CMPE-242
Applied Feedback Control

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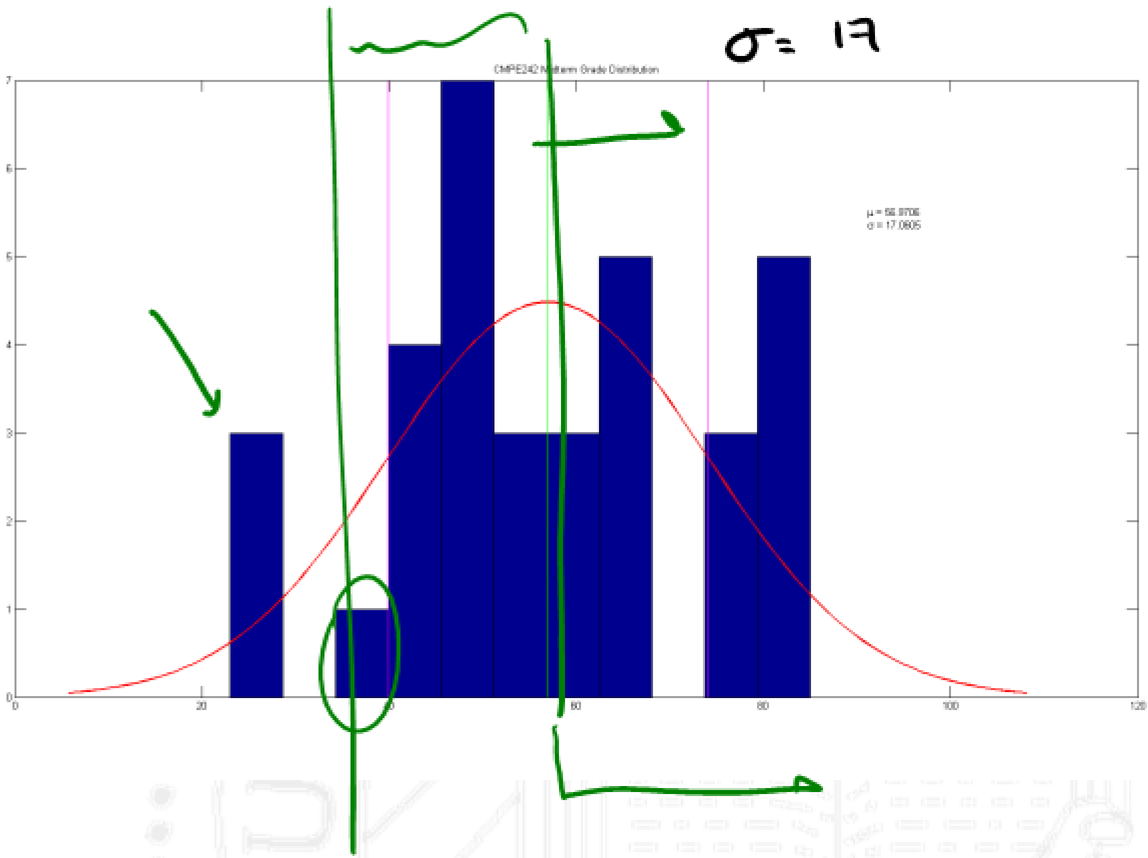
$\mu = 57$

$\sigma = 17$



Questions?

step(Gcl);

eigenvalue: square matrix "A"

$$Ax_0 = \lambda x_0 \qquad x_0 = 0;$$

$$(A - \lambda I)\, x_0 = 0$$

λ's "eigenvalues"

x_0 "eigenvectors"

SVD – Singular Value Decomposition

$$A \to U \Sigma V^T$$

$$\Sigma = \begin{bmatrix} \sigma_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix}$$

$$\left[\begin{array}{c|c} \ddots & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$\mathbb{R}^n \longrightarrow A \longrightarrow \mathbb{R}^m$$

$$\mathbb{R}^n \longrightarrow V \longrightarrow \Sigma \longrightarrow U^T \longrightarrow \mathbb{R}^m$$

STATE SPACE CONTROL

$$\dot{x} = Ax + Bu \qquad \underline{u = -Kx}$$
$$y = Cx + Du$$

"Full STATE FEEDBACK"

Choose K to put $\text{eig}(A-BK)$ where !

define them

check $\text{rcond}(\,)$, $\text{rank}(\mathcal{C})$

$$\mathcal{C} = [B \;\; AB \;\; A^2B \;\cdots\; A^{n-1}B]$$

$$\kappa(\mathcal{A}) = \frac{\sigma_{max}(\mathcal{A})}{\sigma_{min}(\mathcal{A})}$$

"condition number"

$1 \to \infty$

rcond $\frac{1}{\kappa} \to 1 \to 0$.

