MATLAB CODE

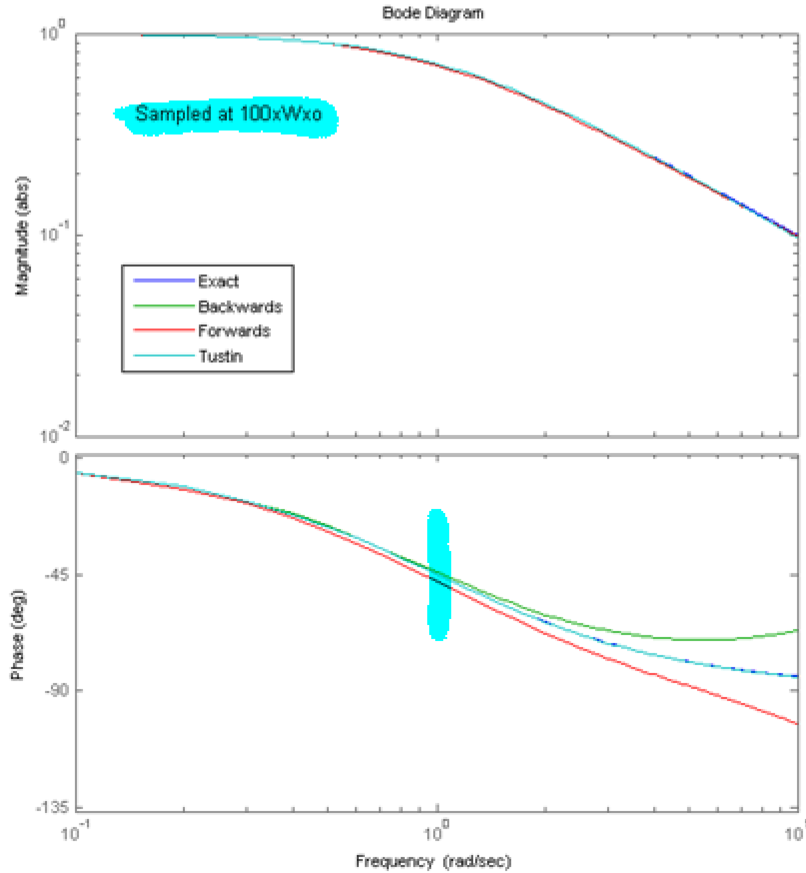
$$\frac{z+2}{1\ z^2+3z+5}$$

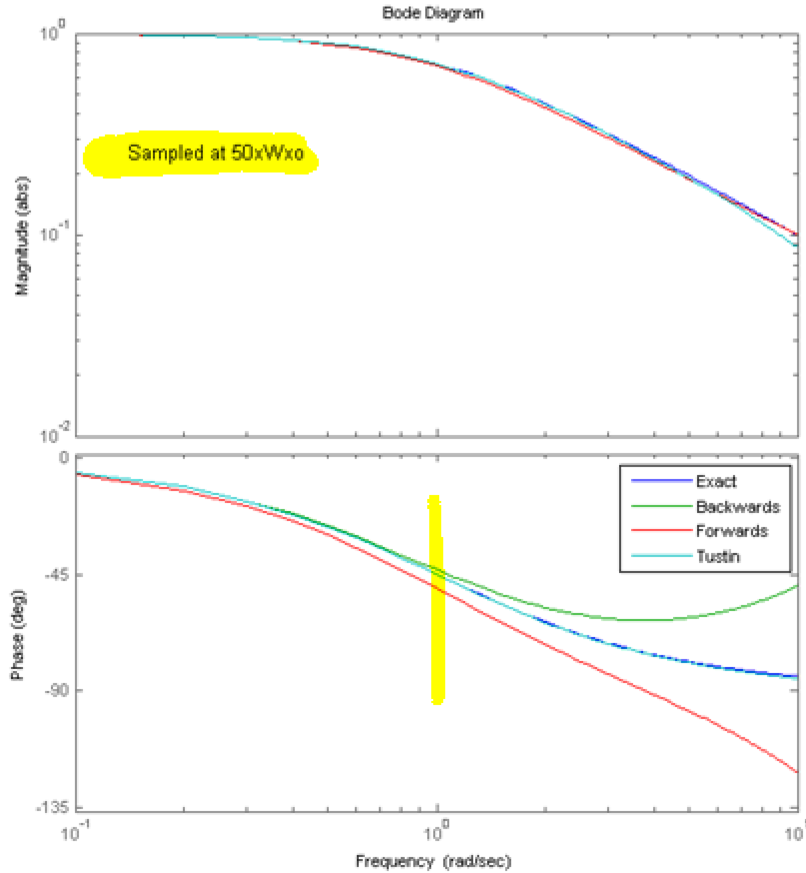
[0, 1, 2], [1 3 5]

