CMPE-242
Applied Feedback Control

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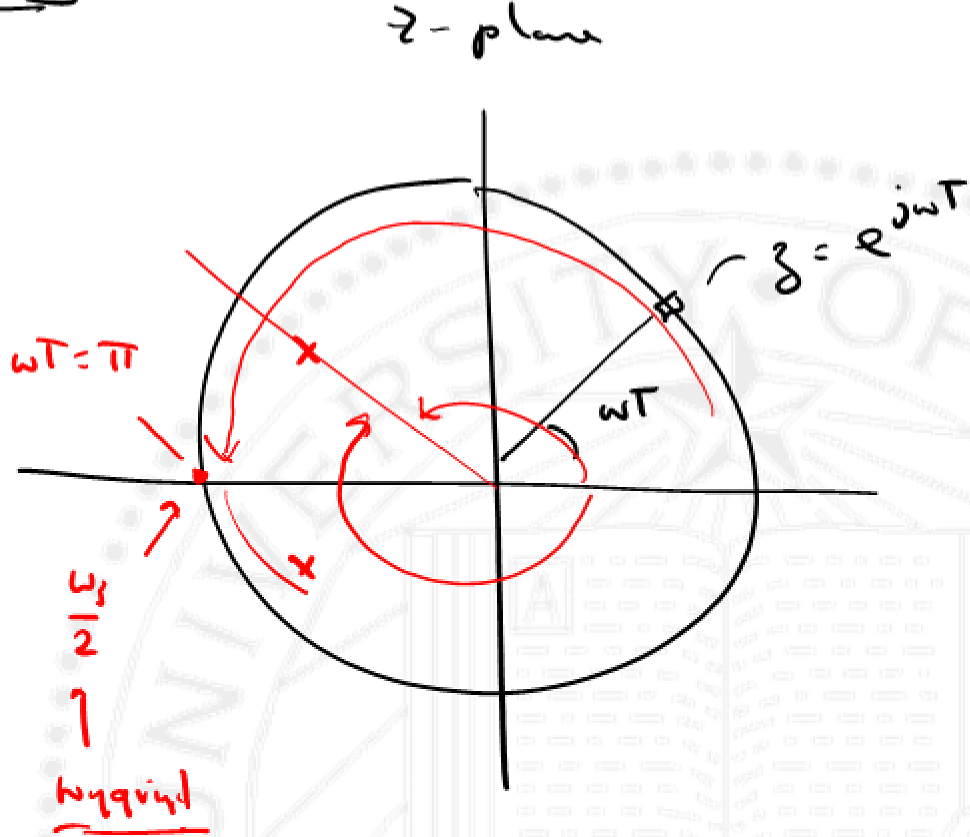
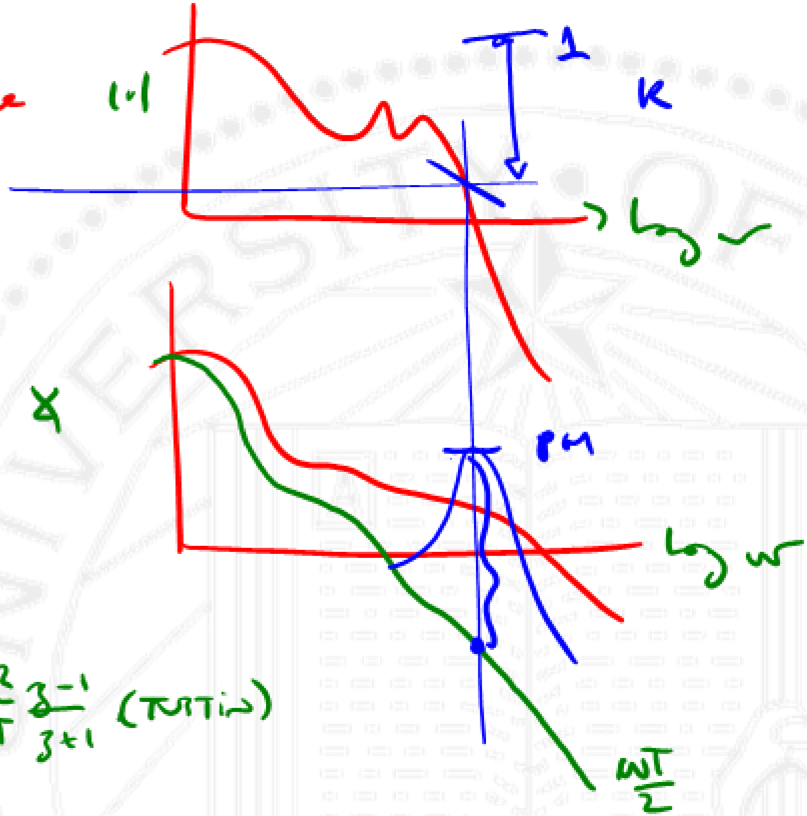
Questions



Don't do discrete bode by hand.