$u \leftarrow \delta_e$

$\dot{h} \leftarrow \delta_T$

MODAL COORDINATES

$$\dot{x} = Ax + Bu \longrightarrow x = Tz \; ; \; z = T^{-1}x$$

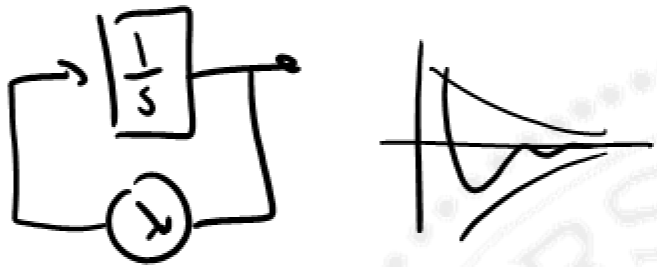
$$T^{-1}AT = \underbrace{\begin{bmatrix} [\;]^{n\times n} & & & \\ & [\;] & \phi & \\ & & [\;] & \\ \phi & & & [\;] \end{bmatrix}}_{\Sigma}$$

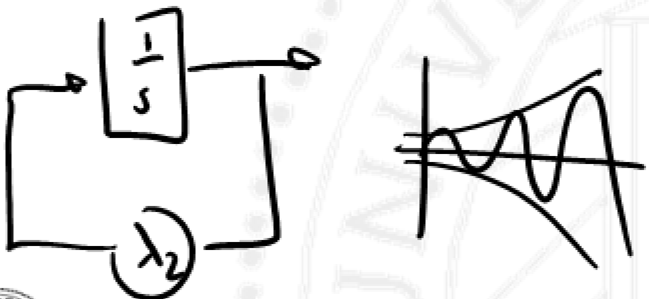
$\longleftarrow$ Block Jordan Form

$$\begin{bmatrix} -2\zeta & \omega_n^2 \\ -\omega_n^2 & 1 \end{bmatrix}$$

2nd order damped oscillator

$$T^{-1}AT = \Sigma \qquad T^{-1}B \text{ — input — } \begin{bmatrix} \boxed{1 \quad 0} \\ 0 \quad \tfrac{1}{2} \\ 0 \quad 0 \\ 0 \quad -\tfrac{1}{2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$





$$\dot{x} = Ax + B\overset{0,\ \forall t}{\cancel{u}} \qquad x(0) \triangleq \underline{x}_0$$

$$\underline{x}(t) = e^{At}\,\underline{x}_0$$

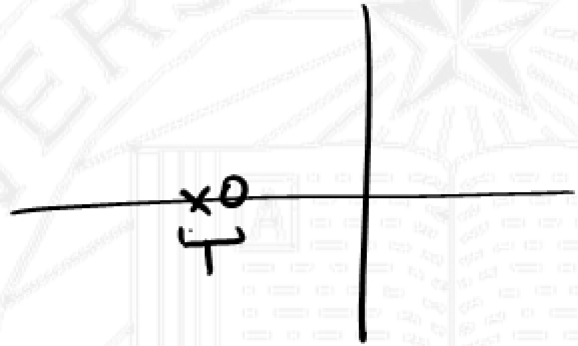
$$e^{At} \triangleq I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots \quad \text{expm}$$

$$e^{\Sigma t} = \begin{bmatrix} e^{\Sigma_1 t} & & 0 \\ & \ddots & \\ 0 & & e^{\Sigma_n t} \end{bmatrix}$$

$$\mathcal{C} — \left[B \quad AB \quad A^2B \cdots A^{n-1}B \right]$$

full rank — $\operatorname{rank}(\mathcal{C}) = n$ "controllable"

$$\kappa(\mathcal{C}) \triangleq \frac{\sigma_{max}(\mathcal{C})}{\sigma_{min}(\mathcal{C})} \to \infty$$ "approaching singular"



$$\dot{x} = Ax + Bu \qquad x = Tz$$

$$\dot{z} = T^{-1}ATz + T^{-1}Bu$$

$$\mathcal{C}_{new} = \left[\, T^{-1}B \quad T^{-1}A\underbrace{\cancel{T\,T^{-1}}}_{I}B \quad T^{-1}A\underbrace{\cancel{T\,T^{-1}}}_{I}A\underbrace{\cancel{T\,T^{-1}}}_{I}B \quad \cdots \,\right]$$

