CMPE-242
Applied Feedback Control

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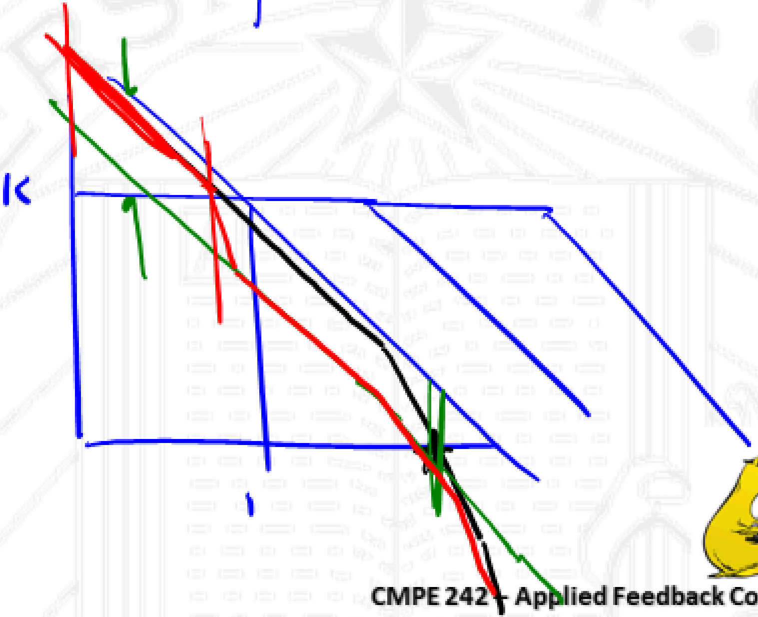
Question

Lag compensation

PM > 40°

$$\frac{K}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{50}+1\right)} \longrightarrow \underbrace{e_{ss}} - \text{ramp} \longrightarrow \underline{K > 100}$$

