CMPE-242
Applied Feedback Control

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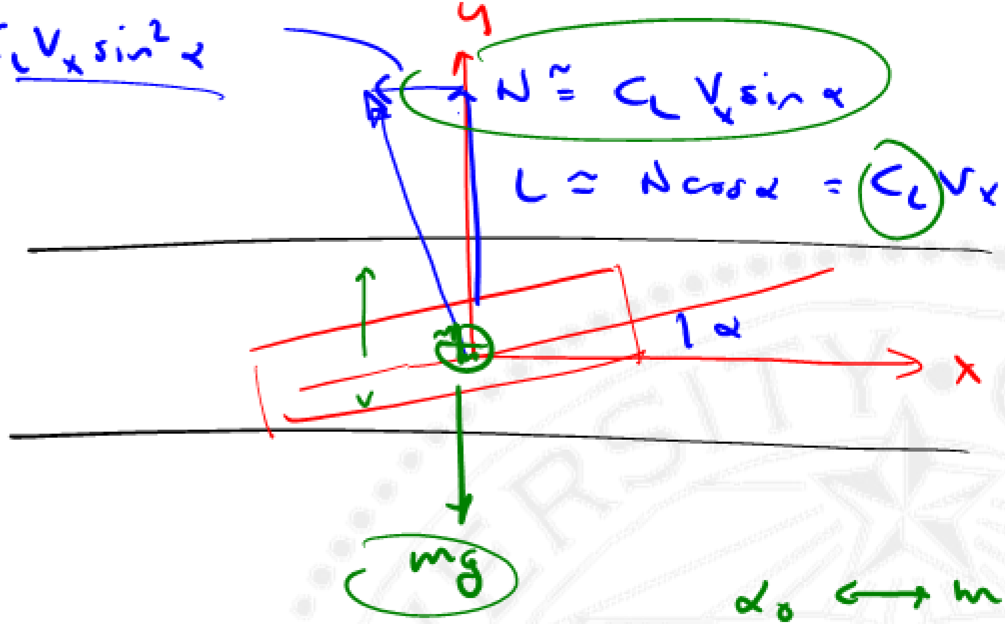
$C_L V_x \sin^2\alpha$

$N \approx C_L V_x \sin\alpha$

$L \approx N\cos\alpha = C_L V_x \sin\alpha\cos\alpha$



$\alpha_0 \longleftrightarrow mg, V_x$

$\alpha \hat{=} \alpha_r + \delta\alpha$

$s^2 q + \ldots$

Questions:

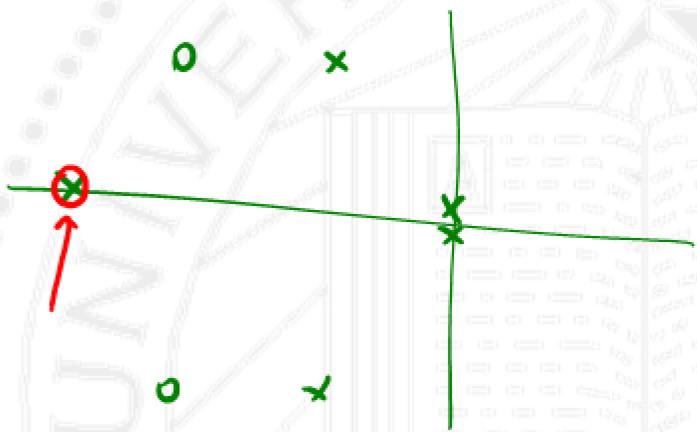
831-334-6047 Sharon

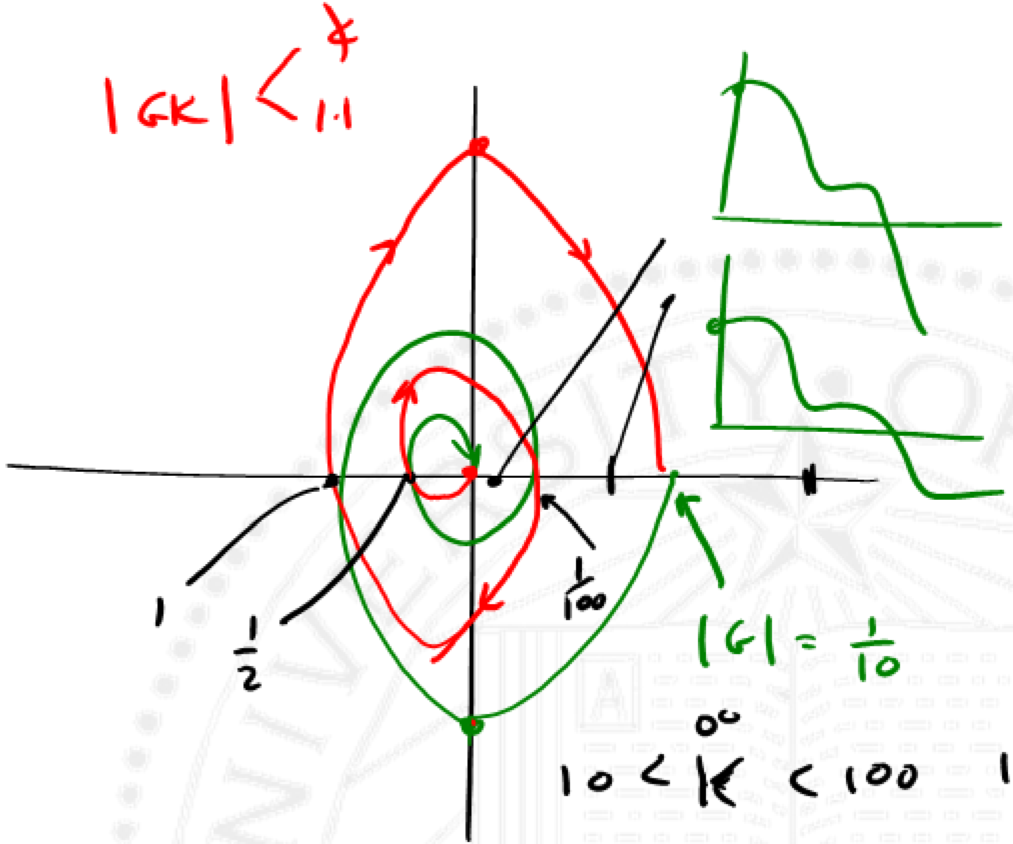
$\boxed{\text{E2-316}}$

3 hour take-home

8PM

2 questions — hard, & parts





0°

$10 < K < 100$ $100 < k$

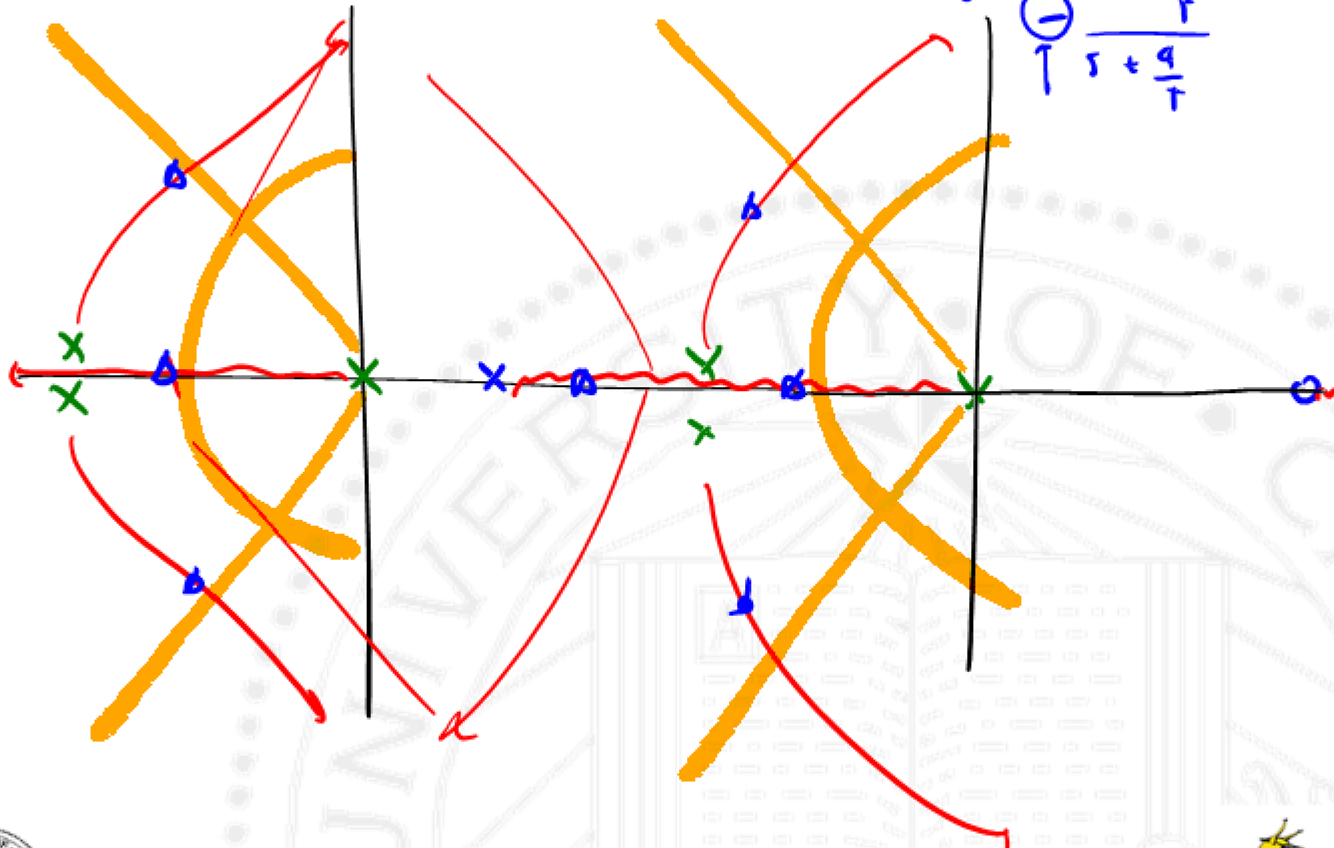
1 encirclement

3 poles in RHP

1 pole

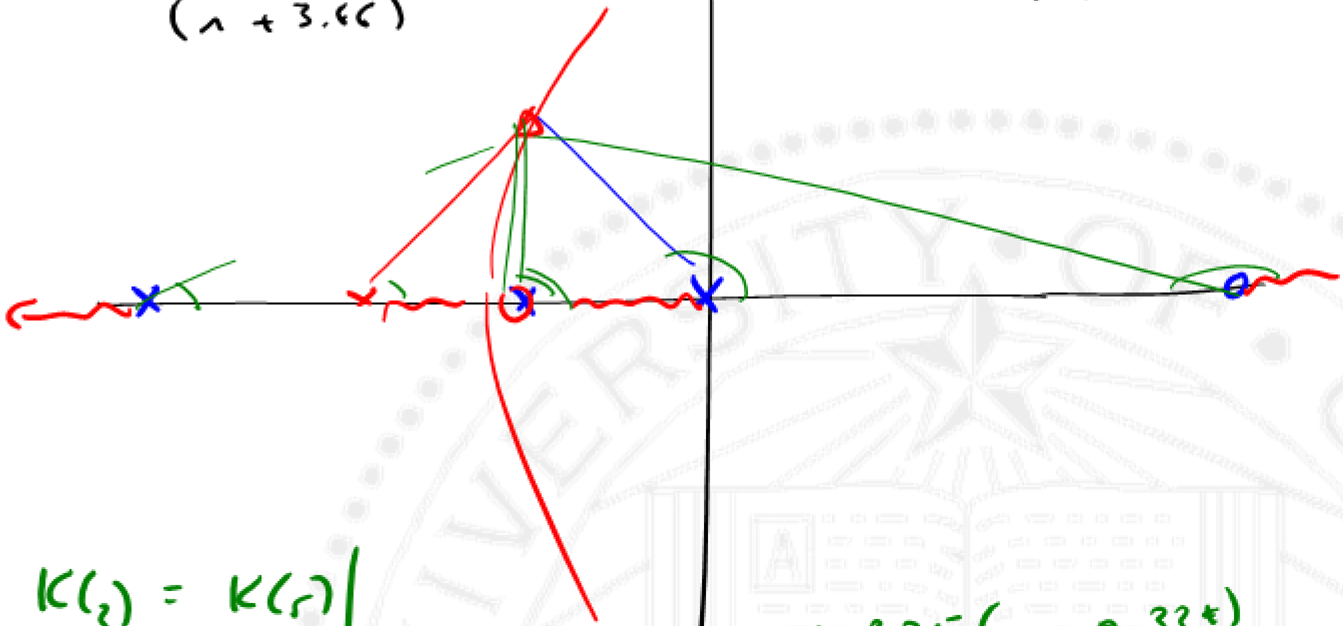
ADD PADÉ

$$-\frac{s - \frac{4}{T}}{s + \frac{4}{T}}$$