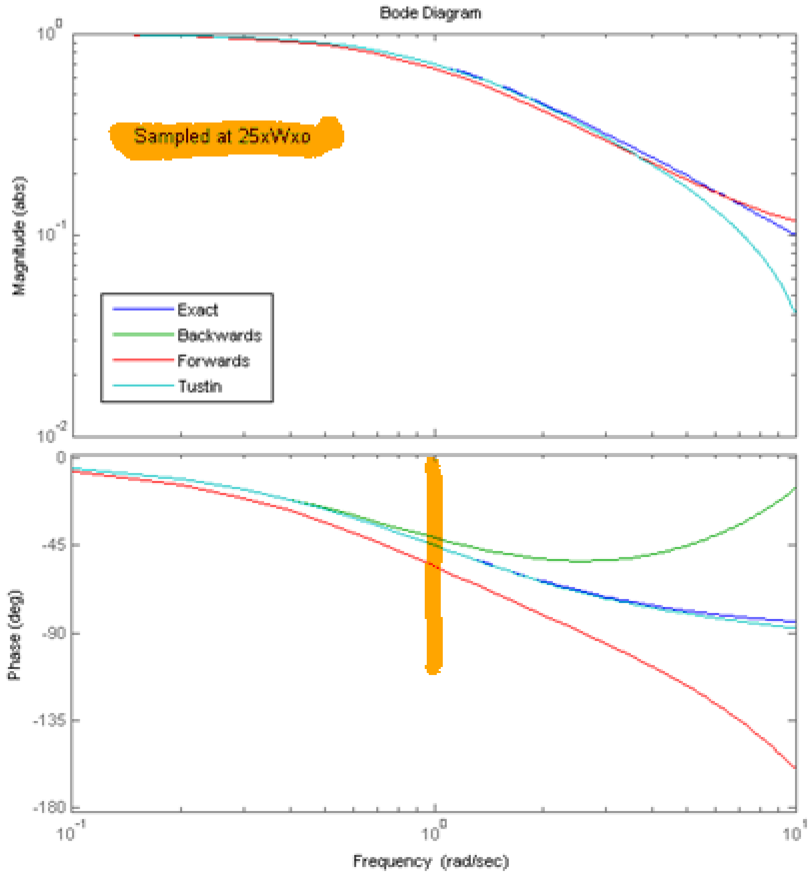
analog

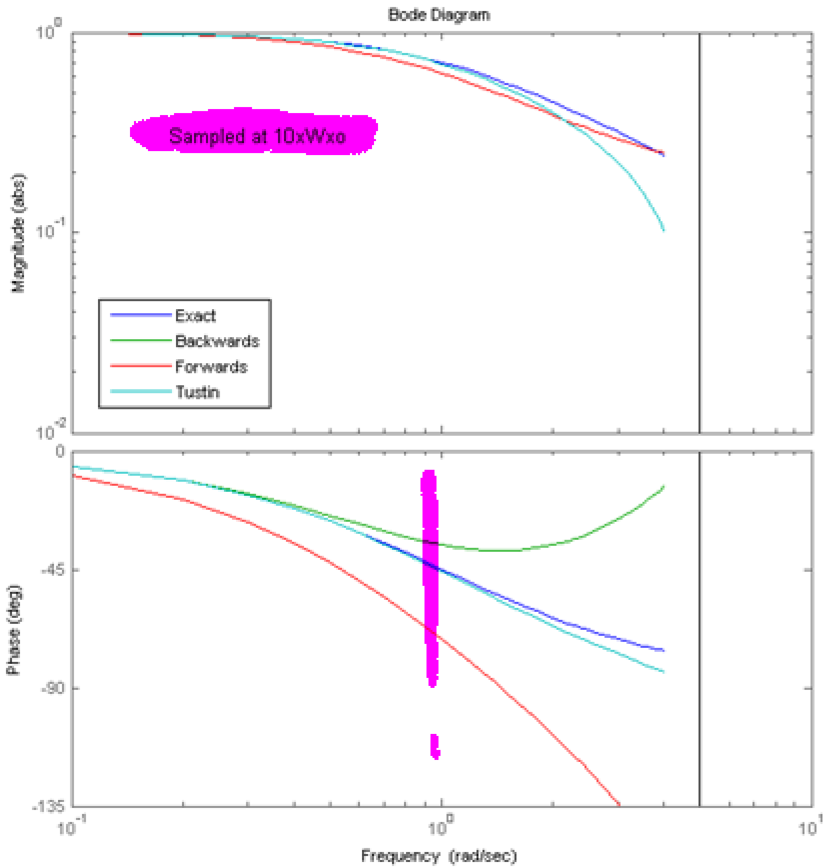
```
w=[0.1:.1:10]';
T=2*pi/100;
a=1;
n=a;
d=[1 a];
n1=[a*T/(1+a*T) 0];
d1=[1 -1/(1+a*T)];
n2 = [0 a*T/(1+a*T)];
n3 = a*T/(2+a*T)*[1 1];
d3 = [1 -(1-a*T/2)/(1+a*T/2)];
bode(n,d,w); hold on
dbode(n1,d1,T,w);
dbode(n2,d1,T,w);
dbode(n3,d3,T,w);
legend('Exact','Backwards','Forwards','Tustin');
```

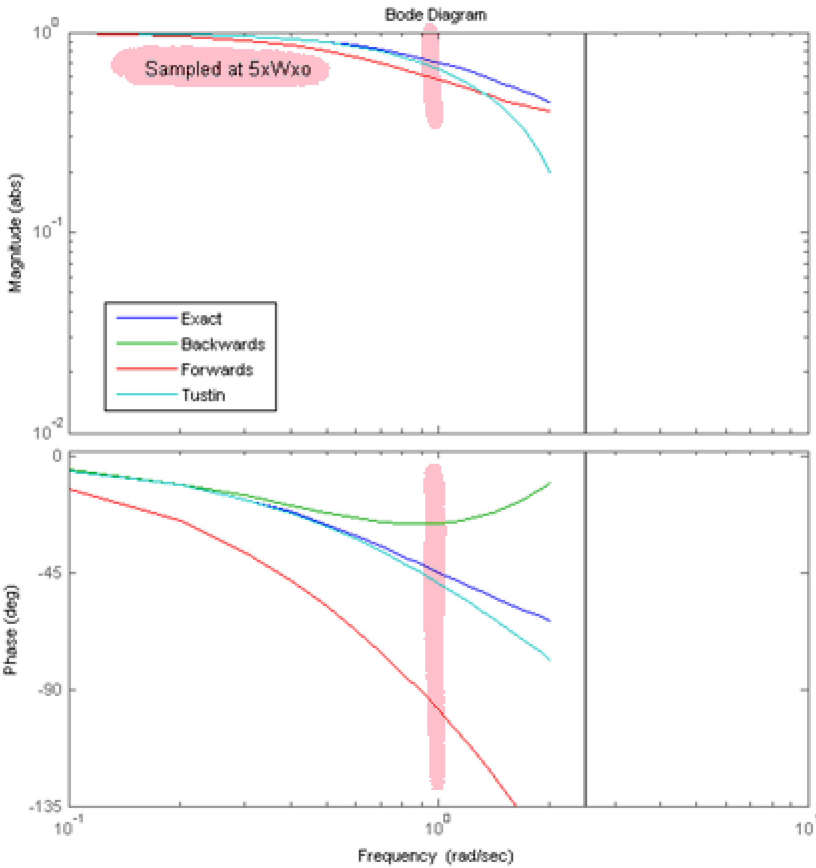
Tustin


Bode Diagram
Sampled at 100xWxo
Magnitude (abs)
10^0
10^{-1}
10^{-2}
Exact
Backwards
Forwards
Tustin
Phase (deg)
0
-45
-90
-135
10^{-1}
10^0
10^1
Frequency (rad/sec)


Bode Diagram
Sampled at 50xWxo
Magnitude (abs)
Phase (deg)
Exact
Backwards
Forwards
Tustin
Frequency (rad/sec)
10^{0}
10^{-1}
10^{-2}
0
-45
-90
-135
10^{-1}
10^{0}
10^{1}


Bode Diagram
Sampled at 25xWxo
Magnitude (abs)
Phase (deg)
Exact
Backwards
Forwards
Tustin
Frequency (rad/sec)
10^0
10^{-1}
10^{-2}
0
-45
-90
-135
-180
10^{-1}
10^0
10^1


Bode Diagram
Sampled at 10xWxo
Magnitude (abs)
Exact
Backwards
Forwards
Tustin
Phase (deg)
Frequency (rad/sec)


Bode Diagram
Sampled at 5xWxo
Magnitude (abs)
Phase (deg)
Frequency (rad/sec)
Exact
Backwards
Forwards
Tustin