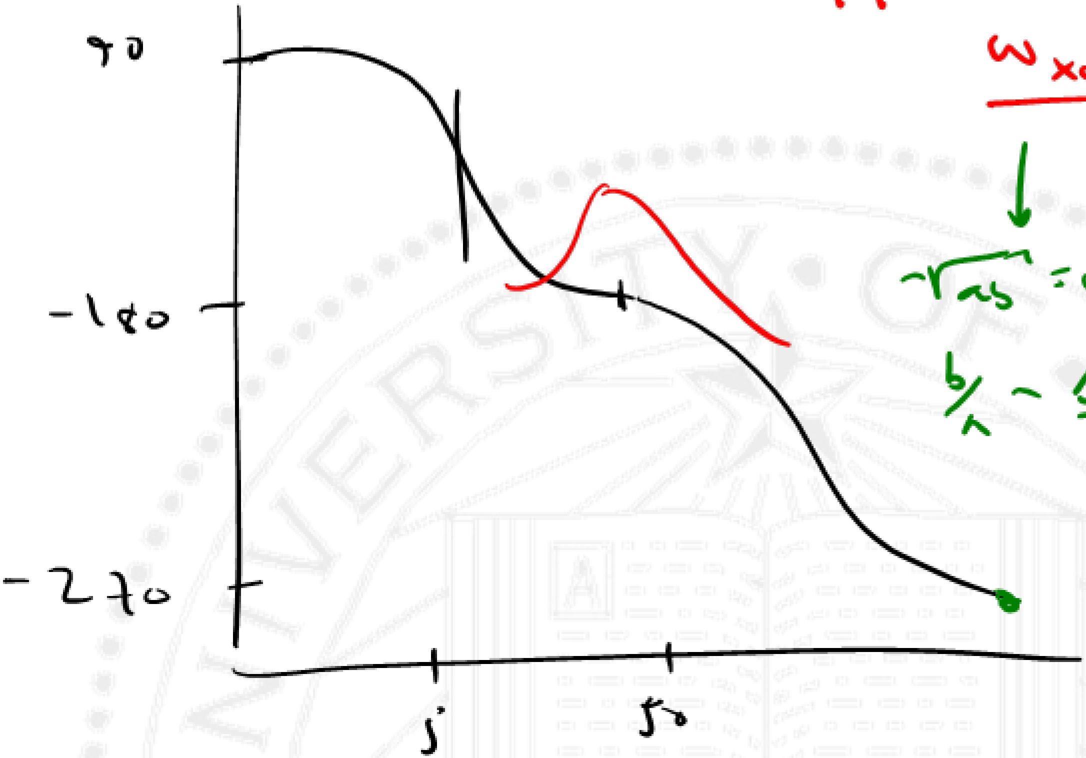
$$\frac{250K}{s(s+5)(s+50)}$$



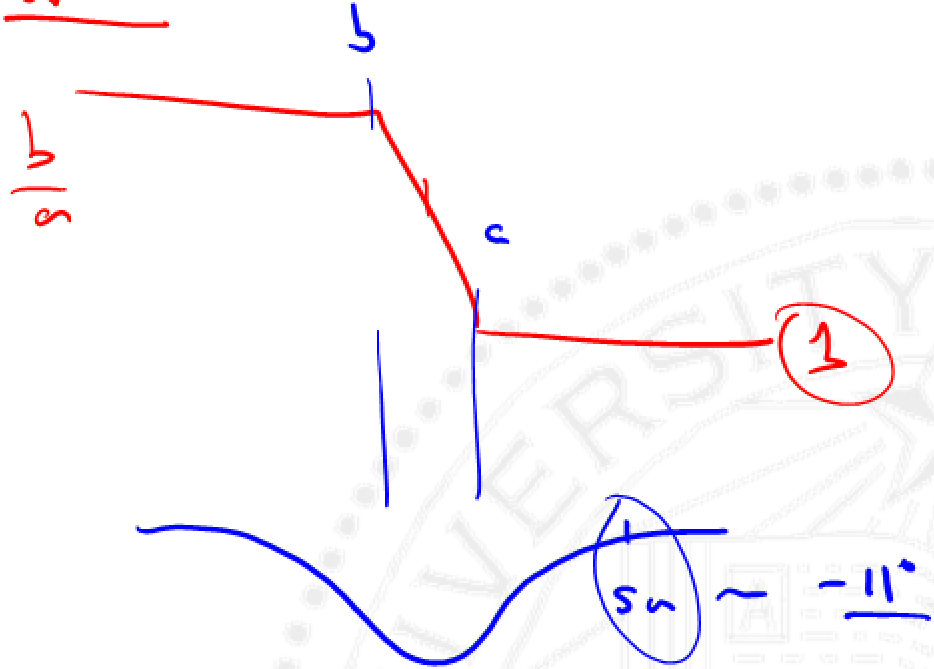
WHERE $|\cdot| = 1$ is $\underline{\omega_{xo}}$