$$K(s) = \frac{2.5(s+1)}{(s+3.55)}$$

Padé : $\dfrac{-(s-4)}{s+4}$



$$K(z) = K(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} \longrightarrow \frac{1.325(z-0.338)}{z+0.2938}$$

$$z_{des} = 0.2 \pm 0.3j$$

$$\downarrow$$

$$s_{des} = -1 \pm j$$

$$D_{cl}\left(K(z)\big|_{Tustin}\right) = 0.25 \pm 0.22j$$

$$\downarrow$$

$$s_{closed\ loop} = -1.08 \pm 0.72j$$


jω
ωs/2
BAD for Analog
G(z)
GOOD
Re
BAD
-ωs/2

Root locus in z

Harder — $G(z) = \frac{z-1}{z} \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$

$$z_{des} = e^{-s_{des} T}$$

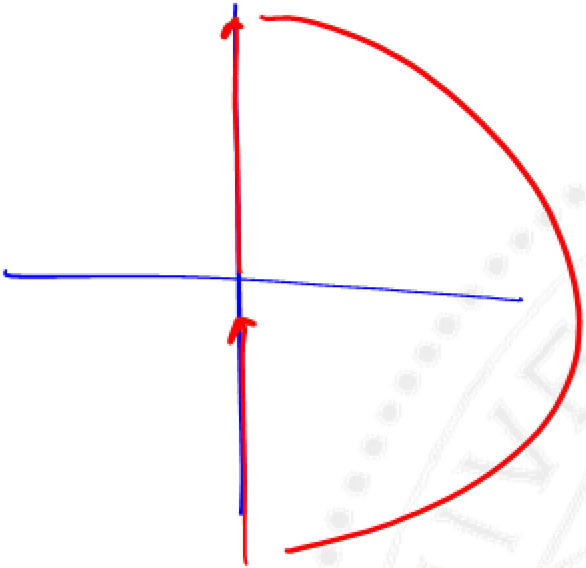
Bode in z?

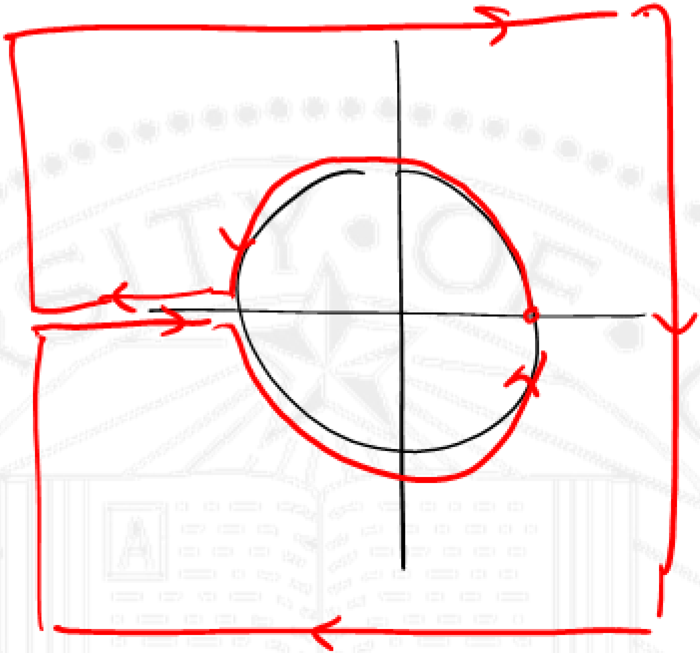
$s = j\omega$

$z = e^{j\omega T}$ — lost my straight line asymptotes

$s \rightarrow z \rightarrow \frac{\omega}{T}$

Nyquist in z?