Analog Bode



$K(s) \rightarrow K(z)$

$s = \frac{2}{T}\frac{z-1}{z+1}$ (Tustin)

Prove that z^{-1} is unit delay

$$\mathcal{Z}\{f_{k-1}\} = \sum_{0}^{\infty} f(k-1)\, z^{-k}$$

let $j = k-1$

$-k = -(j+1)$

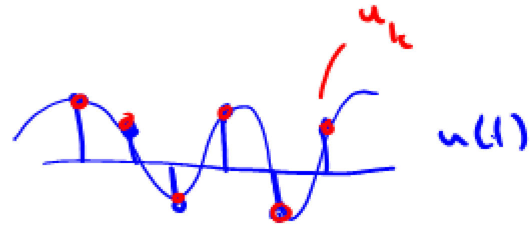
$$= \sum_{0}^{\infty} f(j)\, z^{-(j+1)} = \sum_{0}^{\infty} f(j)\, z^{-j} z^{-1}$$

$$= z^{-1} \underbrace{\sum_{0}^{\infty} f(j)\, z^{-j}}_{F(z)}$$

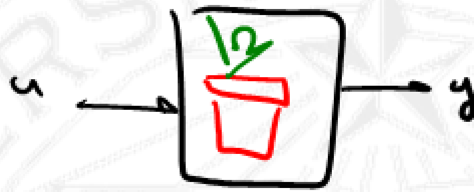
$$\mathcal{Z}\{f_{k-1}\} = z^{-1} F(z)$$

Convolution

$$u_k = u_0\delta_0 + u_1\delta_1 + u_2\delta_2 \cdots$$



$H(z)$ = pulse response of the system



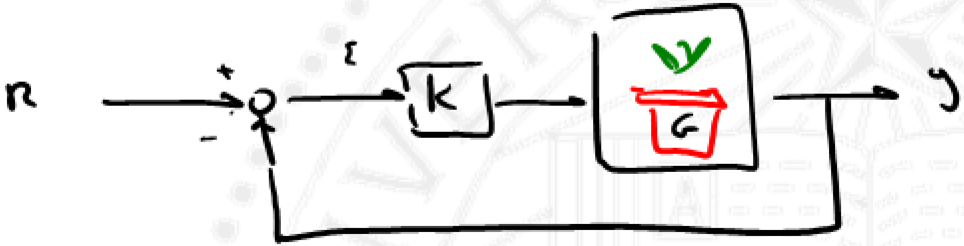
$$y(z) = u_0 H(z) + z^{-1} u_1 H(z) + z^{-2} u_2 H(z) + \cdots$$

$$Y(z) = \underbrace{[u_0 + u_1 z^{-1} + u_2 z^{-2} + u_3 z^{-3} + \cdots]}_{U(z).} H(z)$$

$$\boxed{Y(z) = U(z)\,H(z)}$$

convolution

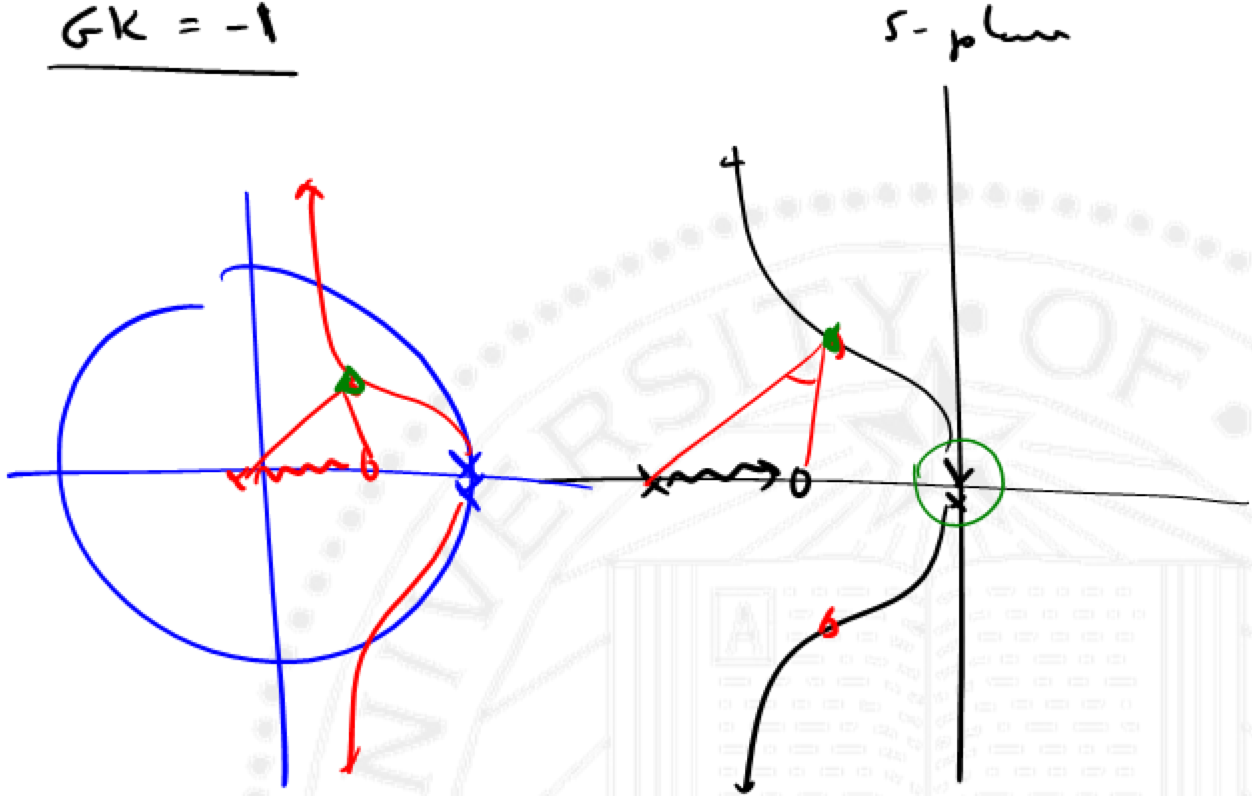
"x"



