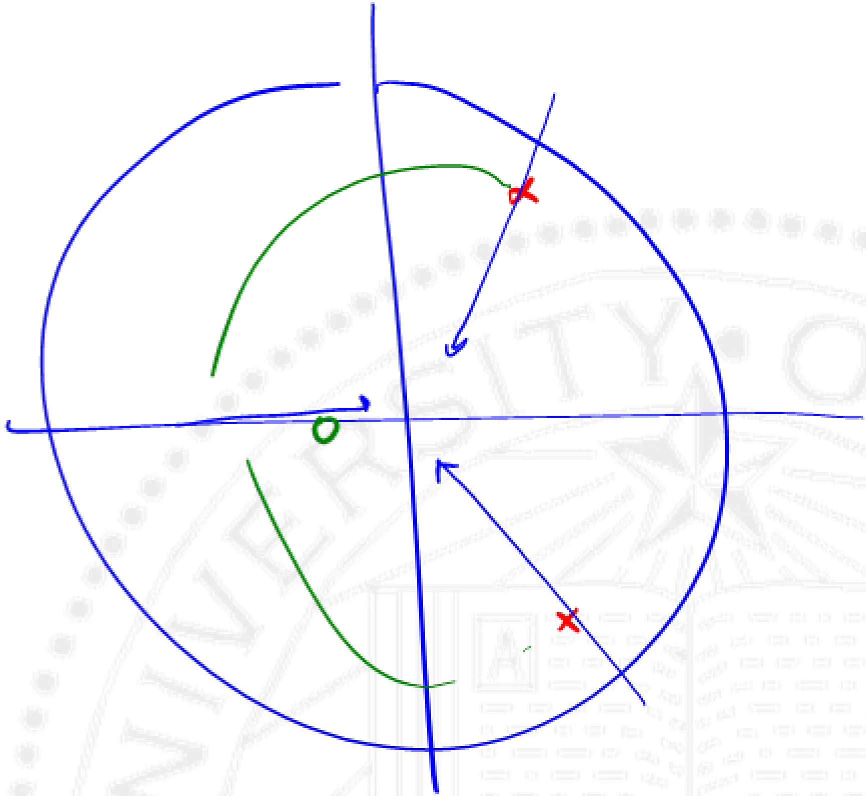
$$\hat{x}_{k+1} = [\Phi - LH]\hat{x}_k + \Gamma u_k + L \underline{y}_k$$

$$u_{k+1} = -K(\hat{x}_{k+1} - \underline{x}_{k+1}^c)$$

ΔT

$$\hat{x}_k \leftarrow \hat{x}_{k+1}$$

$$u_k \leftarrow u_{k+1}$$



Problems w/ Kalman

← Time varying estimator numerically UNSTABLE

Square root form of KF

Information form of KF

$P \longrightarrow S = P^{-1}$

if process noise / measurement noise } ergodic — (stat's don't change over time)

white $-(0, \sigma^2)$

Extended Kalman Filter

$$\dot{x} = f(\underline{x}, \underline{u})$$

at each time step, k,

$$A \simeq \left.\frac{\partial f}{\partial x}\right|_{\substack{x = \underline{x}_k \\ u = u_k}} \qquad B \simeq \left.\frac{\partial f}{\partial u}\right|_{\substack{x = x_k \\ u = u_k}}$$