$$K_p = \lim_{z \to 1} K(z)G(z)$$

$$K_v = \lim_{z \to 1} \frac{z-1}{Tz} K(z)G(z)$$

Digital Bode

$$G(e^{j\omega T}) \begin{cases} |\cdot| \\ \measuredangle \end{cases}$$

dbode

Design lead network in "s"

$$\omega_{x0} = \sqrt{ab}$$

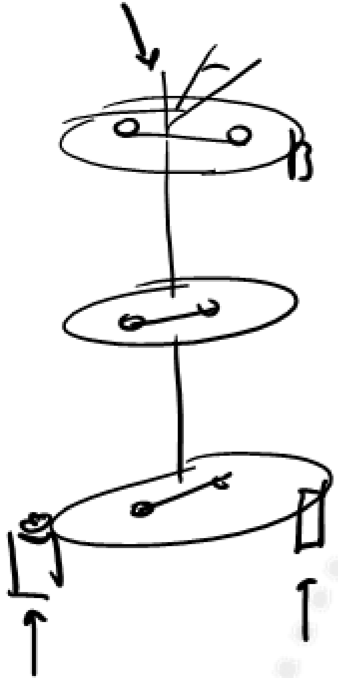
$$\frac{b}{a} \to \Delta\phi$$

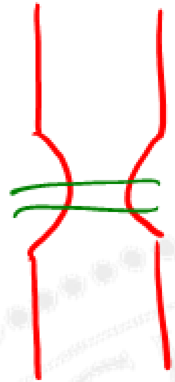
$$K_{lead}(z) = K_{lead}(s)\Big|_{\text{Tustin}}$$

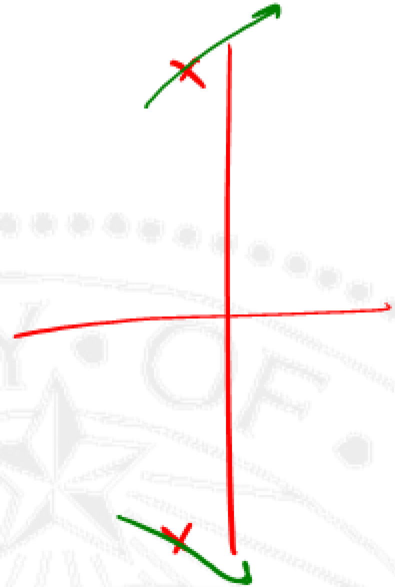
Pre-warp @ ω_{x0}.

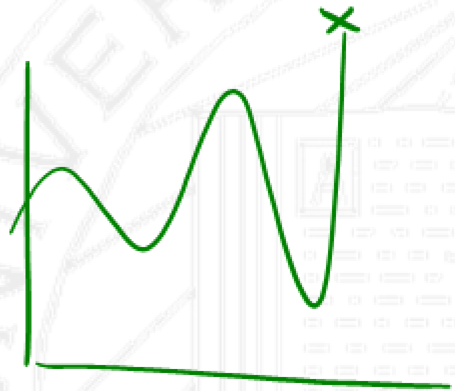
LAG in "s" $\longrightarrow$ DC gain $= \frac{b}{a}$ $\quad \frac{s+b}{s+a}$