CMPE-242
Applied Feedback Control

Gabriel Hugh Elkaim

Announcement — E2-215 @ 11:15

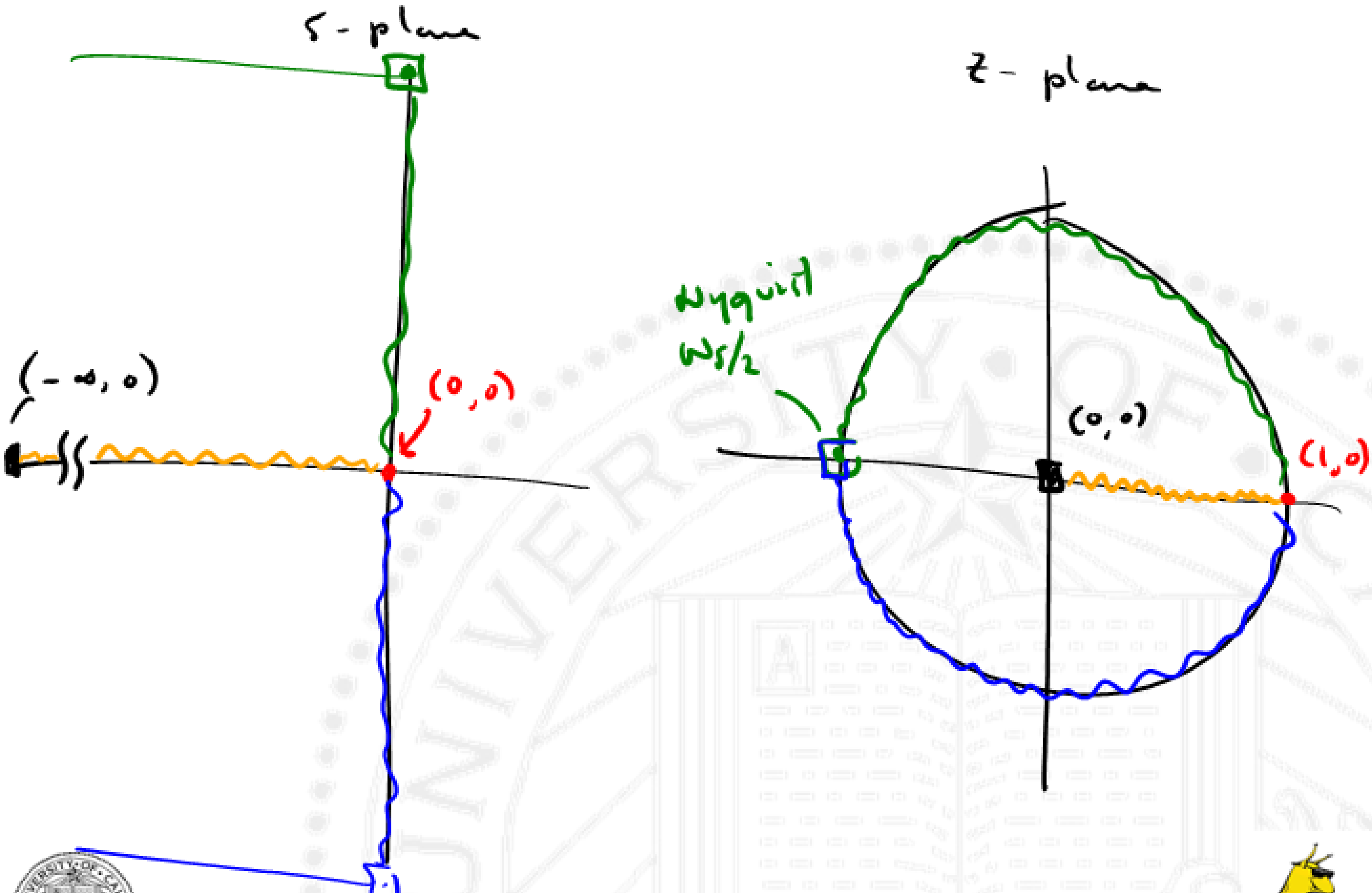
Questions

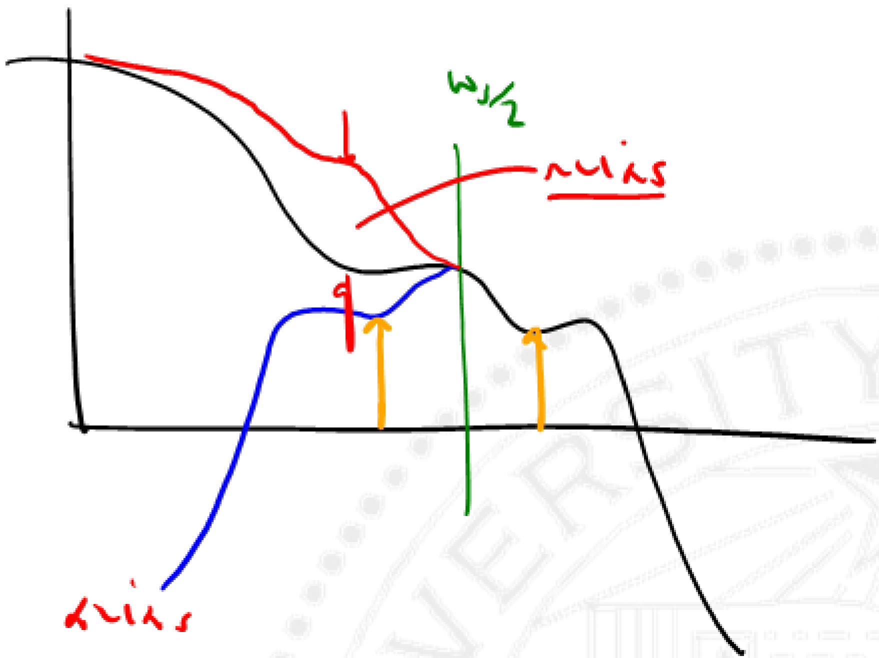
Digital Equivalents

Digital Plant

Bode – $\Delta\phi = -\frac{\omega T}{2}$

R.L – Add a POLE (provide $0°$)

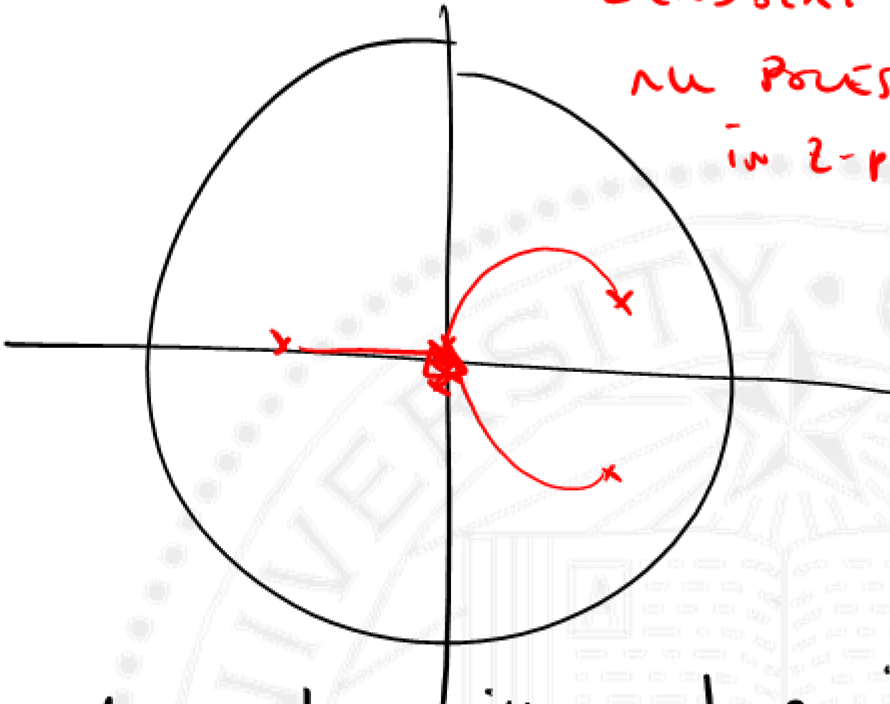

s - plane
(-∞, 0)
(0,0)
z - plane
nyquist
ωs/2
(0,0)
(1,0)