Continuous Time	Digital
$\dot{x} = Ax + Bu$	$x_{k+1} = \Phi x_k + \Gamma u_k$
$y = Cx + Du$	$y_k = Hx_k + Du_k$
$u = -Kx \,/\, -K\hat{x}$	$u_k = -Kx_k \,/\, -K\hat{x}_k$
$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$	$\hat{x}_{k+1} = \Phi\hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k)$
$\hat{y} = C\hat{x} + Du$	$\hat{y}_k = Hx_k + Du_k$
$K = \text{place}(A, B, P_{des})$	$K = \text{place}(\Phi, \Gamma, P_{des})$
$L^T = \text{place}(A^T, C^T, P_{est})$	$L^T = \text{place}(\Phi^T, H^T, P_{est})$
$K = \text{lqr}(A, B, Q, R)$	$K = \text{lqr}(\Phi, \Gamma, Q, R)$
$L = \text{lqe}(A, C, \Gamma, R_w, R_v)$	$L = \text{lqe}(\Phi, H, \Gamma^*, R_w, R_v)$

$$\Phi = e^{AT}$$

$$\Gamma = \int_0^T e^{A\tau}\, d\tau\, B$$

$$\Phi = e^{A\Delta T}$$ — matrix exponential (expm)

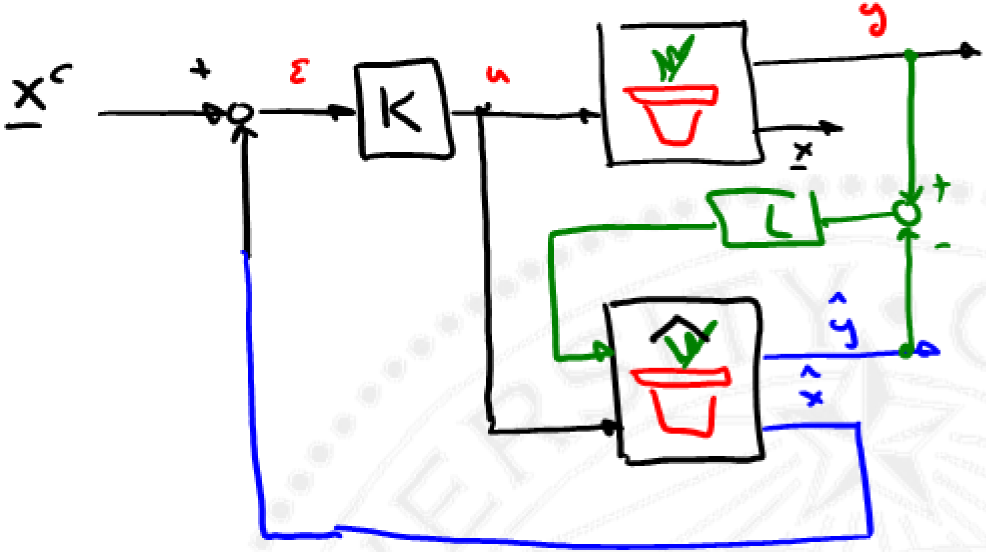
$$e^{At} = I + At + \frac{A^2t^2}{2!} + \dots$$

$$\Gamma = \left[\int_0^{\Delta T} e^{A\tau}\, d\tau\right] B$$ — zoh equivalent for input