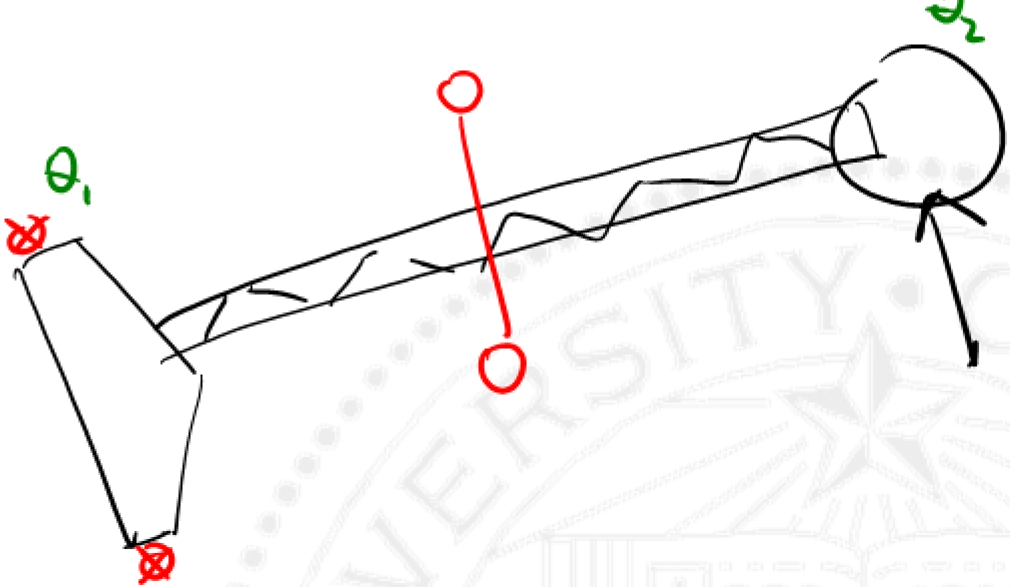
LAG in "z" $\longrightarrow$ DC gain $\frac{1-b}{1-a}$ $\quad \frac{z+b}{z+a}$











Actuate at bottom (θ_1), measure θ_1 collocated

" " " " , measure θ_2 - non-colo

State Space – ch. 7 in FPE

T.F. → Root locus
Bode
Nyquist
Digital

State Space ← linear Algebra

T.F: — n^{th} order differential/difference equation

SS: — N 1st order differential/difference equations

$$\frac{Y}{U} = \frac{1}{s^2+2s+3} \longleftrightarrow \ddot{y} + 2\dot{y} + 3y = u$$

$$\updownarrow$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u$$

$$\underbrace{\begin{aligned}\dot{\underline{x}} &= A\underline{x} + B\underline{u}\\ \underline{y} &= C\underline{x} + D\underline{u}\end{aligned}}$$

west coast
Stanford

$$\underbrace{\begin{aligned}\dot{\underline{x}} &= F\underline{x} + G\underline{u}\\ \underline{y} &= H\underline{x} + D\underline{u}\end{aligned}}$$

east coast
MIT

$$\underbrace{\begin{aligned}\underline{x}_{k+1} &= \Phi\underline{x}_k + \Gamma\underline{u}_k\\ y_k &= H\underline{x}_k + D\underline{u}_k\end{aligned}}$$

DIGITAL

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

$D \in \mathbb{R}^{r \times m}$

DIRECT FEEDTHRU

$A \in \mathbb{R}^{n \times n}$ ← dynamics matrix
state transition "

$B \in \mathbb{R}^{n \times m}$ ← input matrix
(not square)

$C \in \mathbb{R}^{r \times n}$ ← output matrix
(not square)

$\underline{x} \in \mathbb{R}^n$ n×1 vector "state" $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ NOT UNIQUE
n^{th} order system

$\underline{u} \in \mathbb{R}^m$ m×1 vector "inputs" $\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$ m-input

$\underline{y} \in \mathbb{R}^r$ r×1 vector "outputs" $\begin{bmatrix} y_1 \\ \vdots \\ y_r \end{bmatrix}$ r-output

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{array}{c} \\ \begin{matrix} \updownarrow n \\ \updownarrow r \end{matrix} \end{array}
\begin{bmatrix} \dot{x} \\ \hline y \end{bmatrix} =
\begin{array}{cc} n \times n & n \times m \\ \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \\ r \times n & r \times m \end{array}
\begin{bmatrix} x \\ \hline u \end{bmatrix}
\begin{matrix} \updownarrow n \\ \updownarrow m \end{matrix}$$

Proper TF: more x's than o's

$D = \phi$ $\quad \dfrac{s^2+\dots}{s^3+\dots}$

Strictly Proper TF:

x's = o's

$D \neq 0$ $\quad \dfrac{s^3+\dots}{s^3+\dots}$

Improper TF:

more o's than x's.