1.1 1

$$\frac{Y}{R} = \frac{GK}{1+GK} \leftarrow \Delta_{cl} = 1 + GK = 0 \longrightarrow GK = -1 \quad \angle \pm 180°$$

$GK = -1$

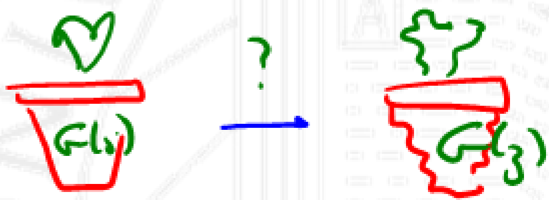


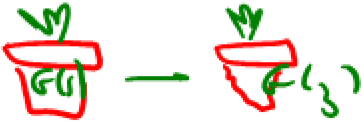
Root locus

Design in root locus:  + PADÉ

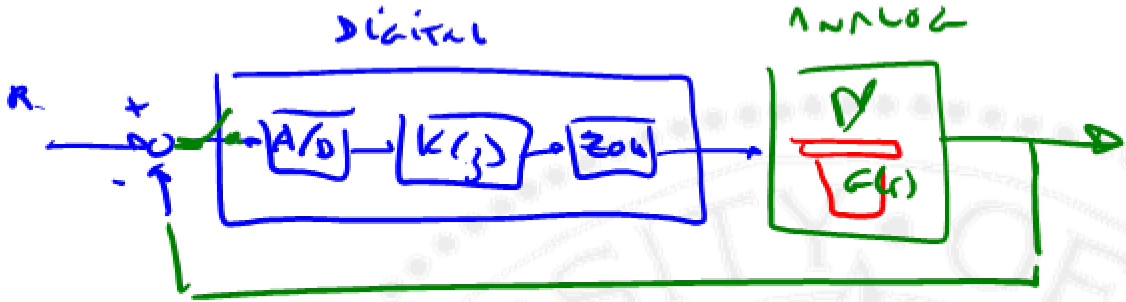
$\underbrace{\text{simulating a } \frac{\Delta T}{2} \text{ delay}}$ $\longrightarrow$ MODIFIED PLANT

$K(s) \longrightarrow K(z)$

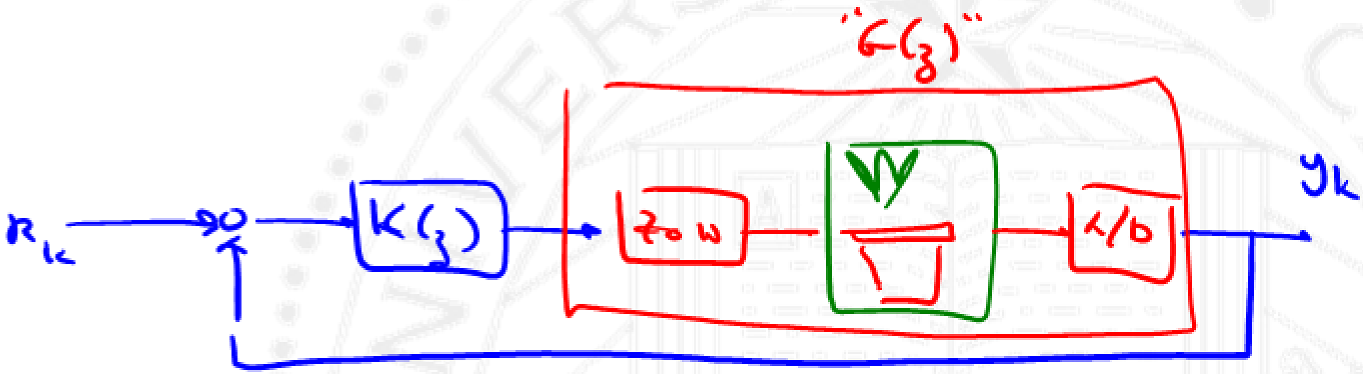




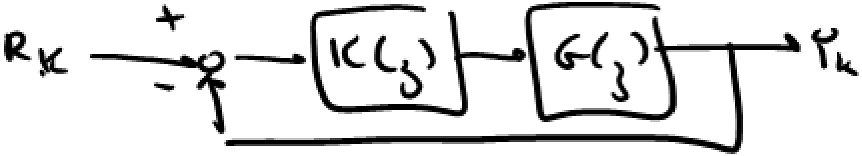
G(s)
G(z)



Digital
Analog
R
+
-
A/D
K(z)
ZOH
G(s)