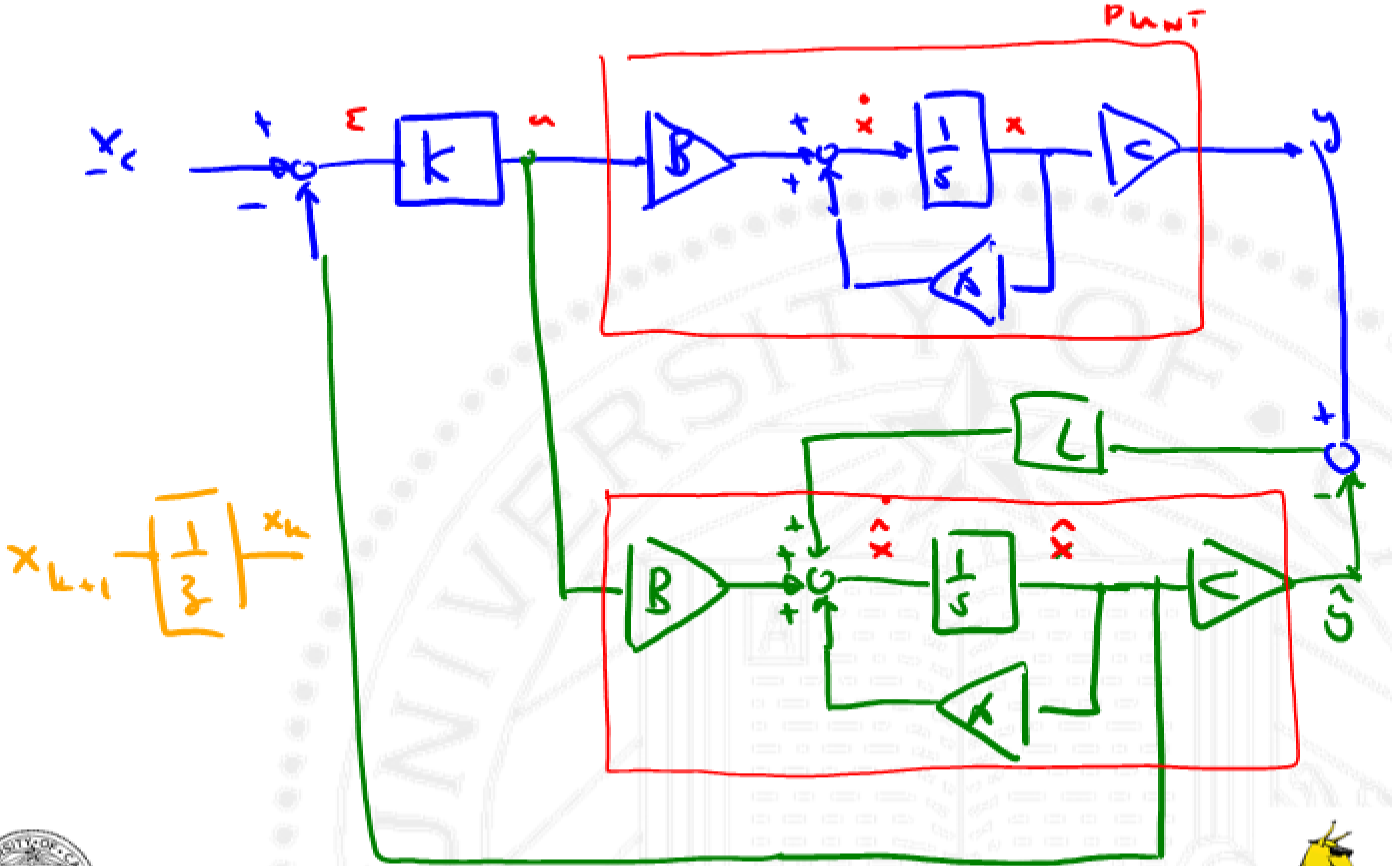
$\dot{x} = Ax \leftarrow x(t) = e^{At}\, x(0)$

c2d('zoh')

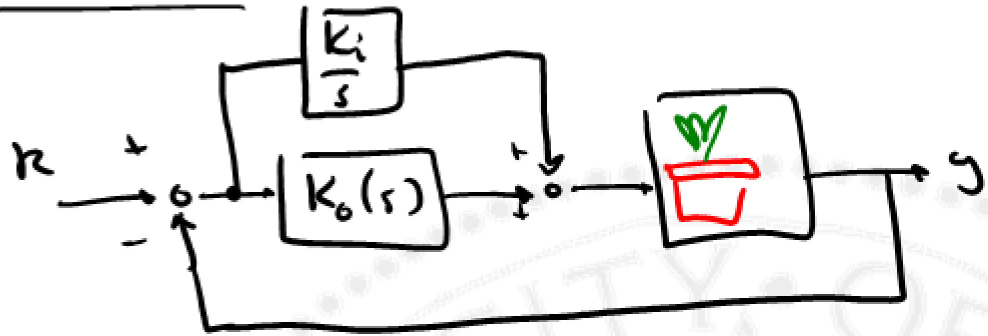
$R_w \neq R_{w_k}$. $\leftarrow$ disrw( )