$$T^{-1}AB \qquad T^{-1}A^{2}B \qquad \cdots$$

$$T^{-1}\,\mathcal{C}_{old}$$

$$\dot{x} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \underline{x} + \underbrace{\begin{bmatrix} \ \\ \ \end{bmatrix}}_{B} \underline{u} \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$\text{I}: \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} \qquad \begin{aligned} \text{rank}(\mathcal{C}) &= 2 \\ \text{cond}(\mathcal{C}) &= 2 \end{aligned}$$

$$C[sI - A]^{-1} B \longrightarrow \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+3 & 2 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{s+3}{(s+2)(s+1)}$$

$$\text{II}: \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{aligned} \text{rank}(\mathcal{C}) &= 1 \\ \text{cond}(\mathcal{C}) &= \infty \end{aligned}$$

$$C[sI - A]^{-1} B \longrightarrow \frac{\cancel{s+2}}{(s+1)\cancel{(s+2)}}$$

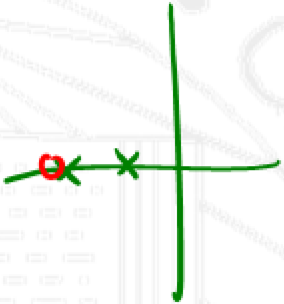
T.M :

$$\begin{bmatrix} -0.999 \\ 1 \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} -0.999 & 0.997 \\ 1 & -0.999 \end{bmatrix}$$

$$\text{rank}(\mathcal{C}) = 2$$

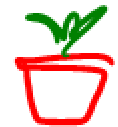
$$\text{cond}(\mathcal{C}) = 4000.$$

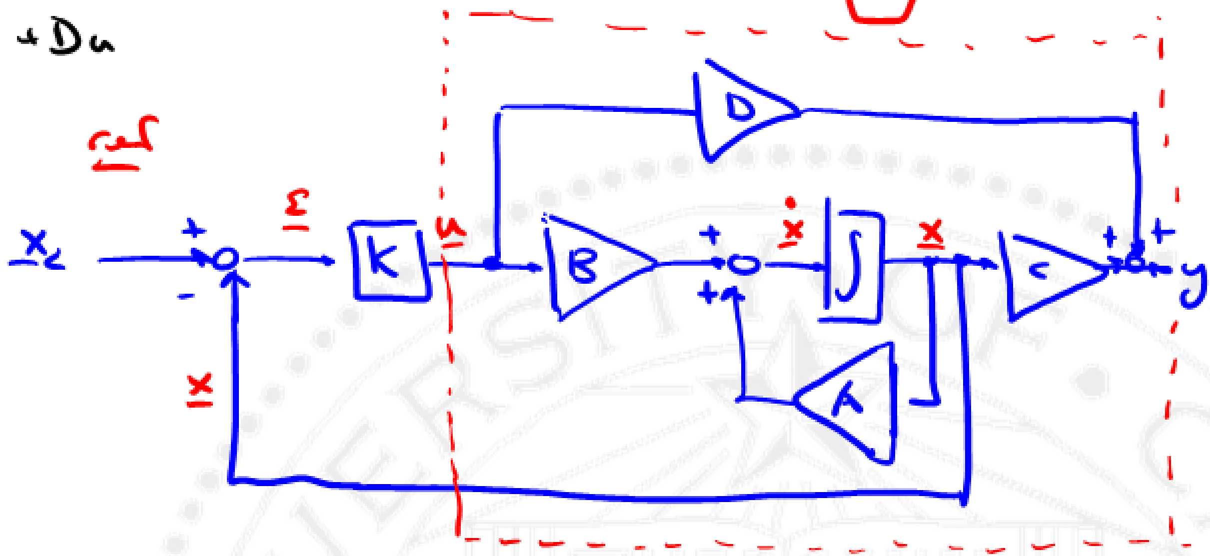
$$C[sI - A]^{-1}B \longrightarrow \frac{s + 2.001}{(s+2)(s+1)}$$



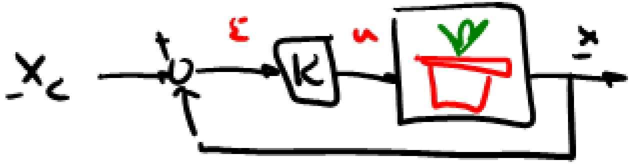
$$\dot{x} = Ax + Bu \qquad u = -K(x - x_c)$$