$\underline{\dot{x}} = A\underline{x} + B\underline{u}$ $\qquad u = -Kx$ control

$\underline{y} = C\underline{x}$

$\underline{\dot{x}} = A\underline{x} + B(-K\underline{x}) = (A - BK)\underline{x}$

eigenvalues of $(A-BK)$ $\leftrightarrow$ poles of closed loop system

$$J = \int_0^\infty (x^T Q x + u^T R u)$$

min J
subj. to $\dot{x} = Ax + Bu$

$u = -Kx$

A.R.E.

LQR

STABLE

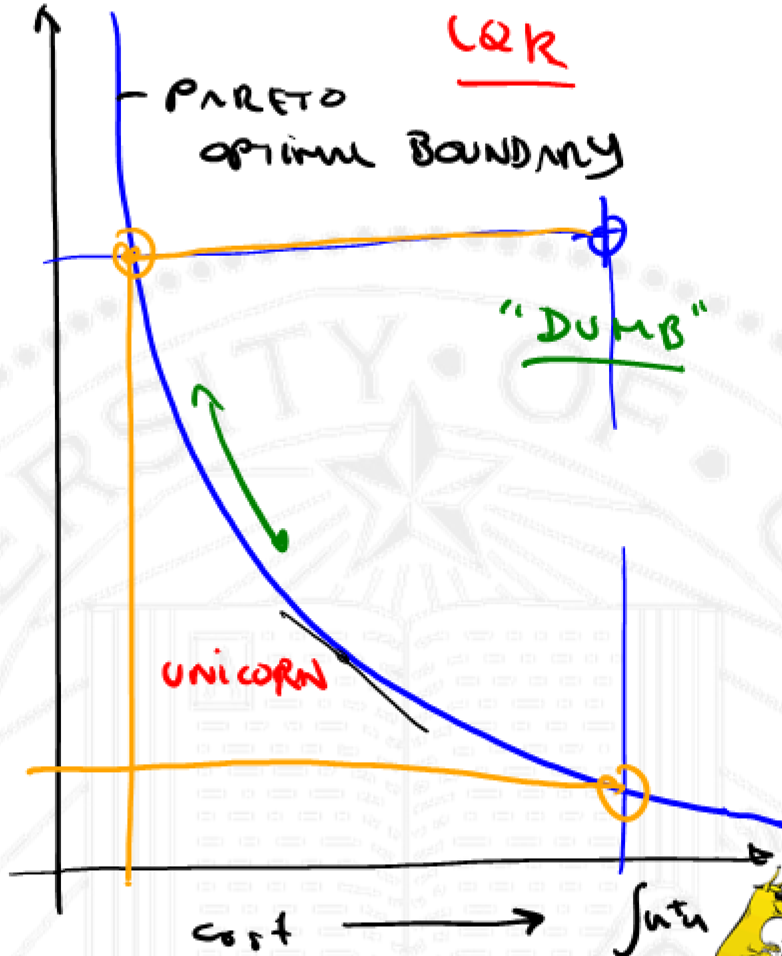
$> 50°$ PM

$\frac{1}{2} <$ GM $< \infty$

$\boxed{\text{MUST MEASURE ALL STATES}}$

$\int x^T x$

performance



$$\frac{Y}{U}(s) = \frac{1}{s^2+2s+3} \longrightarrow [s^2+2s+3]Y = U$$

$$\ddot y + 2\dot y + 3y = u$$

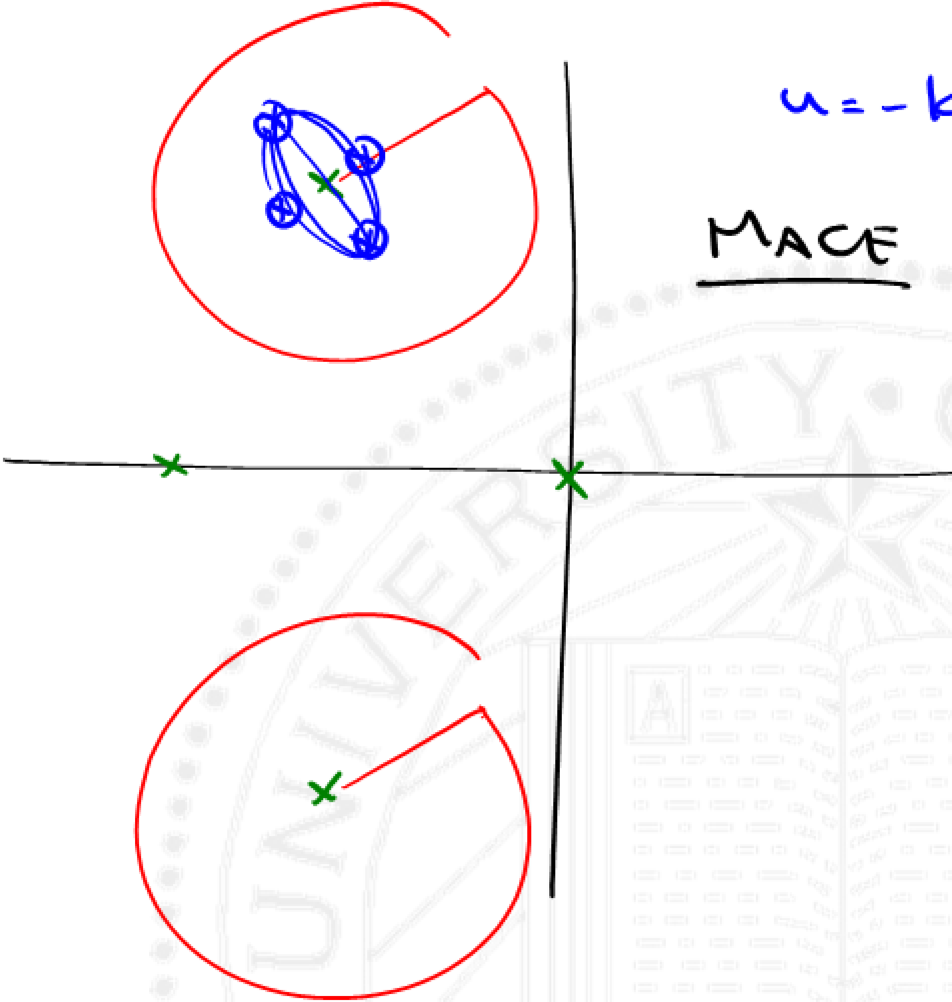
$$\underline{x} = \begin{bmatrix}\dot y \\ y\end{bmatrix}$$

$$\dot{\underline{x}} = \begin{bmatrix}\ddot y \\ \dot y\end{bmatrix} = \underbrace{\begin{bmatrix}-2 & -3 \\ 1 & 0\end{bmatrix}}_{A}\begin{bmatrix}\dot y \\ y\end{bmatrix} + \overbrace{\begin{bmatrix}1 \\ 0\end{bmatrix}}^{B}\underline{u}$$