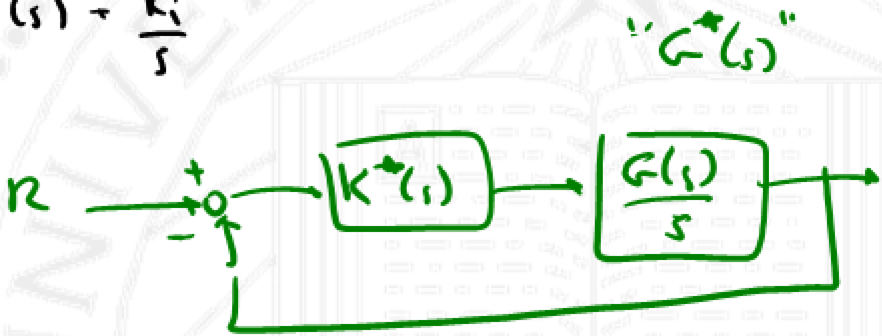
$\underline{x}^c$
+
ε
K
u
y
$\underline{x}$
+
L
-
$\hat{y}$
$\hat{\underline{x}}$


PLANT
K
B
1/s
C
A
L
1/z

Integral Control



$$K^*(s) = K_0(s) + \frac{K_i}{s}$$



$$\dot{x} = Ax + Bu \qquad e = r - y = r - Cx$$

$$y = Cx$$

$$x_I = \int e\,dt \quad \therefore \quad \dot{x}_I = e = r - Cx$$

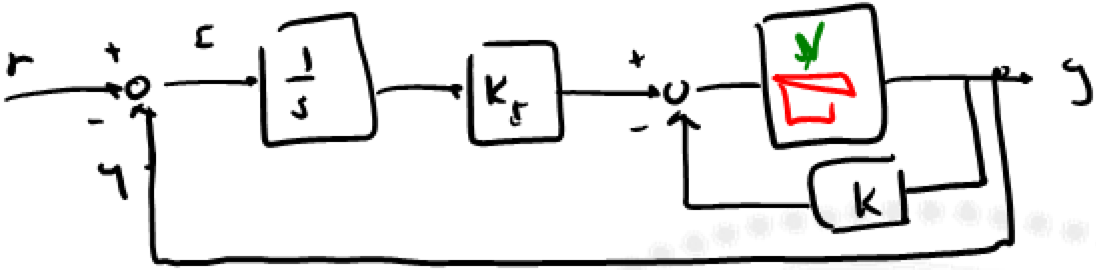
$$\begin{bmatrix} \dot{x} \\ \hline \dot{x}_I \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} A & \phi \\ \hline -C & \phi \end{array}\right]}_{A_{aug}} \begin{bmatrix} x \\ \hline x_I \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ \hline 0 \end{bmatrix}}_{B_{aug}} u + \begin{bmatrix} 0 \\ \hline I \end{bmatrix} r$$

