CMPE-242
Applied Feedback Control

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Announcements

- Controls Seminar tomorrow (Friday) 11-noon.
  Simulation
- This afternoon for additional office hours
  1 pm.

DIGITAL IS <u>NOT</u> FREE.

Time delay $\sim \frac{\Delta T}{2}$

$\mathcal{L}\{\text{time delay}\} = e^{-\frac{\Delta T}{2}s}$

<u>Bode</u> $\left. e^{-\frac{\Delta T}{2}s} \right|_{s=j\omega}$ $\quad e^{-\frac{\Delta T}{2}\omega j}$

$|\cdot| = 1 \;\; \forall \omega$

$\measuredangle = -\frac{\Delta T}{2}\omega$

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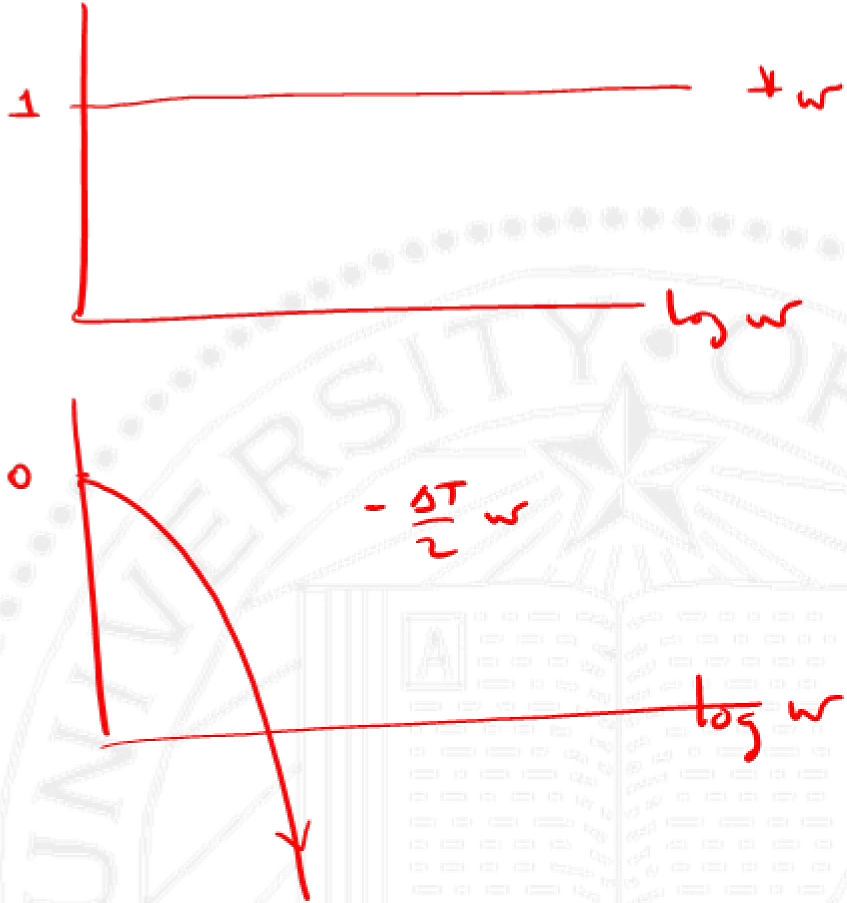
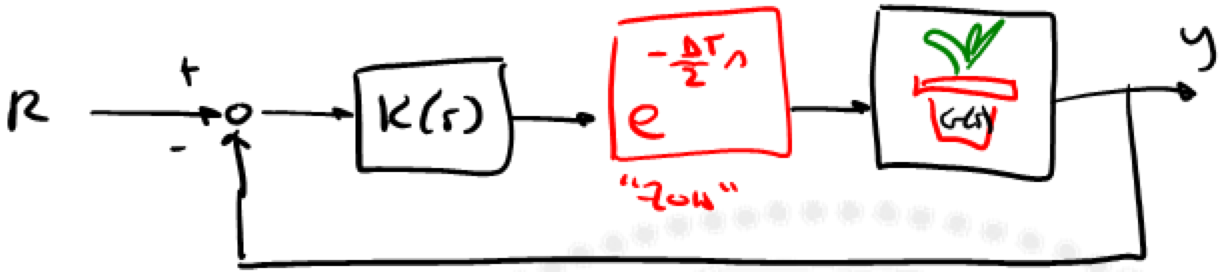
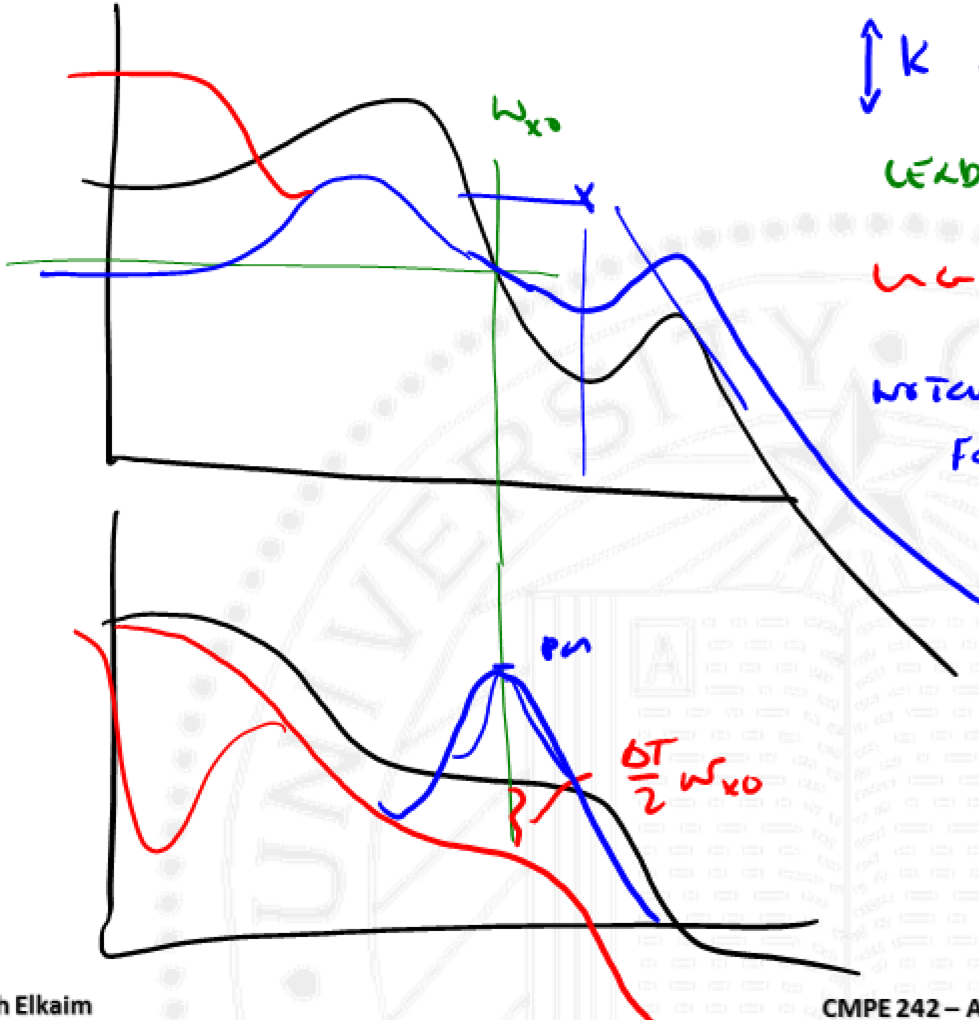
$e^{-\left(\frac{\Delta T}{2}\omega\right)j}$