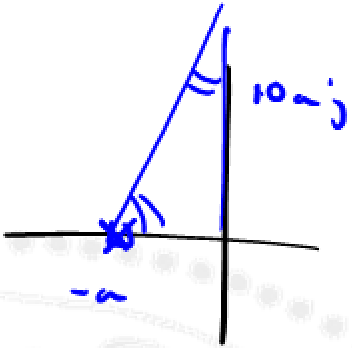
$\sqrt{ab} = \omega_{xo}$

$b/a \sim \underline{\Delta PM}$

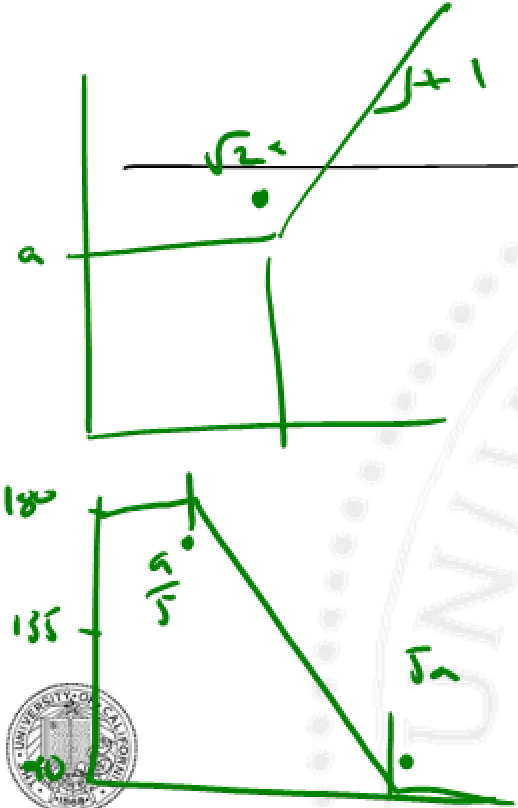


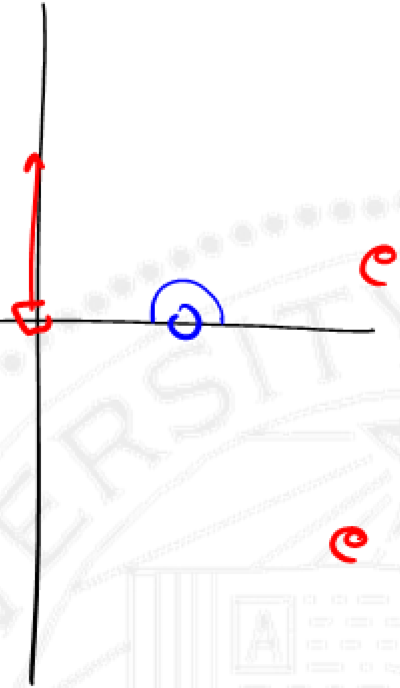
LAG





$G(s) = (s-a)$





@ $w = 0$ $|.| = a$

$\angle = 180$

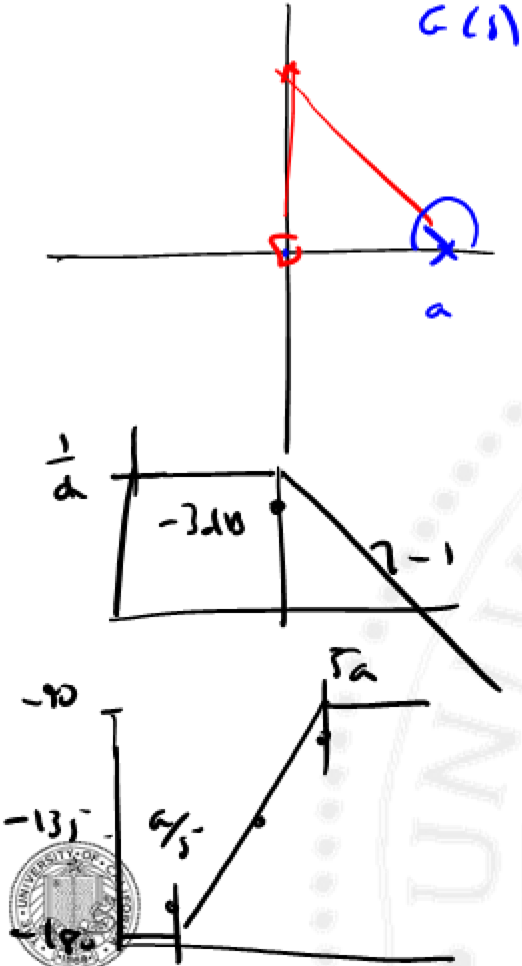
@ $w = a$ $|.| = \sqrt{2}\, a$

$\angle$ 135

@ $w \to \infty$ $|.| \approx w$

$\angle = 90$

$G(s) = \frac{1}{(s-a)}$



@ $\omega = 0 \rightarrow$ $|\cdot| = \frac{1}{a}$

$\measuredangle = -180$

@ $\omega = a$ $|\cdot| = \frac{1}{\sqrt{2}a}$

$\measuredangle = -135°$

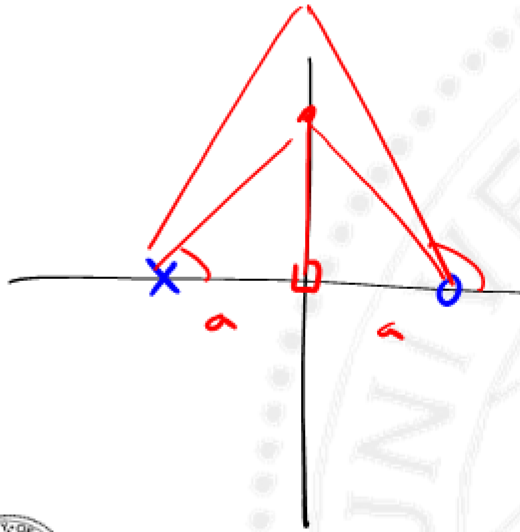
@ $\omega \rightarrow \infty$ $|\cdot| = \frac{1}{\omega}$