$$y = Cx + Du$$

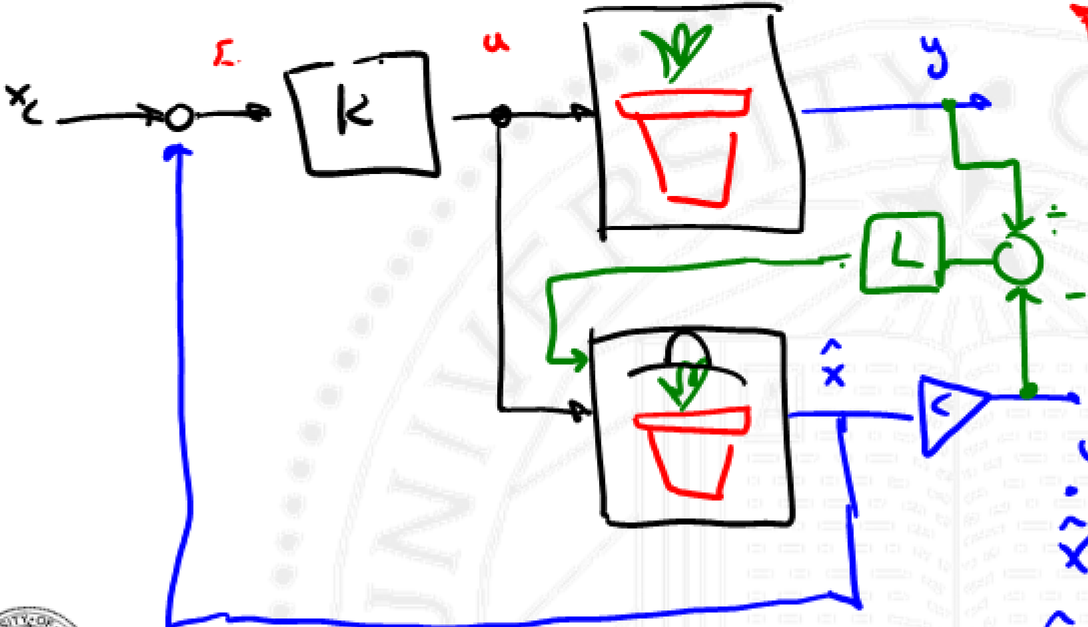




if I have $\underline{x}$ and C is full rank I have very good control



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$u = -K(x - x_c)$$

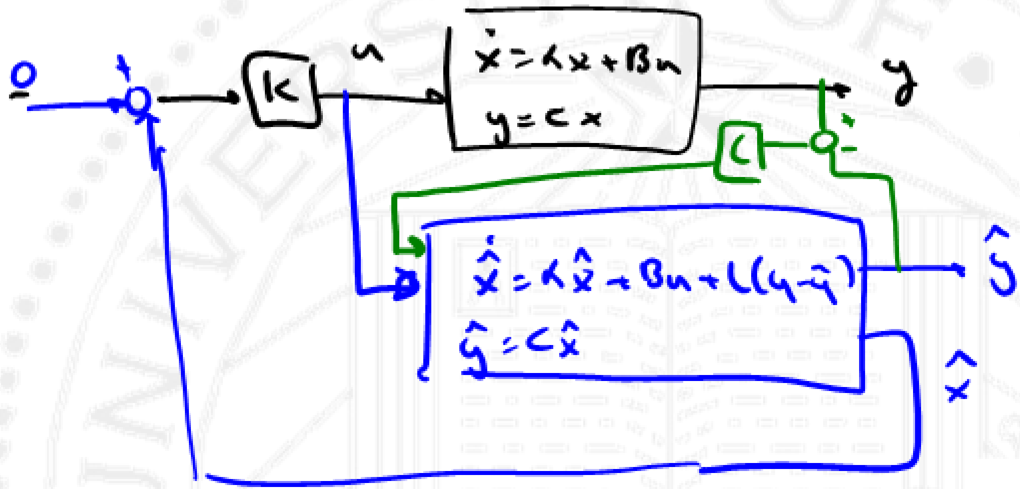


$$u = -K(\hat{x} - x_c)$$
$$\dot{\hat{x}} = A\hat{x} + Bu$$
$$\hat{y} = C\hat{x}$$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$



$$\dot{x} = Ax + Bu \qquad u = -K\hat{x}$$

control gain

$$y = Cx$$

$$\tilde{x} \triangleq x - \hat{x}$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

estimator gain

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\dot{\hat{x}} = \underbrace{(A - BK)}_{\text{stable}}\hat{x} + L(y - \hat{y})$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu} - L(y - \hat{y})$$

$$= A\tilde{x} - LCx + LC\hat{x}$$

$$= A\tilde{x} - LC(x - \hat{x}) = \underbrace{(A - LC)}_{\text{STABLE}}\tilde{x}$$