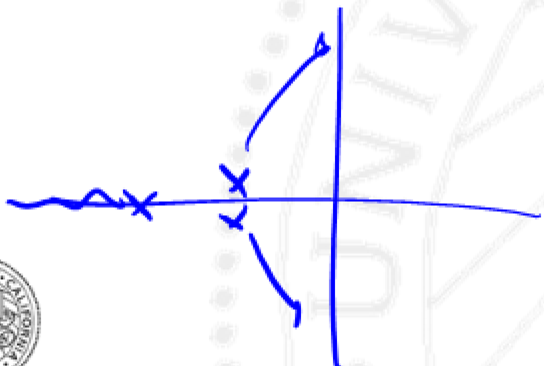


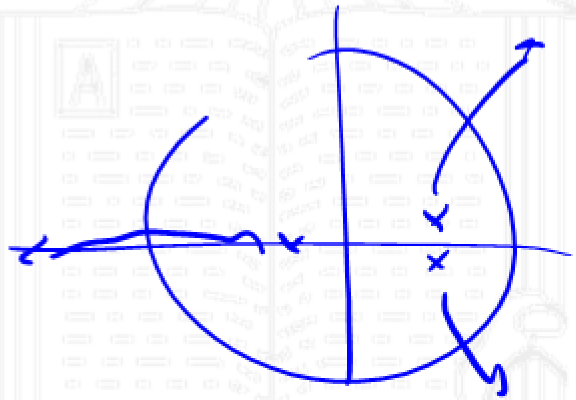
"G(z)"
R_k
K(z)
ZOH
A/D
y_k

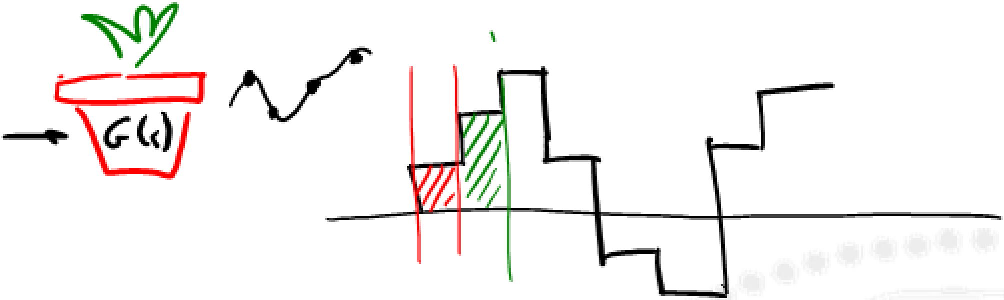


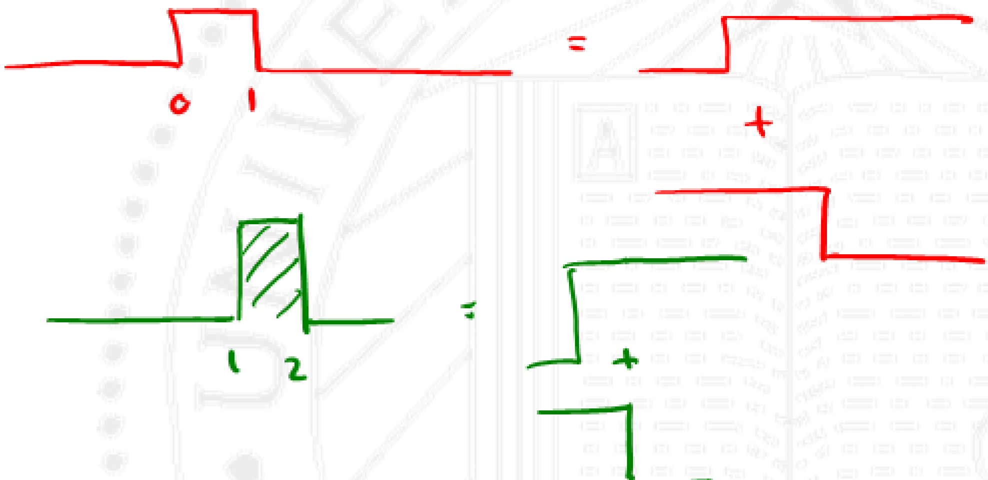
$$\frac{Y_k}{R_k} = \frac{GK(z)}{1+GK(z)} \leftarrow \Delta_{cl} = \emptyset \rightleftharpoons GK(z) = -1 \begin{cases} |\cdot| = 1 \\ \angle 180 \end{cases}$$

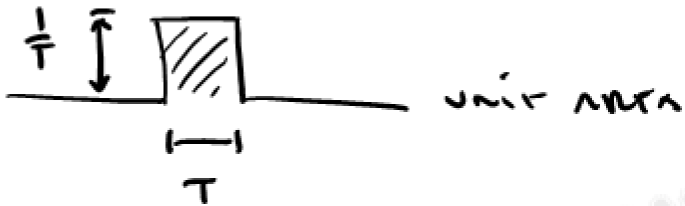
ROOT LOCUS RULES ARE UNCHANGED



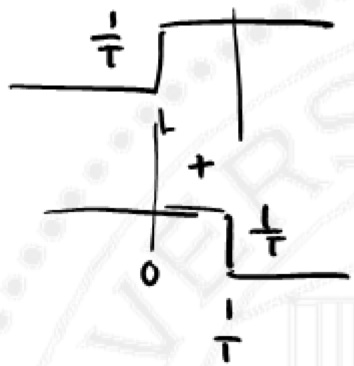



G(s)


0
1
=
+
1
2
=
+



unit area



$$\mathcal{L}\{\text{step } @\ t=0\} = \int_0^\infty 1(t)\, e^{-st}\, dt = \int_0^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st}\Big|_0^\infty = 0 - \frac{-1}{s}$$

$$= \frac{1}{s}$$

$$\mathcal{L}\{\text{step } @\ t=T\} = \int_0^\infty 1(t-T)\, e^{-st}\, dt = \int_T^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st}\Big|_T^\infty$$

$$= 0 - \left(-\frac{e^{-sT}}{s}\right) = \frac{1}{s} e^{-Ts}$$

$$e^{-Ts} \triangleq \text{unit delay}$$

$$\mathcal{L}\{z_{oh}\} = \mathcal{L}\left\{\int_0^t - \int_T^t\right\} = \frac{1}{s} - \left(\frac{e^{-Ts}}{s}\right) =$$

$$\boxed{\mathcal{L}(z_{oh}) = \frac{1 - e^{-sT}}{s}}$$

exact

Approximated $\dfrac{e^{-\frac{T}{2}s}}{}$

$$e^{-\frac{T}{2}\lambda} = 1 - \frac{T}{2}\lambda + \left(\frac{T}{2}\right)^2 \frac{\lambda^2}{2!} - \left(\frac{T}{2}\right)^3 \frac{\lambda^3}{3!} + \dots$$

$$\frac{1}{T}\mathcal{L}\{zoh\} = \frac{1}{T}\,\frac{1-e^{-Ts}}{s} = \frac{\cancel{1} - \left(\cancel{1} - \cancel{Ts}^{\,1} + \frac{T^{\cancel{2}}s^2}{2!} - \frac{T^{\cancel{3}\,2}\cancel{s^3}^{\,2}}{3!}\right)}{\cancel{Ts}}$$

$$= 1 - \frac{T\lambda}{2!} + \frac{T^2\lambda^2}{3!} - \frac{T^3\lambda^3}{4!} + \dots$$