$\measuredangle = -90°$

PADÉ APPROXIMATION

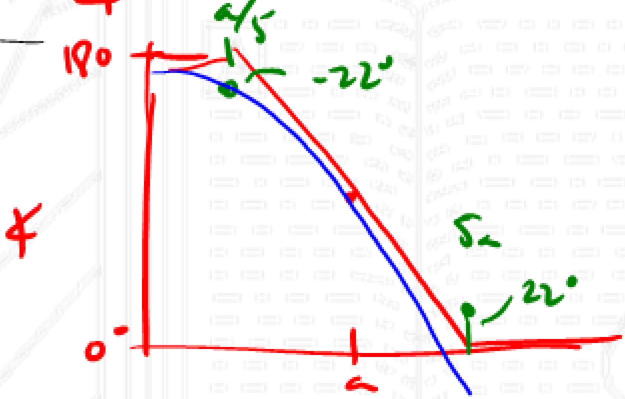
$$G(s) = \frac{s-a}{s+a}$$

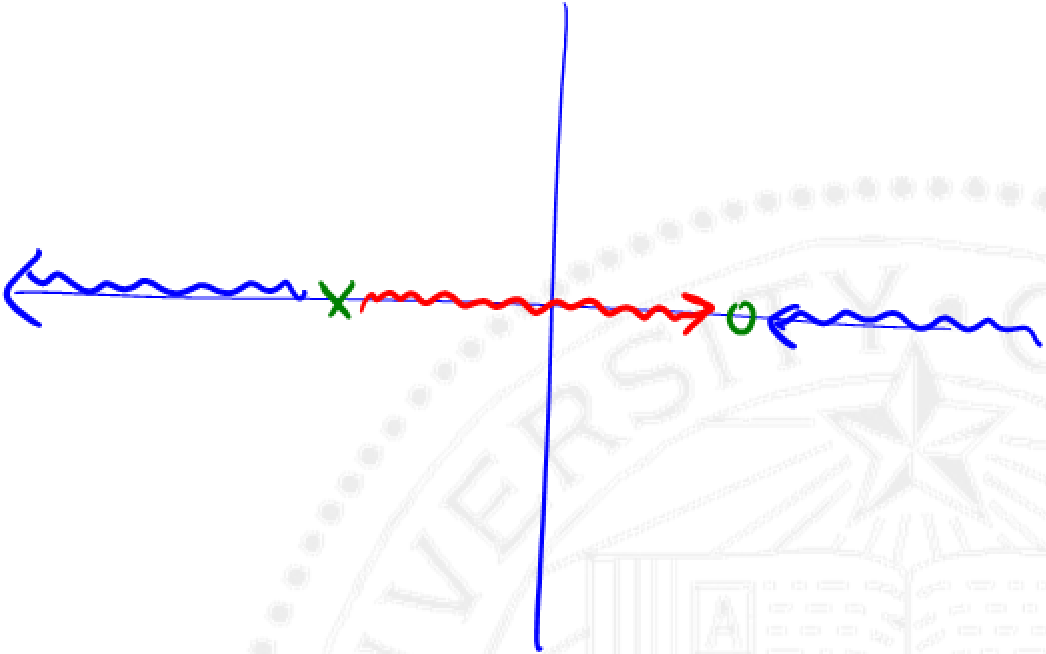
approximating a delay in the system.



$|\cdot| = 1 \ \forall \ \omega$

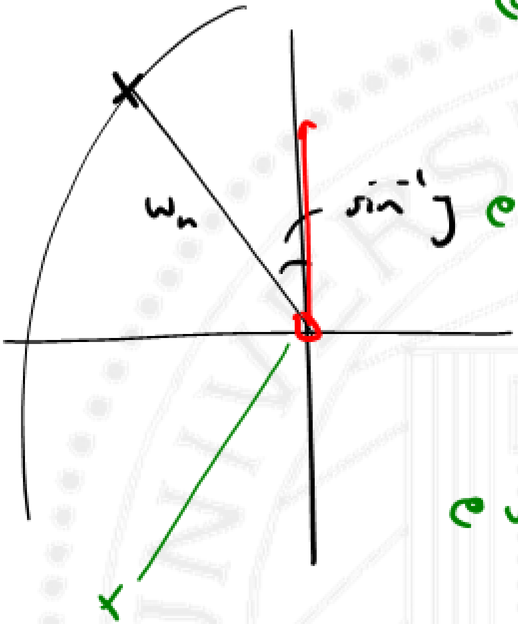
$\measuredangle$: +180 → 90 → 0





$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$(j\omega_n)^2 = -\omega_n^2$$