$(A-BK)$ control gain

$\begin{bmatrix} | \\ B \\ | \end{bmatrix} \begin{bmatrix} - & k & - \end{bmatrix}$

cl poles

$K = \text{place}(A, B, P_{des})$

$(A-LC)$ estimator gain

$\begin{bmatrix} L \end{bmatrix} \begin{bmatrix} - & c & - \end{bmatrix}$

est

$L^T = \text{place}(A^T, C^T, P_{des})$

5-10x faster

$\underbrace{A^T - C^T L^T}$ $\underbrace{\text{"dual" } A - BK}$

For control

$$\mathcal{C} \triangleq \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

full rank controllable

cond ($\mathcal{C}$)

$$\mathcal{C}_{Dual} = \begin{bmatrix} C^T & A^TC^T & (A^2)^TC^T & \cdots & (A^{n-1})^TC^T \end{bmatrix}$$

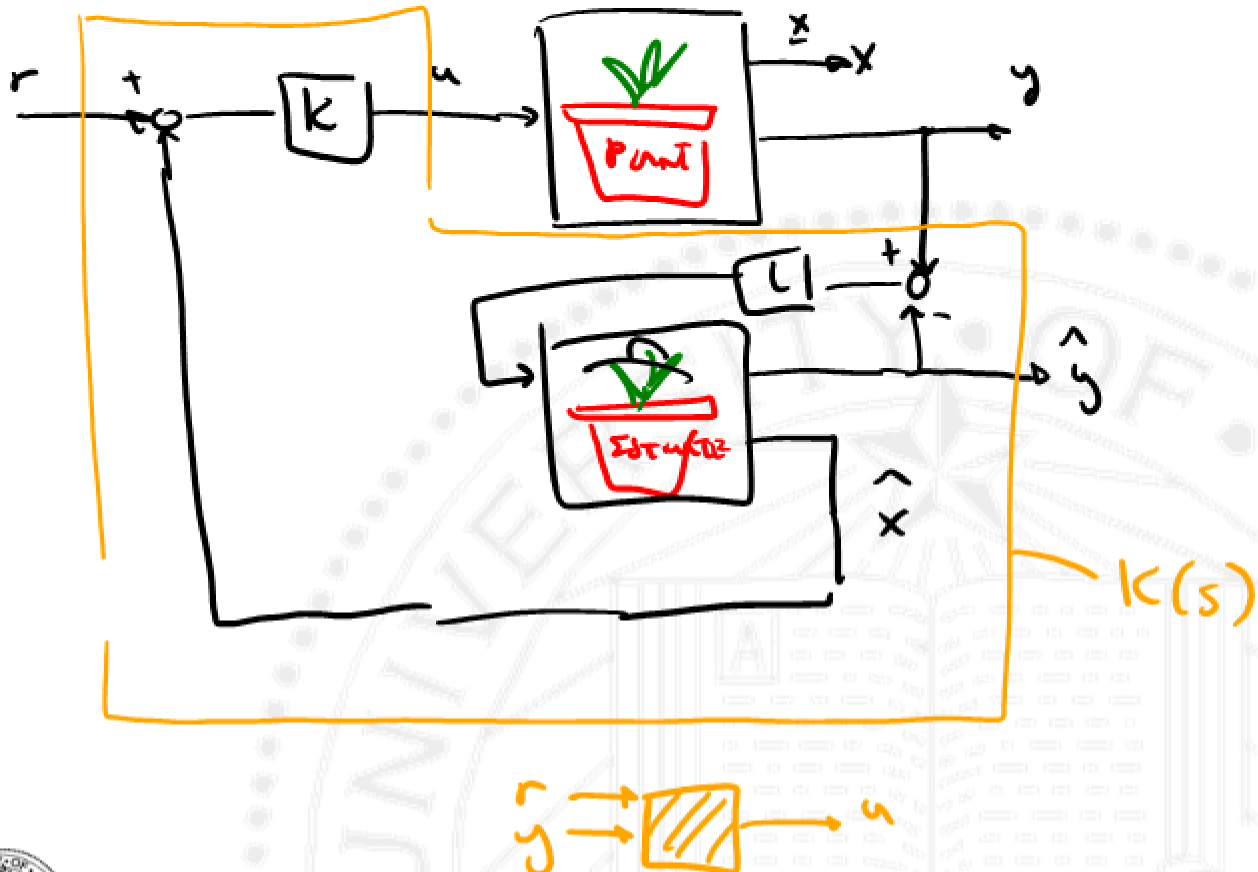
$$\mathcal{O} \triangleq \begin{bmatrix} C \\ AC \\ A^2C \\ \vdots \\ A^{n-1}C \end{bmatrix}$$

observability matrix

obsv (a, c)

full rank – observable

cond ($\mathcal{O}$)