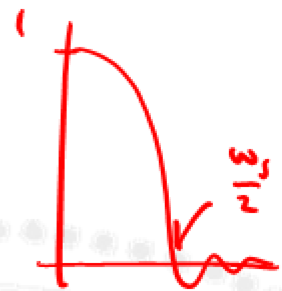
$$e^{-\frac{T}{2}s} \text{ in bode } \begin{cases} |\cdot| = 1 \ \forall \omega \\ \measuredangle = -\frac{\omega T}{2} \end{cases}$$

$$\mathcal{L}\{g_{oh}\}\Big|_{s=j\omega} = \frac{1}{T}\,\frac{1-e^{-j\omega T}}{j\omega} = \frac{1}{T}\,\frac{e^{-j\frac{\omega T}{2}}e^{j\frac{\omega T}{2}} - e^{-j\omega T}}{j\omega}$$

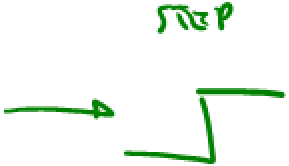
$$= e^{-j\frac{\omega T}{2}}\left[\frac{e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}}{2j}\right]\frac{2}{\omega T}$$

$$= e^{-j\frac{\omega T}{2}}\left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}\right] = e^{-j\frac{\omega T}{2}}\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$$

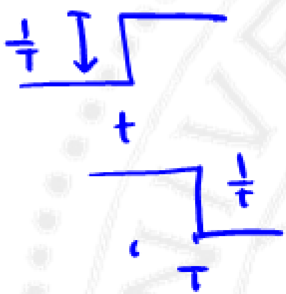
$$\mathcal{L}\{g_0 h\}\Big|_{s=j\omega} = \operatorname{sinc}\left(\frac{\omega T}{2}\right) e^{-j\frac{\omega T}{2}}$$



$$\angle : \frac{\omega T}{2} = \phi$$



$$\frac{G(s)}{s}$$

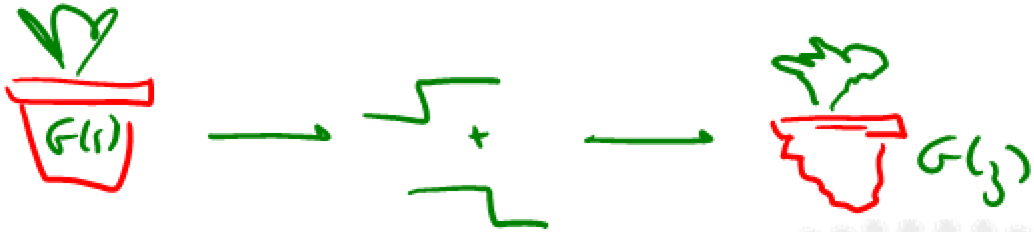


$$\Rightarrow \frac{1}{T}\frac{G(s)}{s}$$

$$-\frac{z^{-1}}{T}\frac{G(s)}{s}$$

$$\frac{1}{T}\left[1-z^{-1}\right]\frac{G(s)}{s}$$

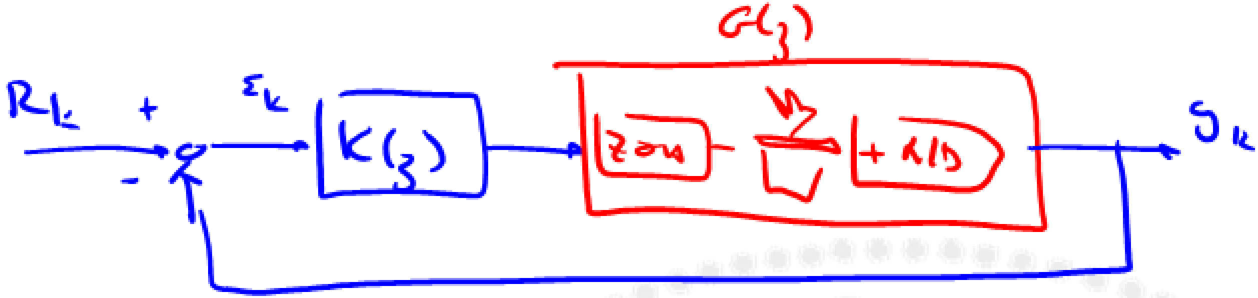
$$\boxed{G(z)} \leftarrow \boxed{\left[\frac{z-1}{Tz}\right]\frac{G(s)}{s}}$$



$$G(z) = \mathcal{Z}\left\{\frac{z-1}{Tz}\frac{G(s)}{s}\right\}$$

$$\boxed{G(z) = \frac{z-1}{Tz}\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}}$$

c2d(sys, T, 'zoh')