ωs/2

"DEADBEAT CONTROL"

All poles @ origin in z-plane



n^{th} order system will go to zero in n-steps.

$$\varepsilon \rightarrow \boxed{K(z)} \rightarrow u$$

$$\frac{\varepsilon}{u} = z + \frac{1}{2} \rightarrow \varepsilon_k = z u_k + \frac{1}{2} u_k$$

$$\varepsilon_k = u_{k+1} + \frac{1}{2} u_k$$

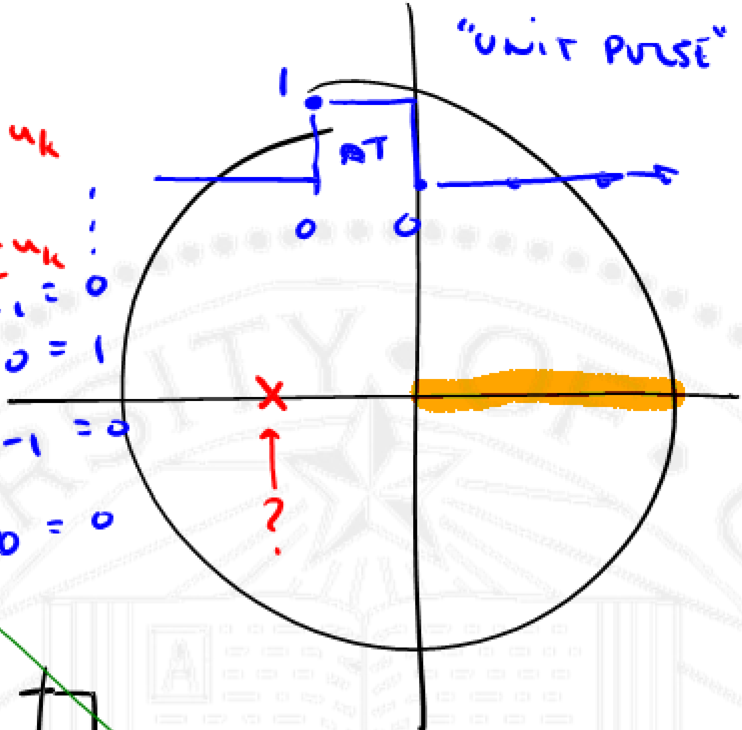
$$\varepsilon_1 = 0$$

$$\varepsilon_0 = 1$$

$$\varepsilon_{-1} = 0$$

$$u_0 = 0$$

$$u_{k+1} = \varepsilon_k - \frac{1}{2} u_k$$



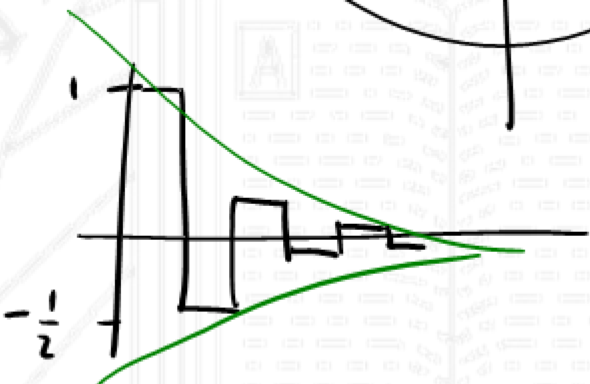
$$u_0 = 0$$

$$u_1 = 1 - \frac{1}{2}(0) = 1$$

$$u_2 = 0 - \frac{1}{2}(1) = -\frac{1}{2}$$

$$\frac{1}{4}$$

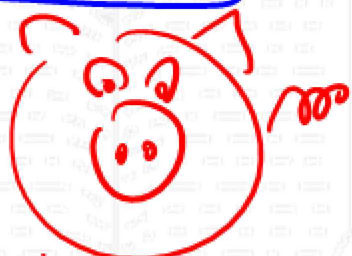
$$-\frac{1}{8}$$



$$s = \frac{1}{T}\ln(z) \longrightarrow z = -\frac{1}{2} \longrightarrow z = \frac{1}{2}e^{\pm\pi j}$$

$$s = \frac{1}{T}\ln\left(\frac{1}{2}e^{\pm\pi j}\right) = \frac{1}{T}\ln\left(\frac{1}{2}\right)\ln\left(e^{\pm\pi j}\right)$$