r
+
K
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Estimator
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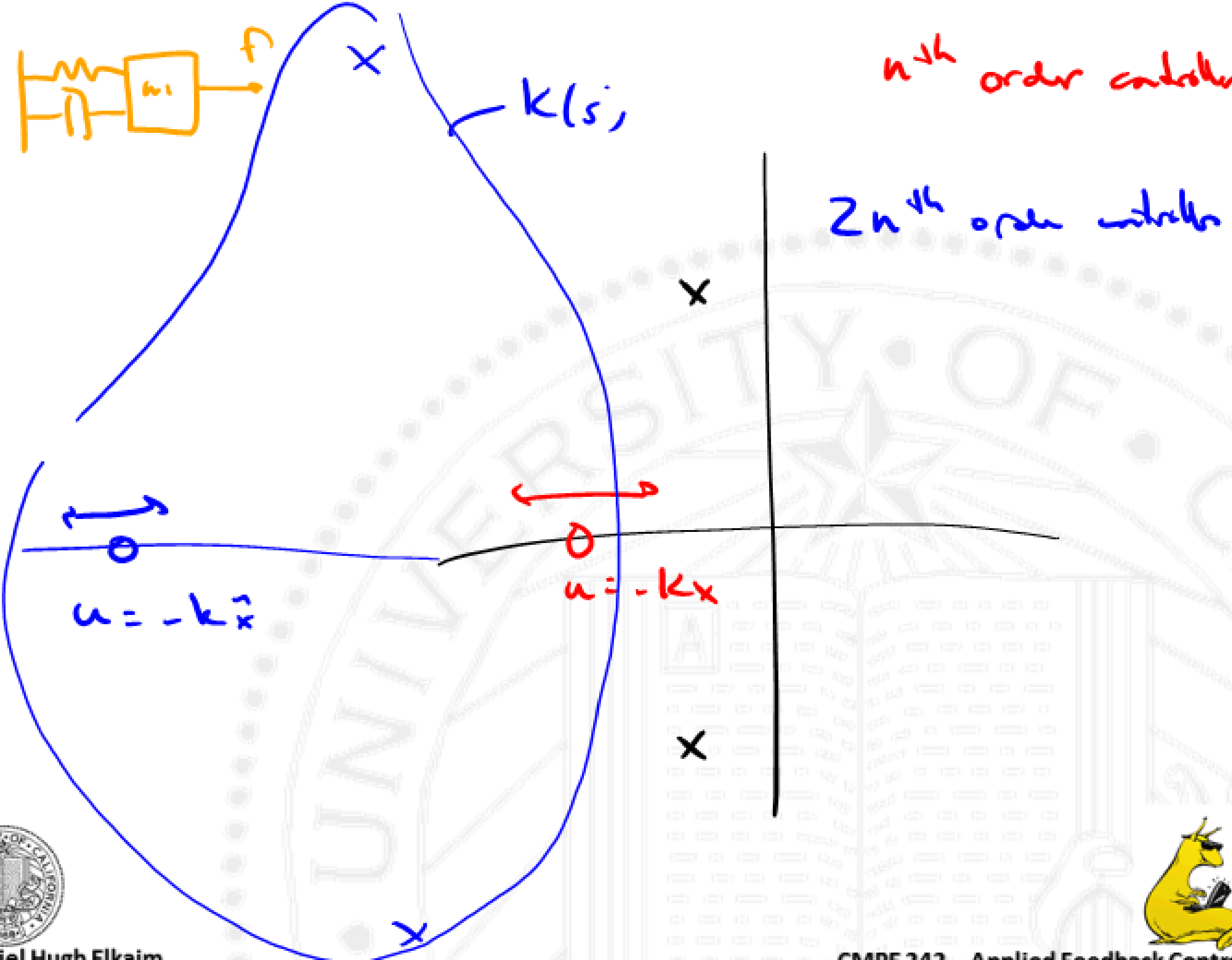


n^{th} order controller
$2n^{th}$ order controller
$K(s)$
$u = -k\hat{x}$
$u = -kx$
m
f

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$u = -K\hat{x}$$

$$\hat{y} = C\hat{x}$$

$$\left[\begin{aligned} \dot{\hat{x}} &= \overbrace{(A - LC - BK)}^{\tilde{A}}\hat{x} + L\overset{\text{input}}{y} \\ \underset{\text{output}}{u} &= -K\hat{x} \end{aligned}\right]$$

$$\frac{\frac{U}{Y}(s)}{K(s)} = -K\left(sI - A + BK + LC\right)^{-1}L$$

$u = -K\underline{x}$ place poles @ P_{des} (choose K)