DESIGN IN BODE AS NORMAL, BUT ACCOUNT FOR $\frac{\Delta T}{2}\omega$ ADDED PHASE LOSS



$\updownarrow$ K for ω_{xo}

LEAD FOR PM

LAG FOR DC gain

NOTCH/LOW PASS FOR HIGH FREQ

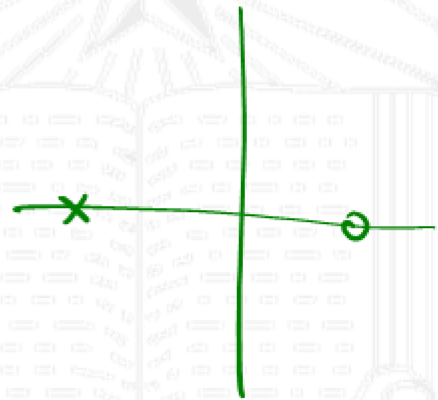
Root Locus

$$e^{-\frac{\Delta T}{2}s}$$

$$e^{-\frac{\Delta T}{2}s} = 1 - \frac{\Delta T}{2}s + \left(\frac{\Delta T}{2}\right)^2 \frac{s^2}{2!} - \left(\frac{\Delta T}{2}\right)^3 \frac{s^3}{3!} + \dots$$

Padé Approximation

$$e^{-\frac{\Delta T}{2}s} \simeq -\frac{(s-a)}{(s+a)}$$



$$-\frac{(s-a)}{s+a} = \frac{1 - s/a}{1 + s/a}$$

$$1 - 2s/a + \frac{2s^2}{a^2}$$

$$1 + s/a \;\overline{\big)\; 1 - s/a}$$

$$\underline{1 + s/a}$$

$$\frac{2s}{a}$$

$$\underline{-\frac{2s}{a} - \frac{2s^2}{a^2}}$$

$$\frac{2s^2}{a^2}$$

$$\frac{2s^2}{a^2} + \frac{2s^3}{a^3}$$

$$\boxed{\therefore\ a = \frac{4}{\delta T}}$$

$$e^{-\frac{\Delta T}{2}\lambda} = 1 - \frac{\Delta T}{2}\lambda + \frac{\Delta T^2}{8}\lambda^2 - \frac{\Delta T^3}{48} + \ldots$$

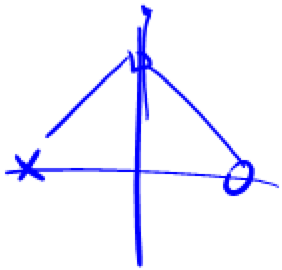
$$\frac{-\left(\lambda - \frac{4}{\Delta T}\right)}{\left(\lambda + \frac{4}{\Delta T}\right)} = 1 - \frac{\Delta T}{2}\lambda + \frac{\Delta T^2}{6}\lambda^2 - \ldots$$