$$\boxed{s = \frac{1}{T}\ln\left(\frac{1}{2}\right) \pm \pi j}$$

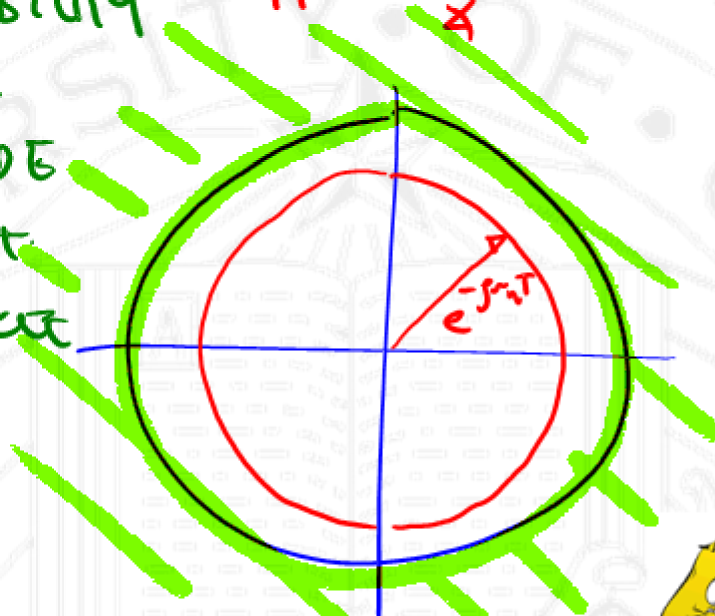


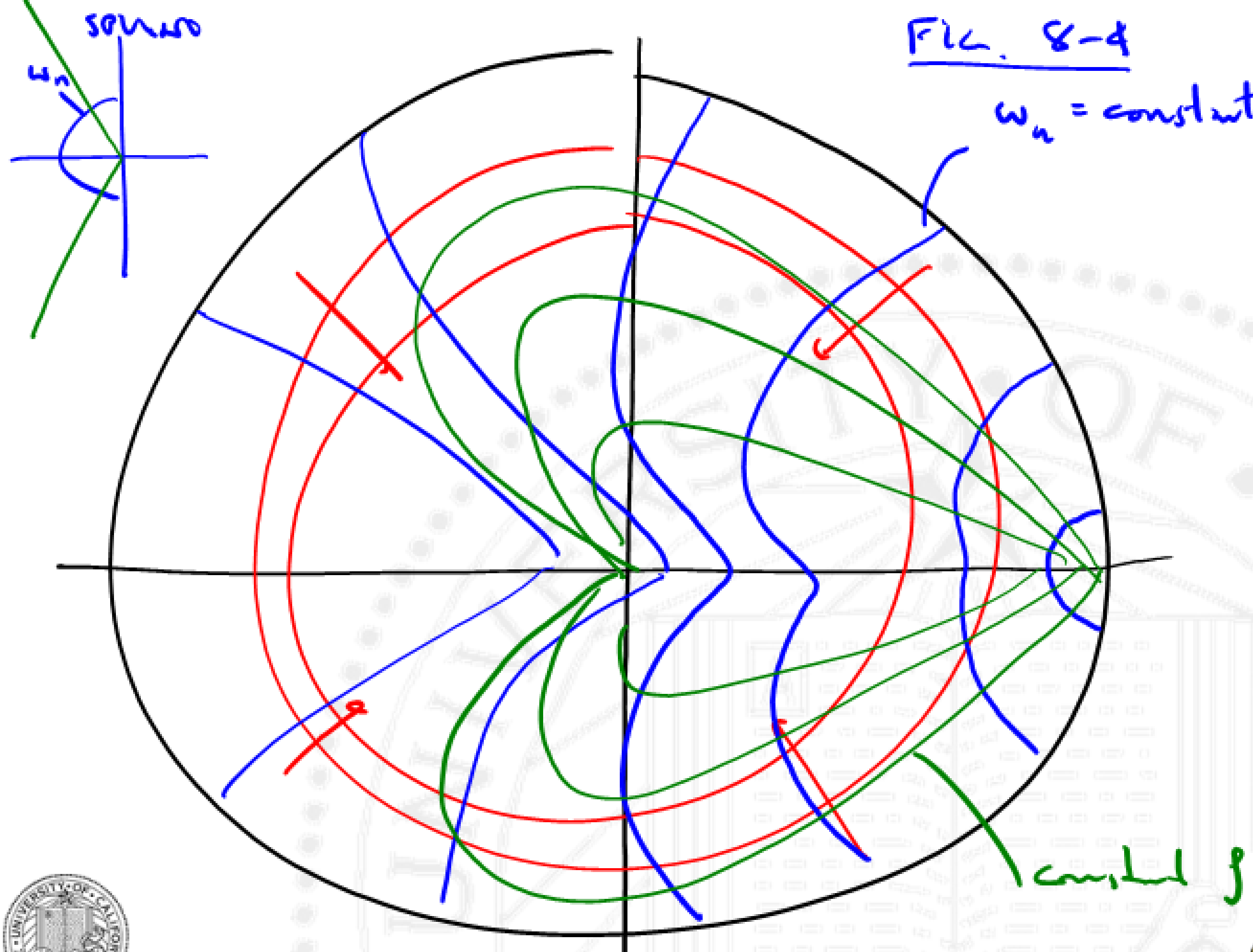
NOT Kosher

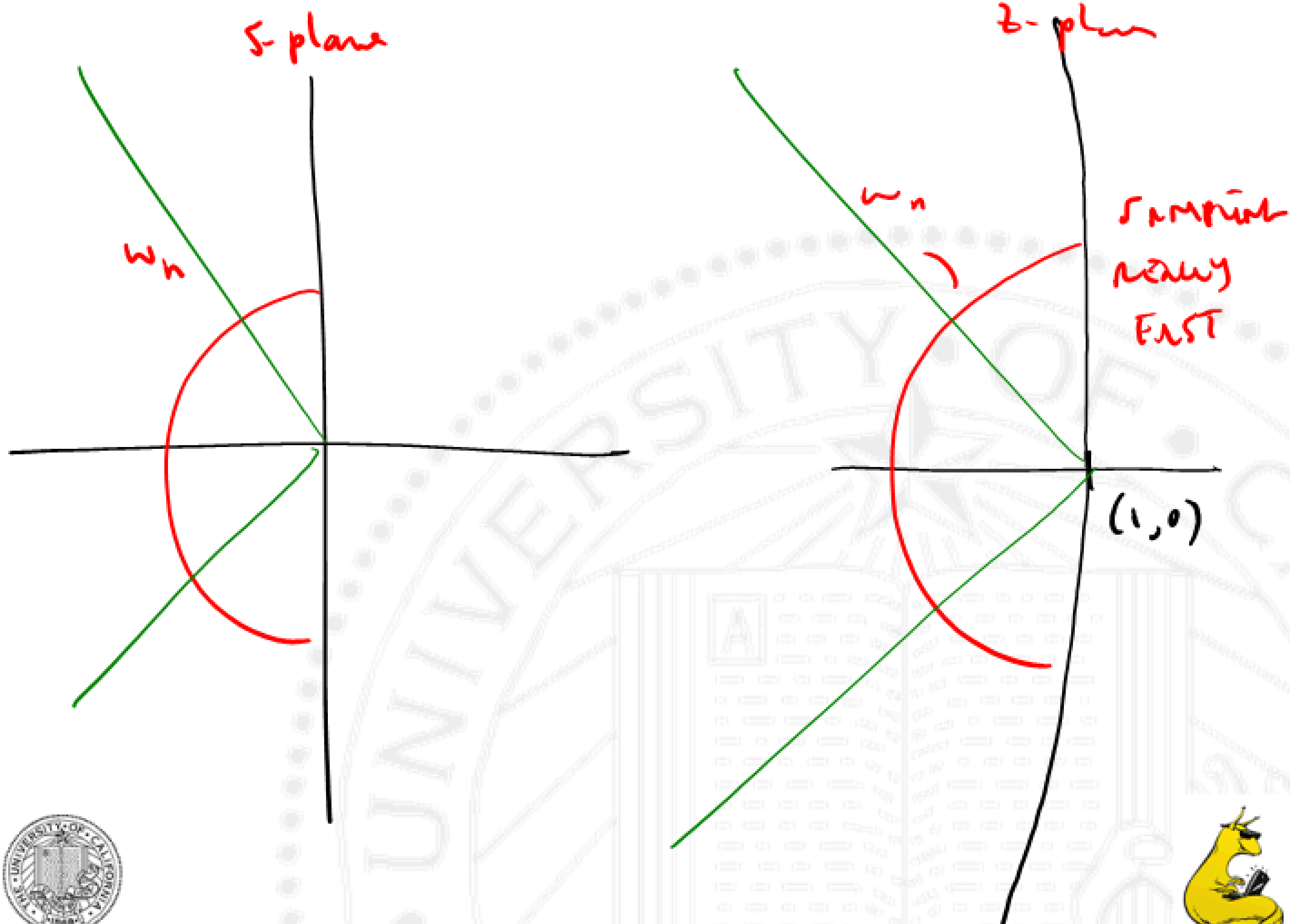
$$s = \frac{1}{T}\ln(z) \longleftrightarrow z = e^{Ts}$$

$$s = -a + bj \longleftrightarrow z = e^{(-a+bj)T} = \underbrace{e^{-aT}}_{|\cdot|}\ \underbrace{e^{jbT}}_{\angle}$$