$L \Rightarrow$ place soln @ P_{est} $\quad \tilde{x} \triangleq x - \hat{x}$

$\hat{x} = x - \tilde{x}$

$\dot{\tilde{x}} = (A - LC)\tilde{x}$

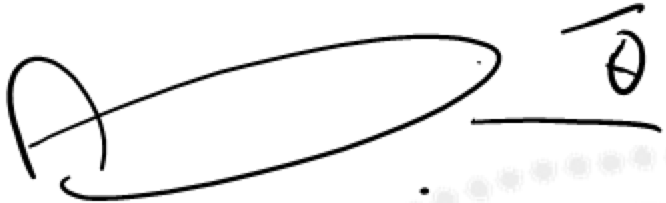
$\dot{\underline{x}} = Ax - BK\hat{x} = Ax - BK(x - \tilde{x})$

$= (A - BK)\underline{x} + BK\tilde{x}$

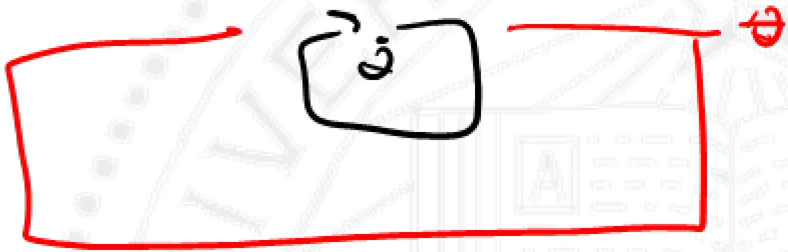
$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{\tilde{x}}} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{\tilde{x}} \end{bmatrix}$$

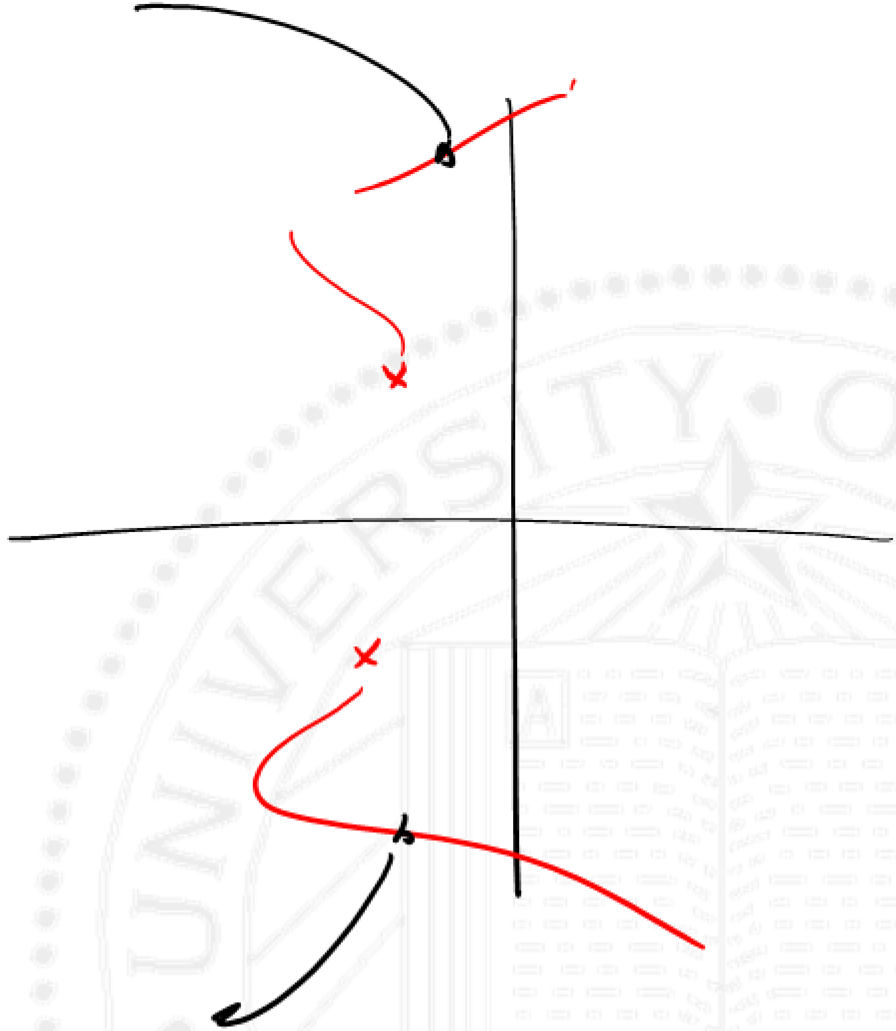
"separation principle"