$$y = \underbrace{\begin{bmatrix}0 & 1\end{bmatrix}}_{C}\begin{bmatrix}\dot y \\ y\end{bmatrix} + \underbrace{\begin{bmatrix}0\end{bmatrix}}_{D}u$$

$$\ddot y = -2\dot y - 3y + u$$

TF2SS $\longleftrightarrow$ SS2TF



$u = -Kx$

MACE

STATE SPACE

(1) Higher order systems — easy to deal with.

(2) Multiple input / Multiple output (MIMO) — easy to handle.

(3) New set of tools (based on Linear Algebra) for control design

$$\overset{n\times 1}{\dot{\underline{x}}} = \overset{n\times n}{A}\,\overset{n\times 1}{\underline{x}} + \overset{n\times m}{B}\,\overset{m\times 1}{u}$$

$$\underset{r\times 1}{y} = \underset{r\times n}{C}\,\underset{n\times 1}{\underline{x}} + \underset{r\times m}{D}\,\underset{m\times 1}{u}$$

$$\left[\begin{array}{c}\dot{x}\\ \hline y\end{array}\right] = \underbrace{\left[\begin{array}{c|c} \quad & \quad \\ \hline \quad & \quad \end{array}\right]}_{\text{NOT SQUARE}} \begin{bmatrix}\underline{x}\\ u\end{bmatrix}$$