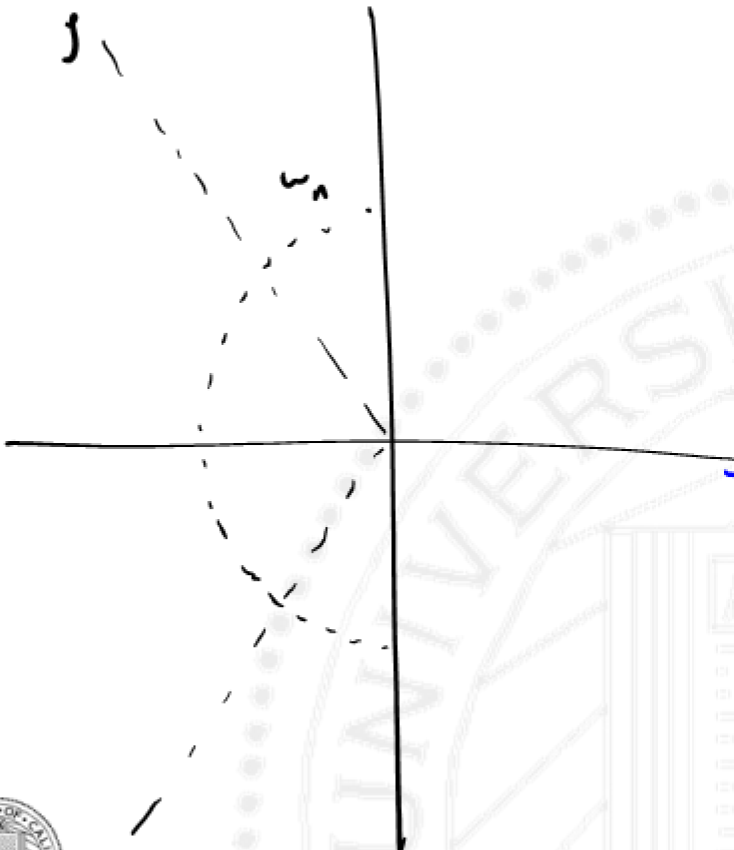
STABILITY
↓
INSIDE
UNIT
CIRCLE



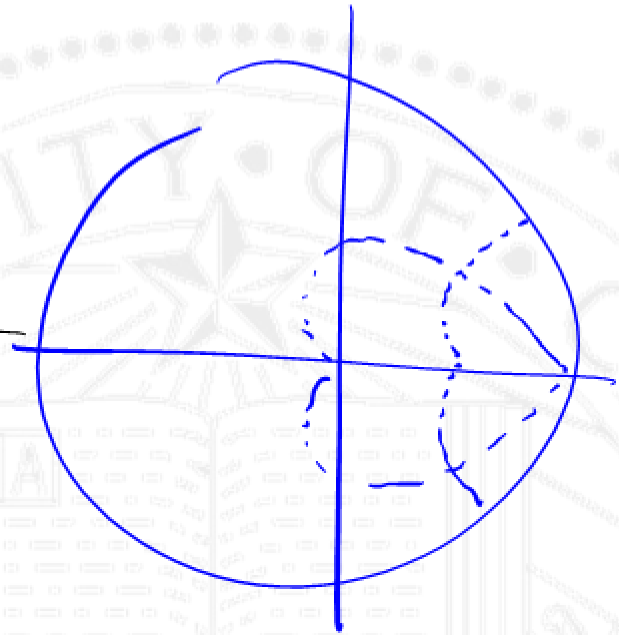

s domain
ω_n
Fig. 8-4
ω_n = constant
constant ζ

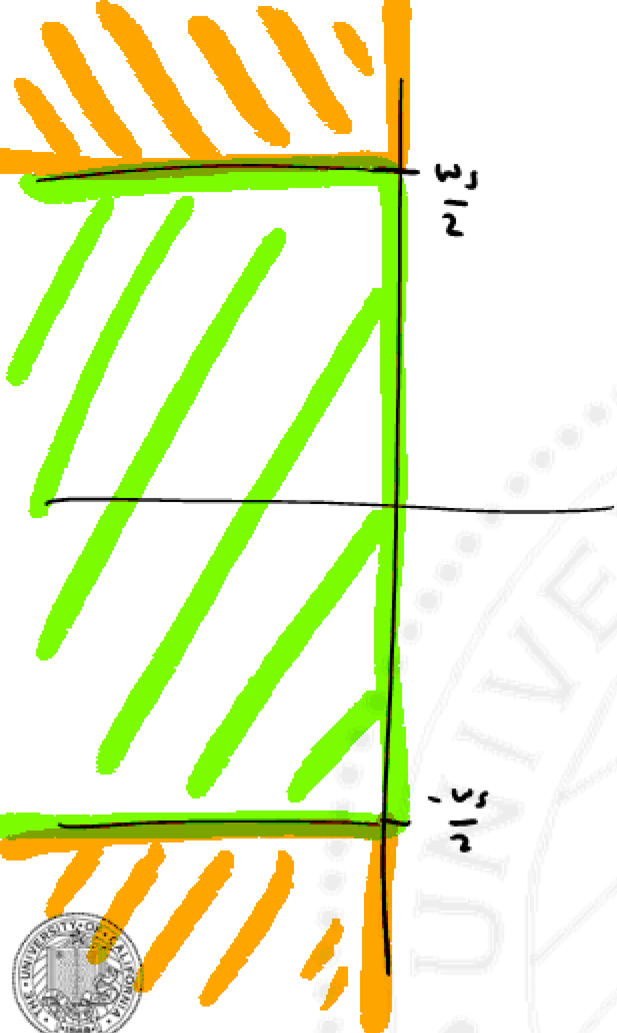

s-plane
ω_n
z-plane
ω_n
SAMPLING
REALLY
FAST
(1,0)

sgrid(ω_n, ζ)