$\det\left[\begin{smallmatrix} & \\ 0 & \end{smallmatrix}\right] = \det(A - BK) \cdot \det(A - LC)$



$$q = \dot{\theta}$$

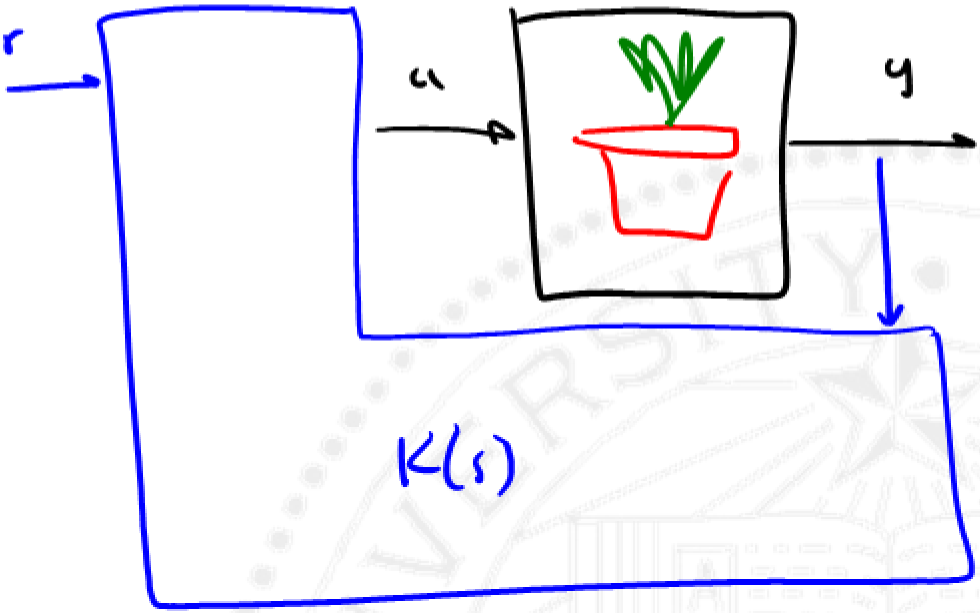




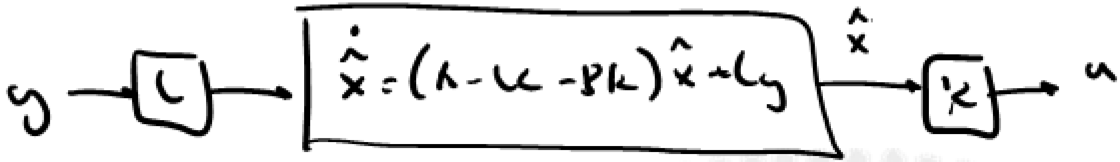
$$\underbrace{\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}}_{\underline{x}} \longrightarrow \quad u = -K\underline{x}$$

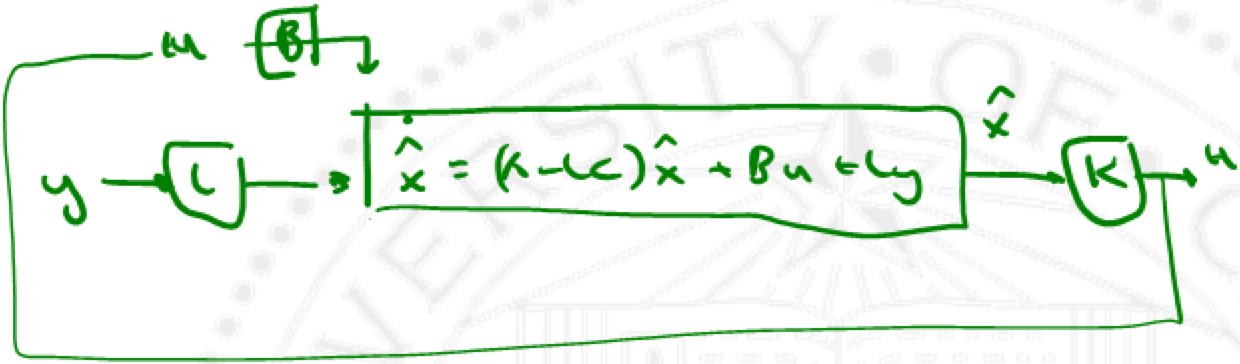
use estimator to get $\dot{\theta}$

$$u = -K\hat{x} \qquad \begin{bmatrix} \hat{\theta} \\ \hat{\dot{\theta}} \end{bmatrix}$$


r
u
y
K(s)

"Theoretical"





(1) Measure Y

(2) $\times L \longrightarrow$ form $\dot{\hat{x}}$

(3) $\dot{\hat{x}} = [2] + [\dots]\hat{x} + [u_{act}]$

(4) $\hat{x}^{+} = \hat{x}^{-} + \dot{\hat{x}}\,\Delta T$

(5) $u_{cmd} = K\hat{x}^{+}$