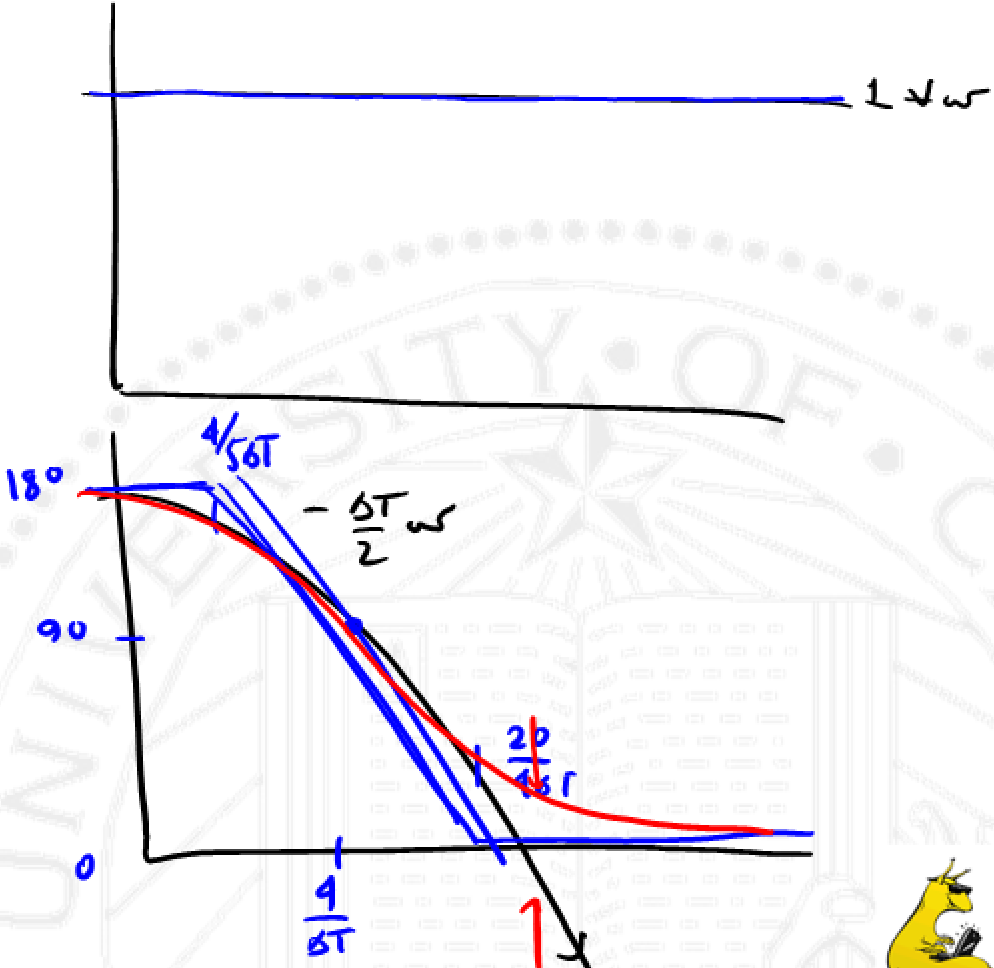
ROOT LOCUS SWITCHES FROM

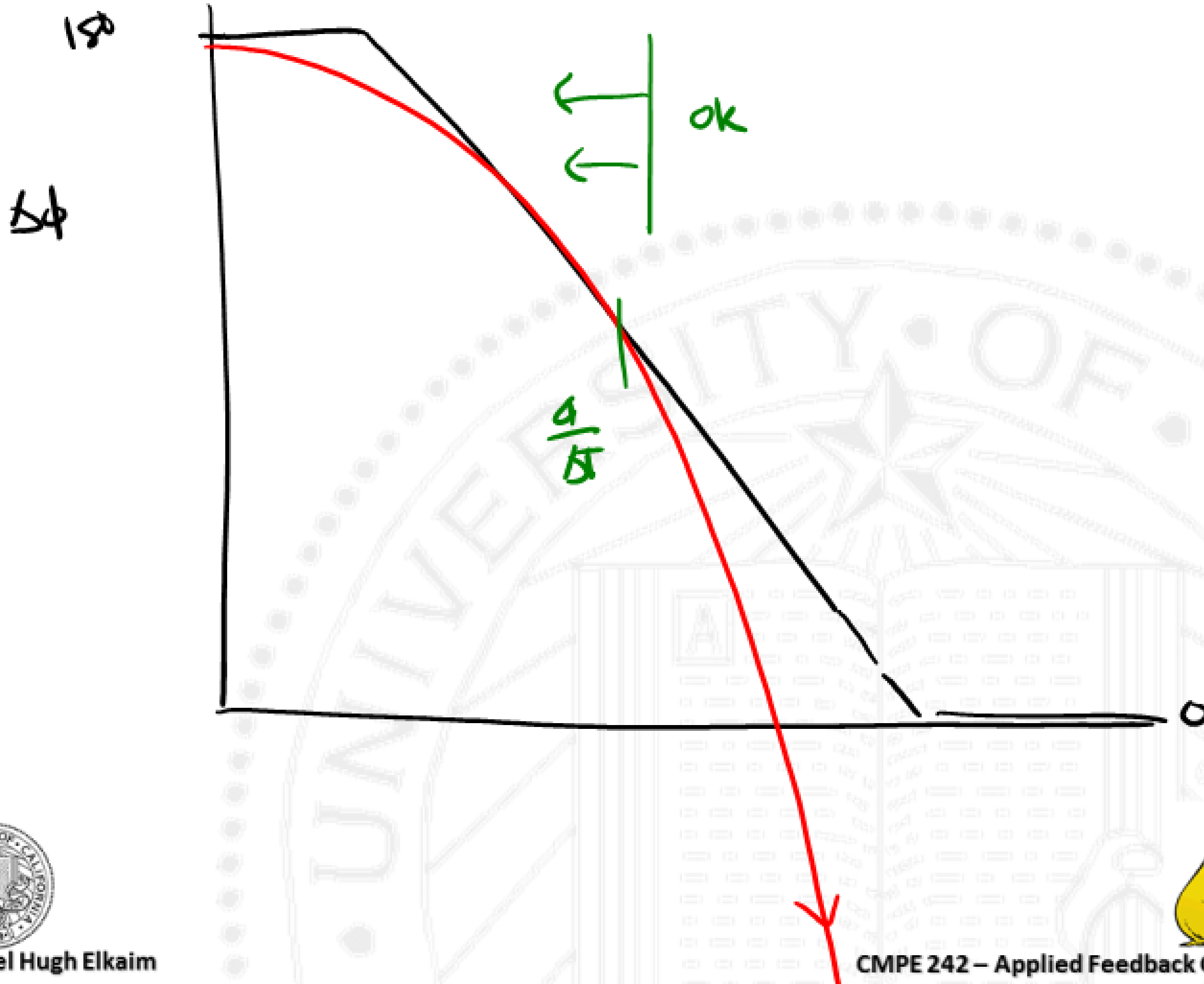
$180^\circ \longrightarrow 0^\circ$

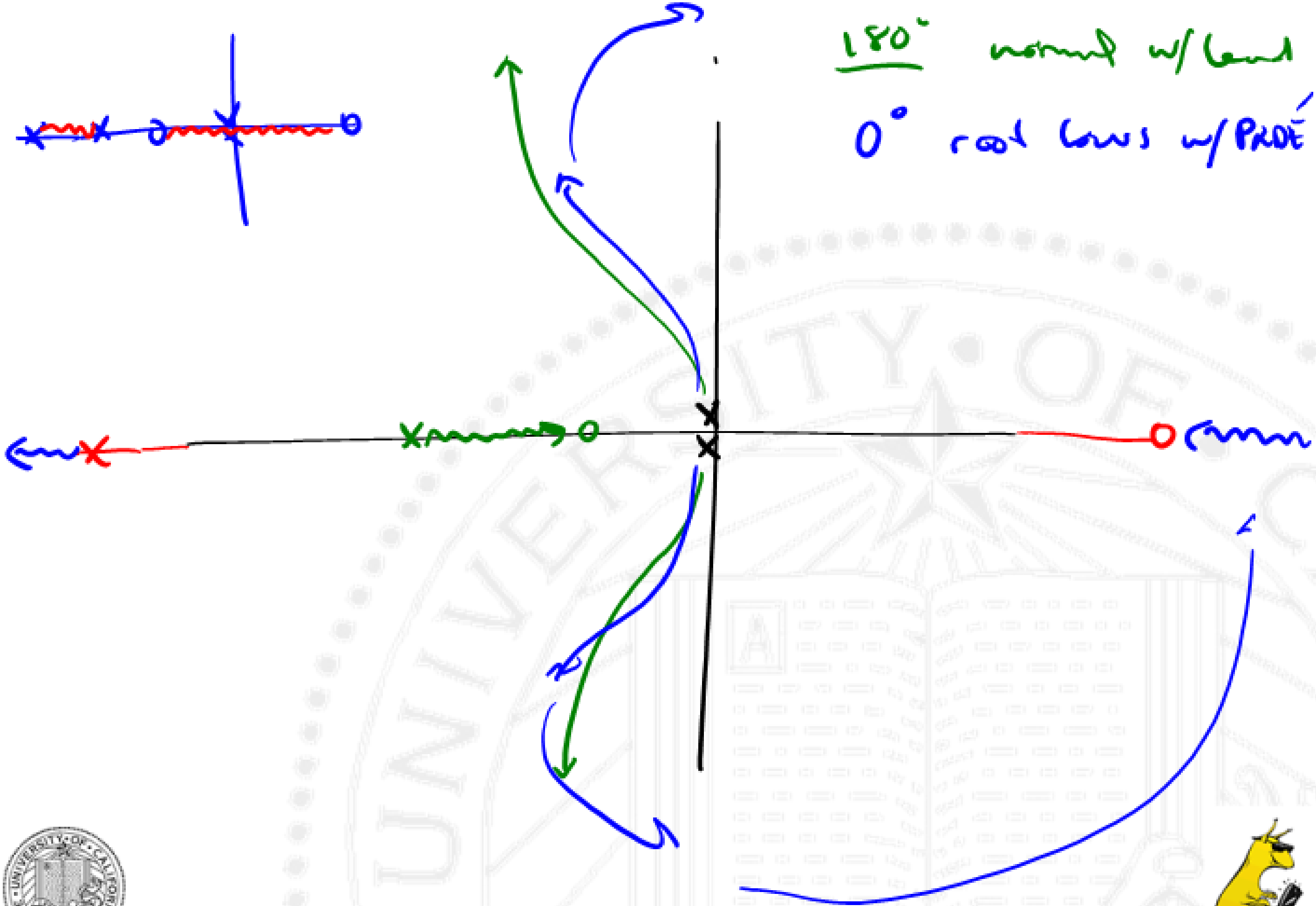
$e^{-\frac{\delta T}{2}s}$

PADÉ






180
Δφ
ok
4/5
0


180° normal w/ lead
0° root locus w/ PADÉ

FOR ROOT LOCUS $\longrightarrow$ ADD A PADÉ

$$-\frac{\left(s-\frac{4}{sT}\right)}{\left(1+\frac{4}{sT}\right)}$$

AND SWITCH $0^\circ \rightarrow 180^\circ$

$180^\circ \rightarrow 0^\circ$

REDO YOUR DESIGN

PID loop

$$u = K_p \varepsilon + K_d \dot{\varepsilon} + K_i \int_0^T \varepsilon \, dt$$

$$U = K_p E + s K_d E + \frac{K_i}{s} E$$

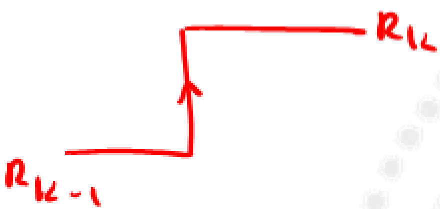
$$\frac{U}{E} = \frac{\left[s^2 K_D + s K_p + K_i \right]}{s} = K(s)$$