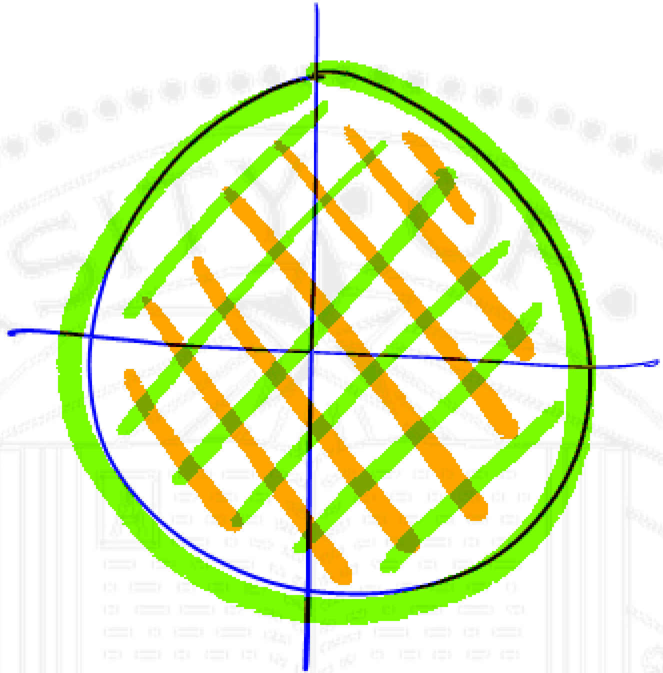


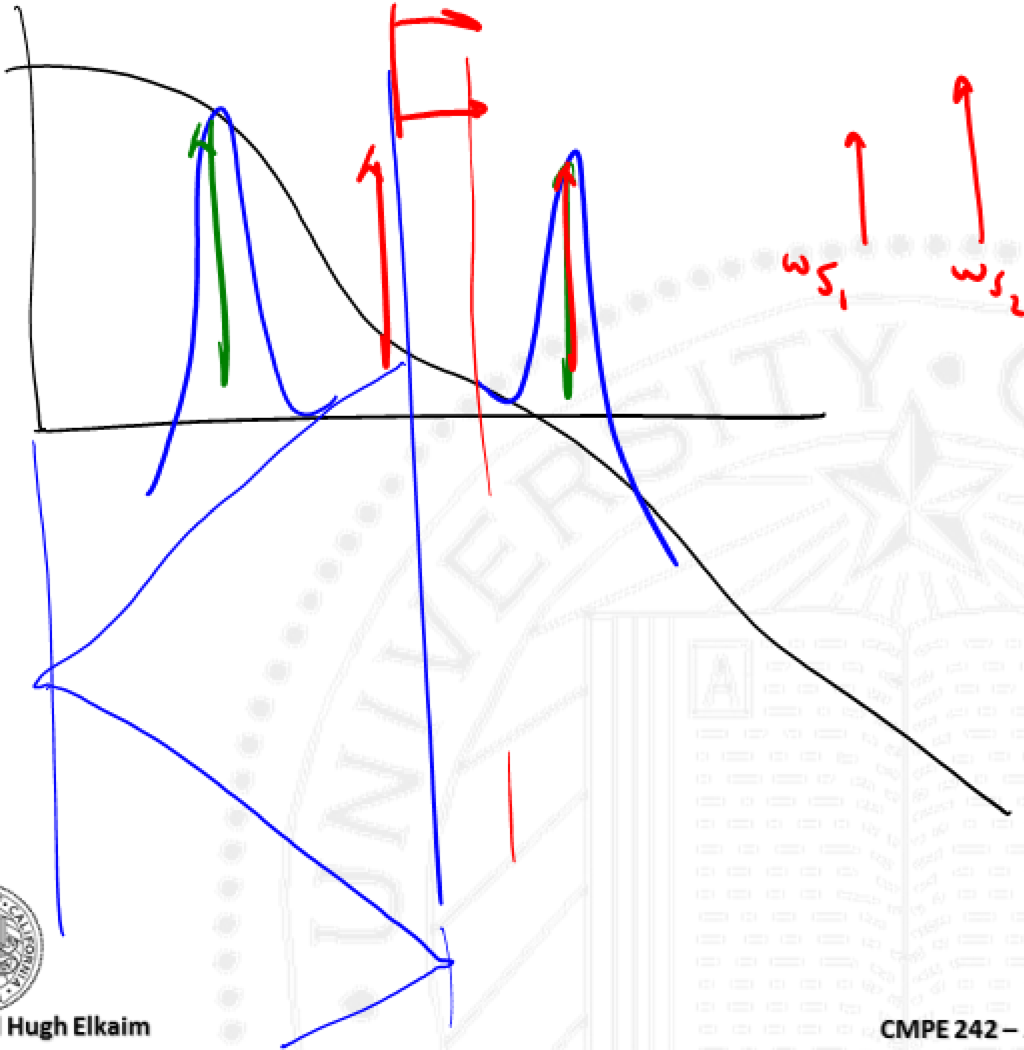
zgrid(ω_n, ζ)





$\frac{\omega_s}{2}$
$-\frac{\omega_s}{2}$





ω_{s_1}
ω_{s_2}

DESIGN IN ANALOG WORLD

ACCOUNT FOR $\frac{\Delta T}{2}$ DELAY

$\downarrow$

CONVERT $K(s) \rightarrow K(z)$ | TUSTIN

DIGITAL CONTROL VIA MAPPING

(1) Account for $\frac{\Delta T}{2}$ delay
- BODE → $\Delta\phi = -\frac{\omega \Delta T}{2}$
- R.L → PADÉ (0°)

(2) Sample @ 20-30x ω_{x0} (ω_n)

(3) Design $K(s)$ using analog tools

(4) Convert $K(s) \rightarrow K(z)$ using TUSTIN

(5) Specific freq of interest (i.e. ω_{x0}) — prewarp

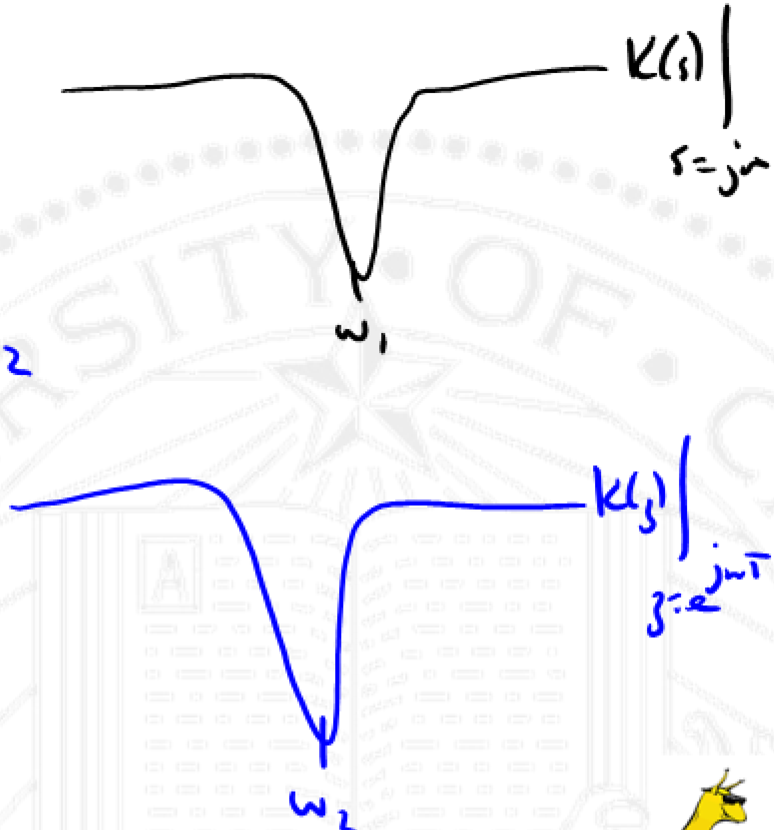
Prewarping

$K(s)$ – "notch"