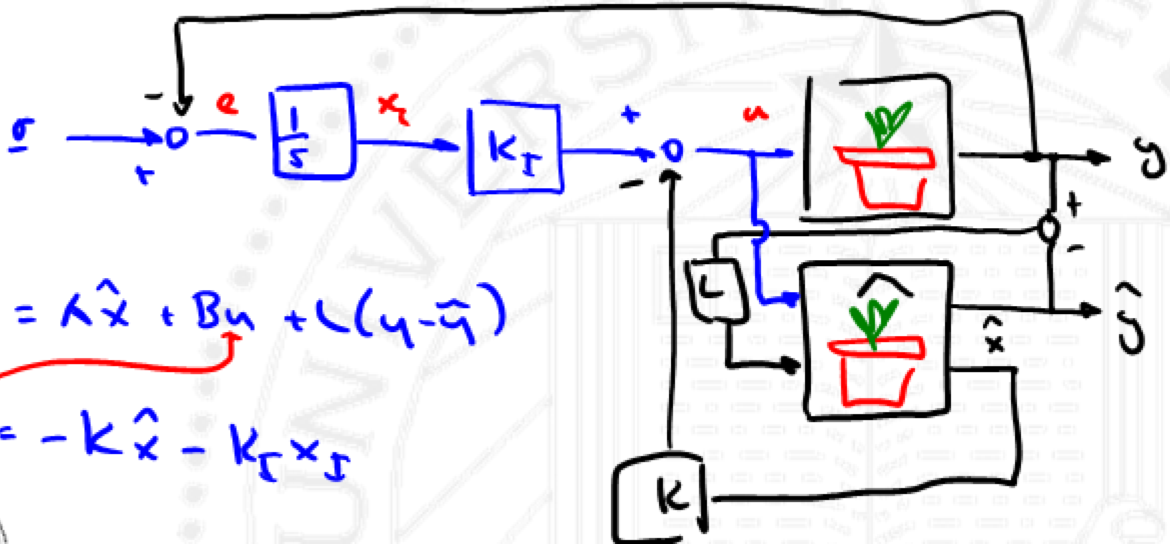
$$u = -\left[K \;\vdots\; K_I\right] \begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix} \qquad \mathrm{eig}\left(A_{aug} - B_{aug}\left[K \;\; K_I\right]\right)$$

$x \in \mathbb{R}^5 \quad u \in \mathbb{R}^2$

Drive as many states to reference
as I have actuators/inputs

Drive as many linear combinations of
states to a reference as I have inputs





$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$u = -K\hat{x} - K_I x_I$$

$$\frac{d}{dt}\begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix} = \underbrace{\begin{bmatrix} A & \phi \\ -C & \phi \end{bmatrix}}_{A_r} \underbrace{\begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix}}_{x_r} + \underbrace{\begin{bmatrix} B & \phi \\ \phi & I \end{bmatrix}}_{B_r} \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{u_r}$$

$$\dot{\underline{x}}_r = A_r \underline{x}_r + B_r \underline{u}_r$$

$$y_r = C_r \underline{x}_r$$

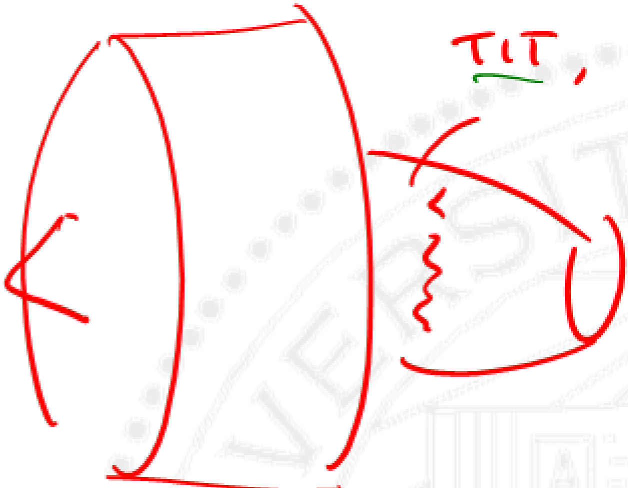
$$\dot{\hat{x}}_r = A_r \hat{x}_r + B_r u_r + L(y_r - \hat{y}_r)$$

$$\hat{y}_r = C_r x_r$$

$$u_r = -K_r x_r = -K[\hat{\underline{x}}] - K_I \hat{x}_I$$

x_I

Reduced Order Estimator



TIT, pruim

$u = -K\hat{x}$

$\hat{x} = \begin{bmatrix} \; \\ \; \end{bmatrix}$

$\hat{x} = \begin{bmatrix} \circ - y_1 \\ \circ - y_2 \end{bmatrix}$