$K(s) \longrightarrow$ O.D.E $\longrightarrow K(z) =$ O.Δ.E

$$U_P = K_P \varepsilon_k$$ ← proportional part

$$\dot{\varepsilon} \simeq \frac{\varepsilon_k - \varepsilon_{k-1}}{\Delta T} \longrightarrow U_D = \frac{K_D}{\Delta T}\left[\varepsilon_k - \varepsilon_{k-1}\right]$$

Derivative Part

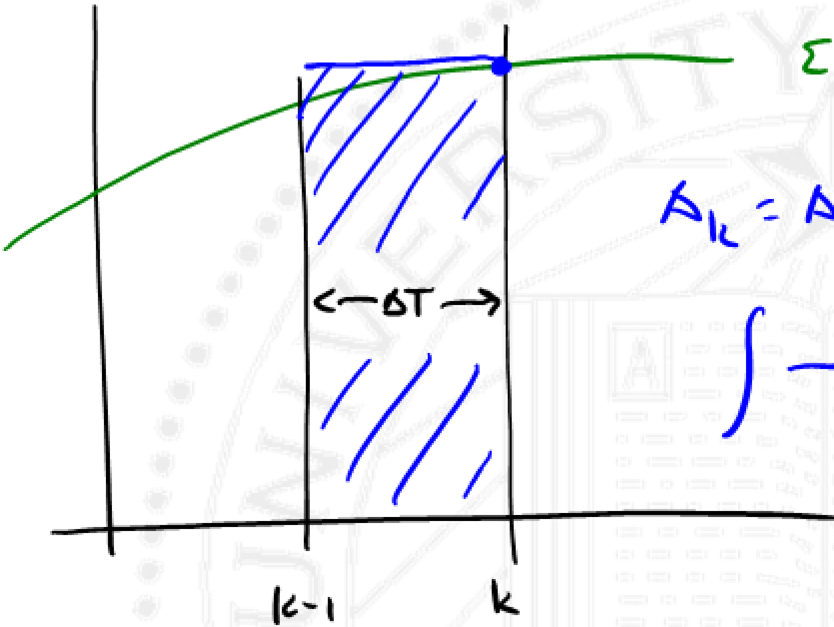


$$\varepsilon_k = R_k - Y_k$$

$$U_D = \frac{K_D}{\Delta T}\left[Y_{k-1} - Y_k\right]$$ ← Derivative Part (output, not error)

$$U_I = K_I\left(A_{k-1} + \varepsilon_k \Delta T\right)$$ ← integral part

accumulation, reset when in saturation



$$A_k = A_{k-1} + \varepsilon_k \Delta T$$

$$\int \rightarrow \Sigma$$

$A_{k-1} = \phi$

$u_{k-1} = \phi$

Read Y_k

Read R_k

$E_k = R_k - Y_k$

$U_P = K_P E_k$

$U_D = \frac{K_D}{\Delta T}(Y_{k-1} - Y_k)$

$A_k = A_{k-1} + E_k \Delta T$

$U_I = K_i A_k$

$U_k = U_p + U_d + U_i$ ← check if $|U| > U_{max}$

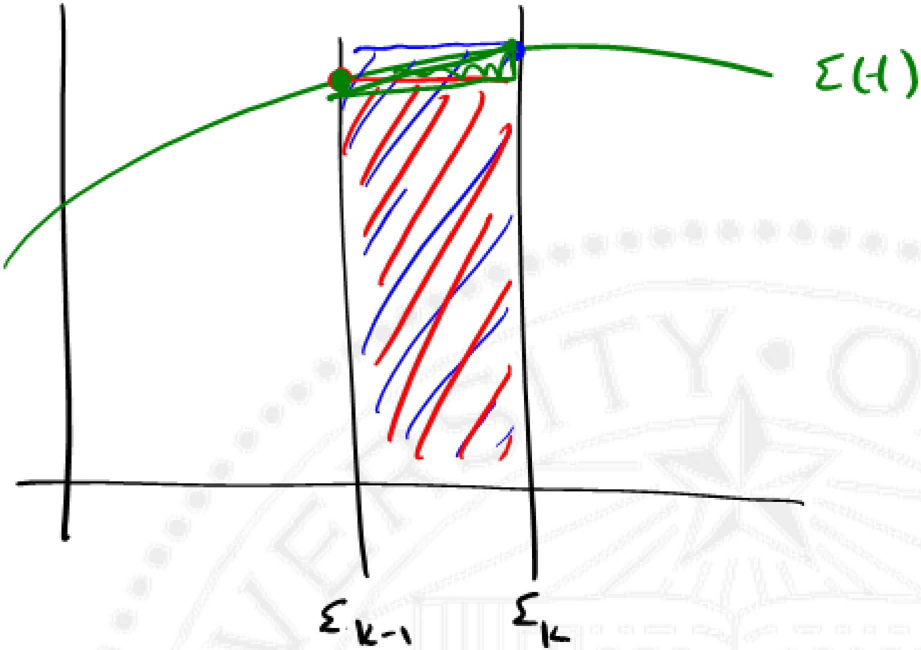
write U_k

update A_{k-1}, U_{k-1}

ANTI WINDUP

$\boxed{A_k = A_k - E_k \Delta T}$

$U_k = U_{max}$



Euler

$A_k = A_{k-1} + \Delta T\, E_k$ (Backwards Integration)

$A_k = A_{k-1} + \Delta T\, E_{k-1}$ (Forwards Integration)

$A_k = A_{k-1} + \left(\frac{E_k + E_{k-1}}{2}\right)\Delta T$ (Trapezoidal / Tustin)