$$G(z) = \frac{z-1}{Tz} \, Z\left\{ \frac{G(s)}{s} \right\}$$

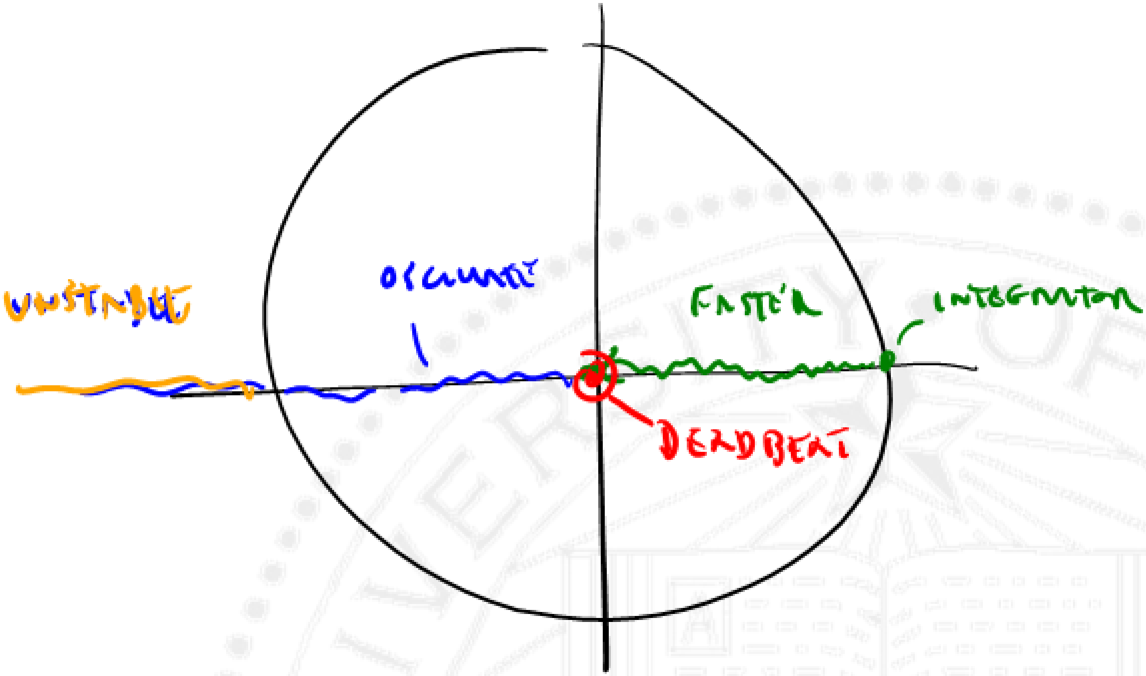
$$\frac{Y_k}{R_k} = \frac{GK}{1+GK} \longleftarrow \Delta(z) = 0 \longrightarrow GK = -1 \quad |\cdot| = 1, \ \angle \pm 180$$