$\Downarrow$ TUSTIN

$K(z)$

$\omega_1 \neq \omega_2$



$$K(s) \longrightarrow K(z)$$

$$s = j\omega_{des} = \frac{2}{T}\,\frac{e^{j\omega_1 T} - 1}{e^{j\omega_1 T} + 1}$$

$$e^{j\omega_1 T} = \cos(\omega_1 T) + j\sin(\omega_1 T)$$

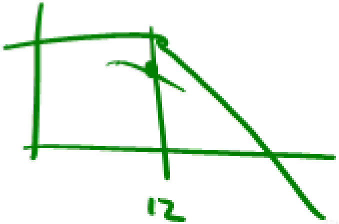
$$j\omega_{des} = \frac{2}{T}\tan\left(\frac{\omega_1 T}{2}\right)$$

$$\boxed{K(z) = K(s)\Big|_{s = \left[\frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}\right]\frac{z-1}{z+1}}}$$

Tustin w/ Prewarp

$$s = \left[\frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}\right]\frac{z-1}{z+1}$$

$$K(s) = \frac{12}{s+12}$$



$$25\,\text{Hz} \rightarrow T = \frac{1}{25}$$

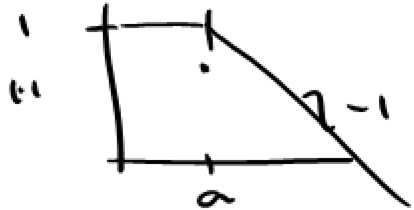
$$K(z) = K(s)\Big|_{s = \left[\frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}\right]\frac{z-1}{z+1}}$$

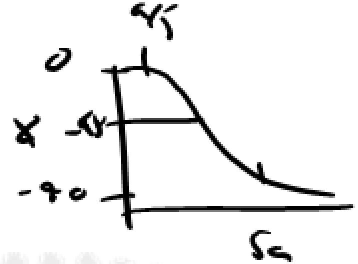
$$\frac{T}{2} = \frac{1}{50} \sim 0.02$$

$$K(z) = \frac{12}{\underline{0.244}\,\frac{z-1}{z+1} + 12}$$

LAG

$$K(s) = \frac{a}{s+a}$$





$$K(z) = \frac{a}{s+a}\Big|_{s=\frac{z-1}{T}} = \frac{a}{\frac{z-1}{T}+a} = \frac{aT}{z-1+aT} = \frac{aT}{z-(1-aT)}$$ "Forward"

$$K(z) = \frac{a}{s+a}\Big|_{s=\frac{z-1}{Tz}} = \frac{a}{\frac{z-1}{Tz}+a} = \left[\frac{aT}{1+aT}\right]\frac{z}{z-\left(\frac{1}{1+aT}\right)}$$ "Backwards Euler"

$$K(z) = \frac{a}{s+a}\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{a}{\frac{2}{T}\frac{z-1}{z+1}+a} = \left[\frac{\frac{aT}{2}}{1+\frac{aT}{2}}\right]\frac{z+1}{z-\left[\frac{1-aT/2}{1+aT/2}\right]}$$ "Tustin"

Z-transform

$$\mathcal{L}\{f(t)\} \triangleq \mathbb{F}(s) = \int_0^\infty f(t)\,e^{-st}\,dt$$

conv → x

i.c.'s

$\dot{x} = sX - x(0)$

$$\mathcal{Z}\{f(kT)\} \triangleq \mathbb{F}(z) = \sum_0^\infty f_k\, z^{-k}$$

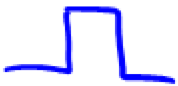
f_k

conv → x

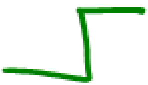
$x_{k-1} = z^{-1} x_k$

unit delay

 impulse $\mathcal{Z}\{\delta\} = 1$

 unit pulse $Z\{0, 1, 0, 0, 0\} = \sum_{0}^{\infty} f_k z^{-k} = 1z^{0} + 0z^{-1} + \dots$

$= 1$

 step $\mathcal{Z}\{\text{step}\} = \frac{1}{s}$

$$\boxed{Z\{0, 1, 1, 1\}} = 1z^{-0} + 1z^{-1} + 1z^{-2} + \dots$$

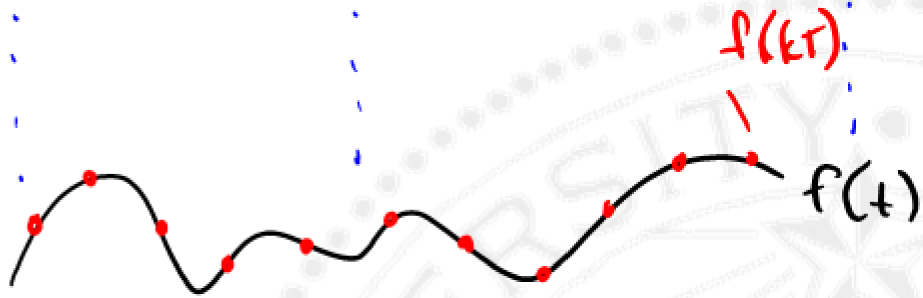
$$= \sum_{0}^{\infty} z^{-k} = \frac{1}{1 - z^{-1}} = \boxed{\left(\frac{z}{z-1}\right)}$$

$\alpha = z^{-1}$

$$\sum_{0}^{\infty} \alpha^k = \frac{1}{1-\alpha} \quad \forall \; |\alpha| < 1.$$

$$\mathcal{Z}\{\Gamma\} = \frac{1}{1-z^{-1}} = \boxed{\left(\frac{z}{z-1}\right) = \mathcal{Z}(\Gamma)}$$

$f(t)$ $\qquad \mathcal{L}\{f(t)\}$ $\qquad \mathcal{Z}\{f(kT)\}$



$$\mathcal{L}\{f(t)\} \rightarrow F(s) \longrightarrow \mathcal{L}^{-1}\{F(s)\} \rightarrow f(t)$$

$$\mathcal{Z}\{f(kT)\} \rightarrow F(z) \longrightarrow \mathcal{Z}^{-1}\{F(z)\} \longrightarrow f_k$$

$$\mathcal{Z}^{-1}\left\{\frac{z}{z-1}\right\}$$

f_k

$$\begin{array}{r|l} & 1 + 1z^{-1} + 1z^{-2} + \dots \\ \hline z-1 & z \\ & \underline{z-1} \\ & +1 \end{array}$$