LG

$$\frac{u}{\varepsilon} = K\frac{s+a}{s+b} \longrightarrow \dot{u} + bu = K(\dot{\varepsilon} + a\varepsilon)$$

$$\frac{u_k - u_{k-1}}{\Delta T} + b u_k = K\left[\frac{\varepsilon_k - \varepsilon_{k-1}}{\Delta T} + a\varepsilon_k\right]$$

$$u_k = \frac{\left[u_{k-1} + K(1 + a\Delta T)\varepsilon_k - K\varepsilon_{k-1}\right]}{\left[1 - b\Delta T\right]}$$

$\dot{u} \rightarrow s\,u$ (Laplace)

$u_{k-1} \longrightarrow z^{-1} u_k$ (Z-transform)

z^{-1} unit delay

$q = z^{-1}$

$s\,e = \dot{e} \rightarrow \dfrac{e_k - e_{k-1}}{\Delta T} \rightarrow \dfrac{e_k - z^{-1} e_k}{\Delta T} = \left(\dfrac{z-1}{z\Delta T}\right) e_k$

$s \simeq \dfrac{z-1}{z\Delta T}$

mapping

"Backwards Integration"

$$\frac{U}{\mathcal{E}} = \left.\frac{K(s+a)}{(s+b)}\right|_{s=\frac{z-1}{z\Delta T}} = \frac{K\left(\frac{z-1}{z\Delta T}+a\right)}{\left(\frac{z-1}{z\Delta T}+b\right)} = \frac{K(z-1+a\Delta T z)}{z-1+b\Delta T z}$$

$$\frac{U}{\mathcal{E}} = \frac{K\left[(1+a\Delta T)z-1\right]}{(1-b\Delta T)z-1}$$

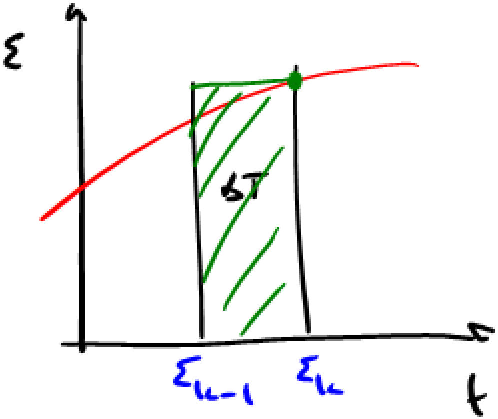
$z^{-1} \rightarrow z$

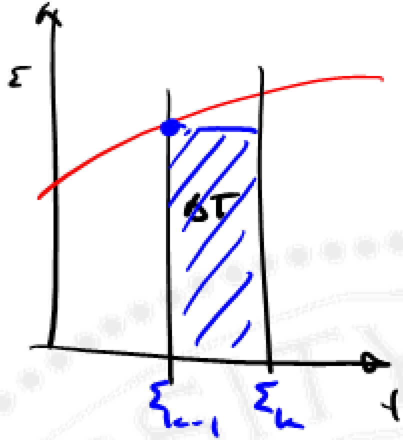
z^{-1}: unit delay

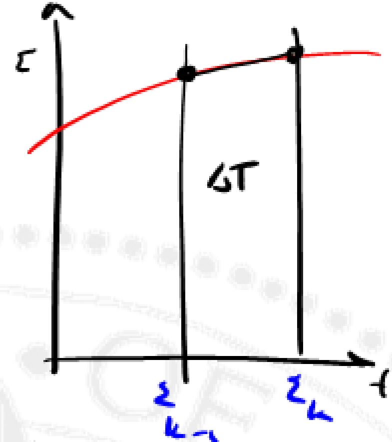
z: unit advance

$$\frac{u_k}{\mathcal{E}_k} = \frac{K\left[(1+aT)-z^{-1}\right]}{(1+bT)-z^{-1}} \rightarrow (1+bT)u_k - u_{k-1} = K\left[(1+aT)\mathcal{E}_k\right] - K\mathcal{E}_{k-1}$$

$$\boxed{u_k = \frac{1}{1+bT}\left[u_{k-1} + K(1+aT)\mathcal{E}_k - K\mathcal{E}_{k-1}\right]}$$







$$A_k = A_{k-1} + \epsilon_k \delta T$$

$$\boxed{\lambda = \frac{z-1}{z\,\delta T}} \quad ①$$

"Backwards Euler"

$$A_k = A_{k-1} + \epsilon_{k-1} \delta T$$

$$(1-z^{-1})A_k = z^{-1}\epsilon_k \delta T$$

$$\frac{A}{\epsilon} = \frac{z^{-1}\delta T}{1-z^{-1}} = \frac{\delta T}{z-1} = \frac{1}{\lambda}$$

$$\therefore \boxed{\lambda = \frac{z-1}{\delta T}} \quad ②$$

"Forwards"

$$A_k = A_{k-1} + \frac{\delta T}{2}(\epsilon_k + \epsilon_{k-1})$$

$$(1-z^{-1})A_k = \frac{\delta T}{2}(1+z^{-1})\epsilon_k$$

$$\frac{A}{\epsilon} = \frac{\delta T}{2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{1}{\lambda}$$

$$\therefore \boxed{\lambda = \frac{2}{\delta T}\left(\frac{z-1}{z+1}\right)} \quad ③$$

"Tustin"

Trapezoid

① $$\frac{1}{\lambda} = \frac{z\,\Delta T}{z-1} \leftrightarrow \lambda = \frac{z-1}{z\,\Delta T} \quad \therefore \quad z = \frac{1}{1-\Delta T\lambda}$$ "Backwards/Euler"

② $$\frac{1}{\lambda} = \frac{\Delta T}{z-1} \leftrightarrow \lambda = \frac{z-1}{\Delta T} \quad \therefore \quad z = 1 + \Delta T\lambda$$ "Forwards"