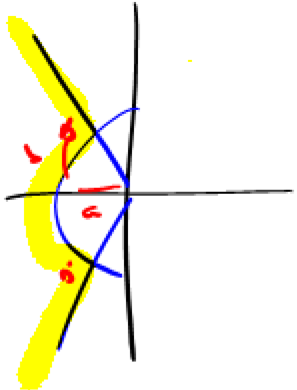
normal

$\lambda_{des} \longrightarrow$

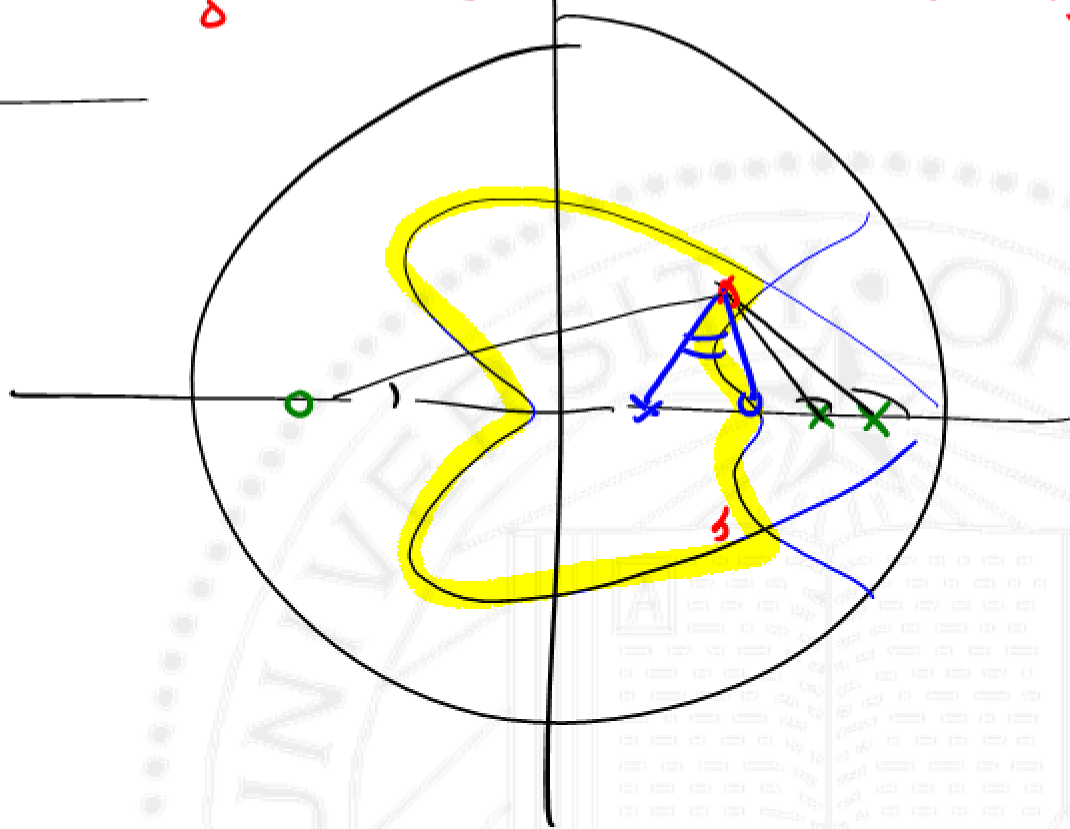
discrete

z_{des}


UNSTABLE
OSCILLATE
FASTER
INTEGRATOR
DEADBEAT



$$z = e^{sT} = e^{-aT} e^{\pm jbT} = e^{-aT}\left(\cos(bT) \pm j\sin(bT)\right)$$



$$u_k \longrightarrow \boxed{H(z)} \longrightarrow y_k$$

$H(z) \triangleq$ unit pulse response from zero i.c.'s

STEP :

$$\frac{z}{z-1} H(z)$$

FVT :

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$$

$$\lim_{k \to \infty} f(kT) = \lim_{z \to 1} \frac{z-1}{z} F(z)$$

$$\text{DC gain} = \lim_{s\to 0} s\frac{F(s)}{s} = F(s)\Big|_{s=0}$$

$$\lim_{z\to 1} \frac{z-1}{z}\cdot\frac{z}{z-1}F(z) = F(z)\Big|_{z=1}$$

$$H(z) = \frac{3z^2 + 2z + 1}{4z^3 + 5z^2 + 2z + 4} \longrightarrow \text{DC gain: } \frac{6}{15}$$

in matlab add in leading 0's.

$$\frac{z+3}{z^2+2z+1}$$

tf([1 3],[1 2 1])

tf([0 1 3],[1 2 1])

$$z = e^{j\omega T} = \cos(\omega T) + j\sin(\omega T)$$

$$G(z) = \frac{z-1}{z}\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

step response of plant

$G(z)$ is <u>exact</u> for a 'zoh'.

Analysis / Design directly in z-domain

DIGITAL CONTROL

ZOH ~ $e^{-\frac{T}{2}s}$ ½ sample delay

Bode → $\Delta\phi = -\frac{\omega T}{2}$

Root Locus → PADÉ

$$e^{-\frac{T}{2}s} \simeq -\frac{\left(s-\frac{4}{T}\right)}{s+\frac{4}{T}}$$

0

USE ANALOG TOOLS → $K(s)$

$$K(z) = K(s)\Big|_{s=\text{TUSTIN}}$$

$$s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$G(z) = \frac{z-1}{Tz}\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

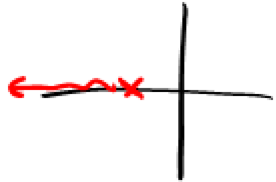
$G(z)$ = 'exact'

$$\frac{Y}{R} = \frac{GK}{1+GK} \quad \therefore\ GK = -1 \quad |\cdot|=1,\ \angle 180$$

DESIGN IN z

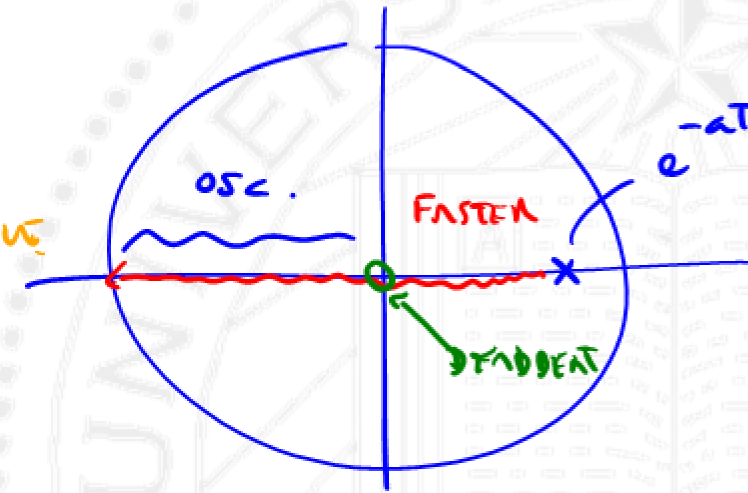
$$z_{des} = e^{s_{des}T}$$

$$G(s) = \frac{a}{s+a}$$



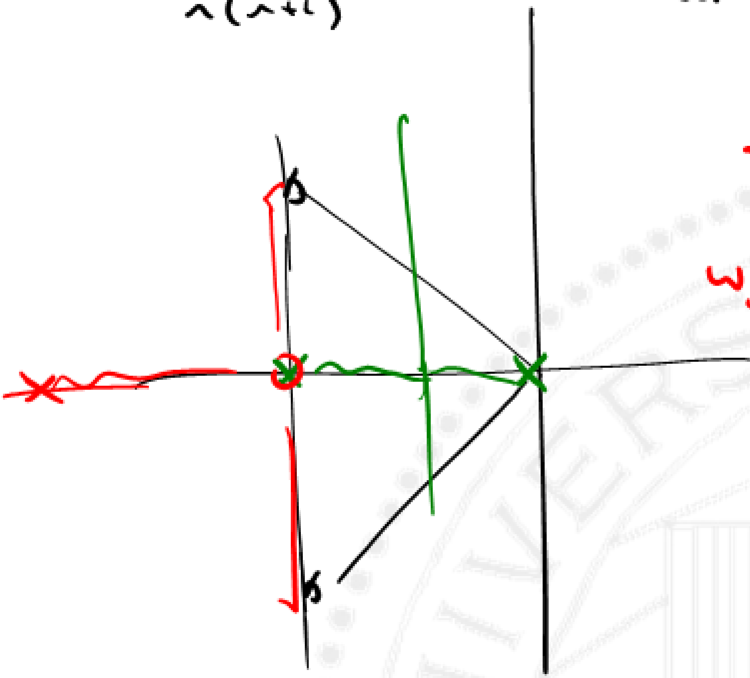
$$\mathcal{Z}\left\{\frac{a}{s(s+a)}\right\} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$$