③ $$\frac{1}{\lambda} = \frac{\Delta T}{2}\frac{(z+1)}{(z-1)} \leftrightarrow \lambda = \frac{2(z-1)}{\Delta T(z+1)} \quad \therefore \quad z = \frac{1+\frac{\Delta T}{2}\lambda}{1-\frac{\Delta T}{2}\lambda}$$ "Tustin/Trapezoid"

④ $$e^{-\Delta T\lambda} = z^{-1} \leftrightarrow \lambda = \frac{1}{\Delta T}\ln(z) \quad \therefore \quad z = e^{\Delta T\lambda}$$ "Exact"

④ $z = e^{\Delta T \lambda} = 1 + \Delta T \lambda + \frac{\Delta T^2}{2!}\lambda^2 + \frac{\Delta T^3}{3!}\lambda^3 + \dots$

③ $z = \frac{1 + \frac{\Delta T}{2}\lambda}{1 - \frac{\Delta T}{2}\lambda} = 1 + \Delta T \lambda + \frac{\Delta T^2 \lambda^2}{2} + \frac{\Delta T^3 \lambda^3}{4} + \dots$

② $z = 1 + \Delta T \lambda = 1 + \Delta T \lambda + \phi$

① $z = \frac{1}{1 - \Delta T \lambda} = 1 + \Delta T \lambda + \Delta T^2 \lambda^2 + \Delta T^3 \lambda^3 + \dots$

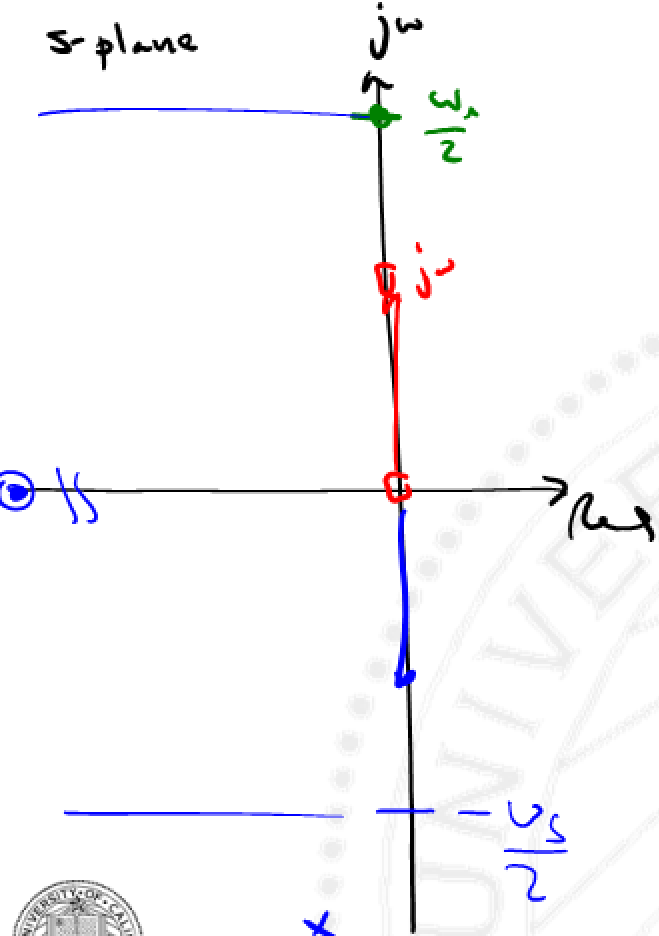
$$K(s) = K\,\frac{s+a}{s+b}\Bigg|_{s=\frac{1}{\Delta T}\ln(z)} = \frac{K\left(\frac{1}{\Delta T}\ln(z) + a\right)}{\frac{1}{\Delta T}\ln(z) + b}$$ "hard to implement"

Exact is useful → where are my equivalent poles.

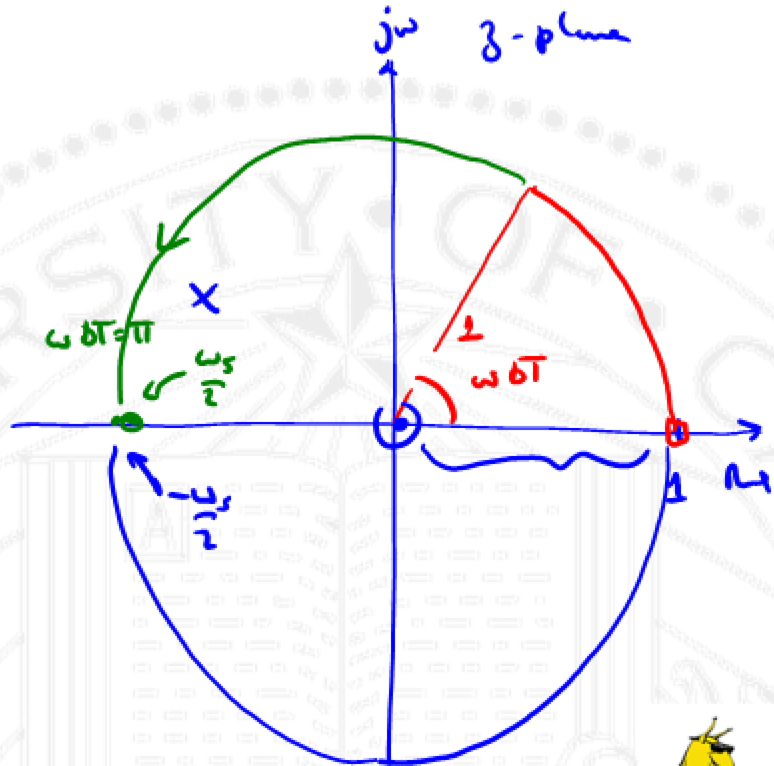
$\text{ODE} \rightarrow K(s) \rightarrow s = j\omega$ − Bode $<$ $|\cdot|$, $\measuredangle$