$$G(z) = \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{\cancel{z}(1-e^{-aT})}{\cancel{(z-1)}(z-e^{-aT})} = \frac{1-e^{-aT}}{z-e^{-aT}}$$



$G(s) = \frac{1}{s(s+1)}$

$s_{des} = -1 \pm j$



$T = 1$ sec.

$\omega_s = 1$ Hz $= 2\pi$ rad/sec.

$\frac{\omega_s}{2} = \pi$.

$$z_{des} = e^{s_{des}T} = e^{-1 \pm j} = e^{-1} e^{\pm j}$$

$$z_{des} = 0.2 \pm 0.3j$$

Find $G(z) = \mathrm{zoh}\{G(s)\} = \frac{z-1}{z} Z\left\{\frac{G(s)}{s}\right\}$

$\longrightarrow$ c2d(G, T, 'zoh')

$$G(z) = \frac{z-1}{z} Z\left\{\frac{1}{s^2(s+1)}\right\} = \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{\cancel{z}}{(z-1)^{\cancel{2}}} \left[\frac{e^{-T} z + (1-2e^{-T})}{z-e^{-T}}\right]$$

$T = 1s$

$$= 0.37 \frac{(z+.72)}{(z-1)(z-0.37)}$$

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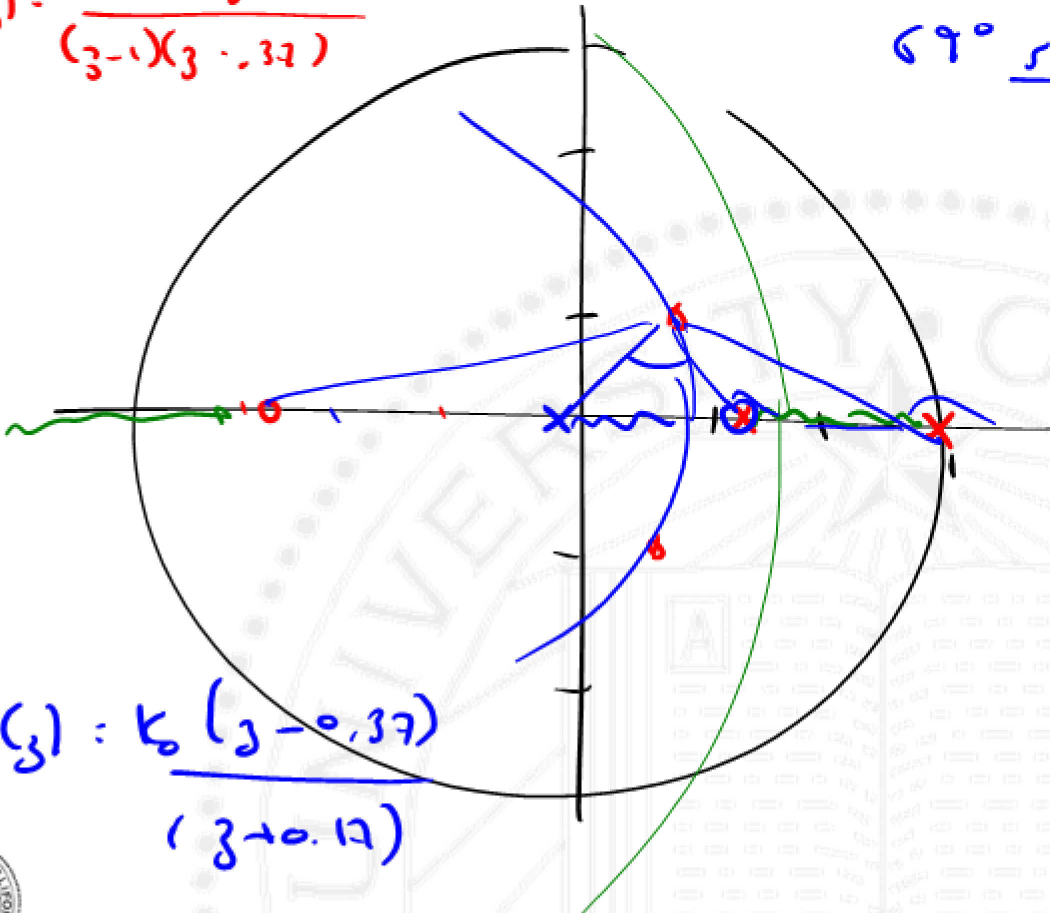
$$G(z) = \frac{0.37\,(z+.72)}{(z-1)(z-.37)}$$

69°



$$K(z) = K_0\,\frac{(z-0.37)}{(z+0.17)}$$

$$GK(z) = K_0 \frac{(0.37)(z+0.72)\cancel{(z-0.37)}}{(z+0.17)(z-1)\cancel{(z-0.37)}}$$

$$z = 0.2 \pm 0.3j$$

$$\therefore \underline{K_0 \cong 1.2}$$