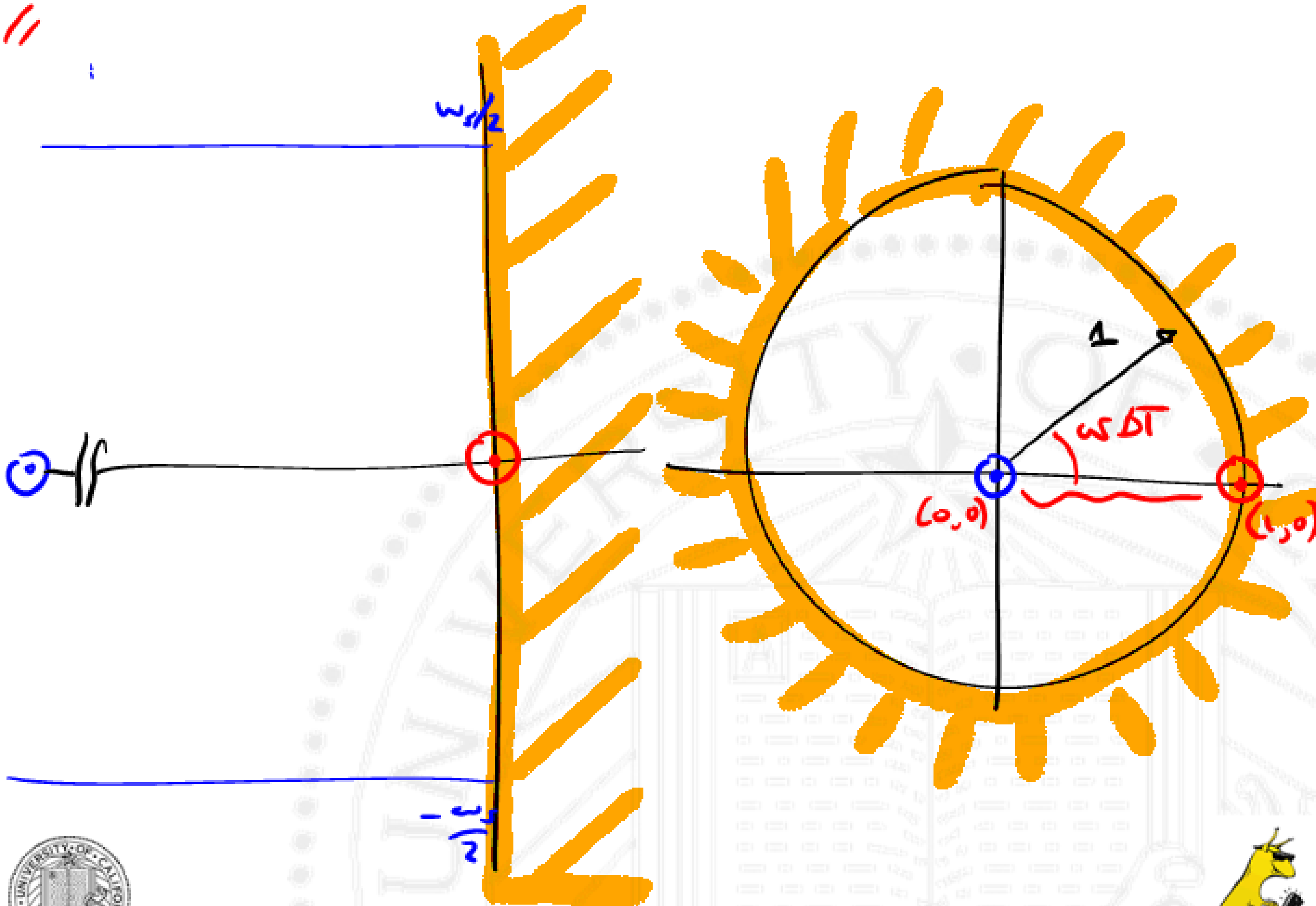
$\text{ODE} \rightarrow K(z) \quad z = e^{\Delta T s}$ − poles/zeros

$K(z)\Big|_{z = e^{j\omega\Delta T}} <$ $|\cdot|$, $\measuredangle$ dBode

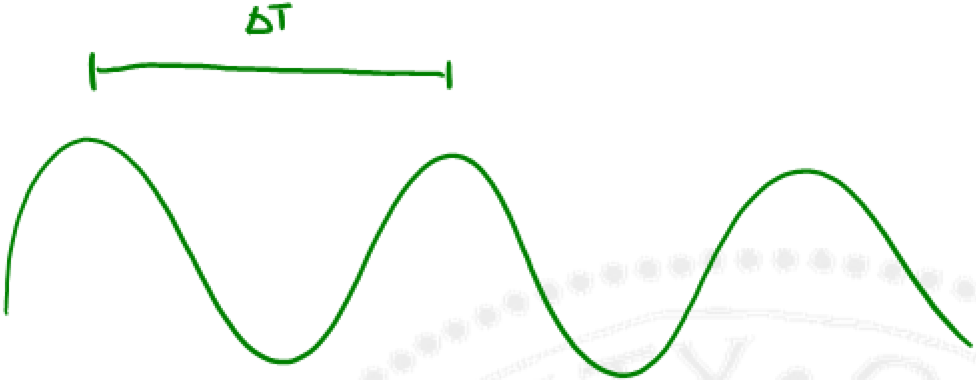


$z = e^{j\omega \delta T}$





ωs/2
-ωs/2
1
ωΔT
(0,0)
(1,0)



$$\omega_s = \frac{2\pi}{T}$$

$$\omega \Delta T = \pi$$

$$\frac{\omega_s}{2} = \omega$$ ← Nyquist