@ $\omega = 0 \rightarrow |\cdot| = 1$

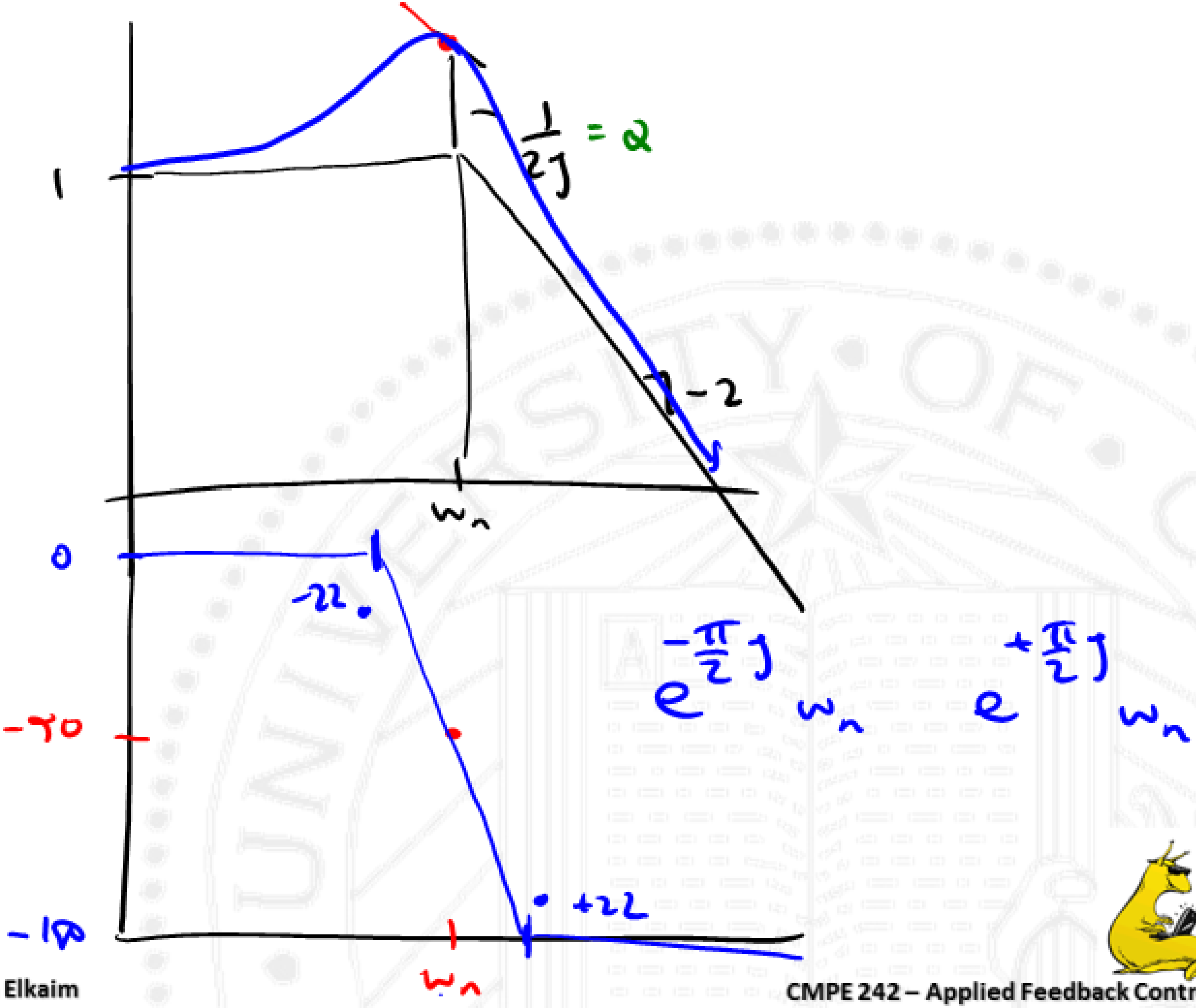
$\measuredangle = 0^\circ$

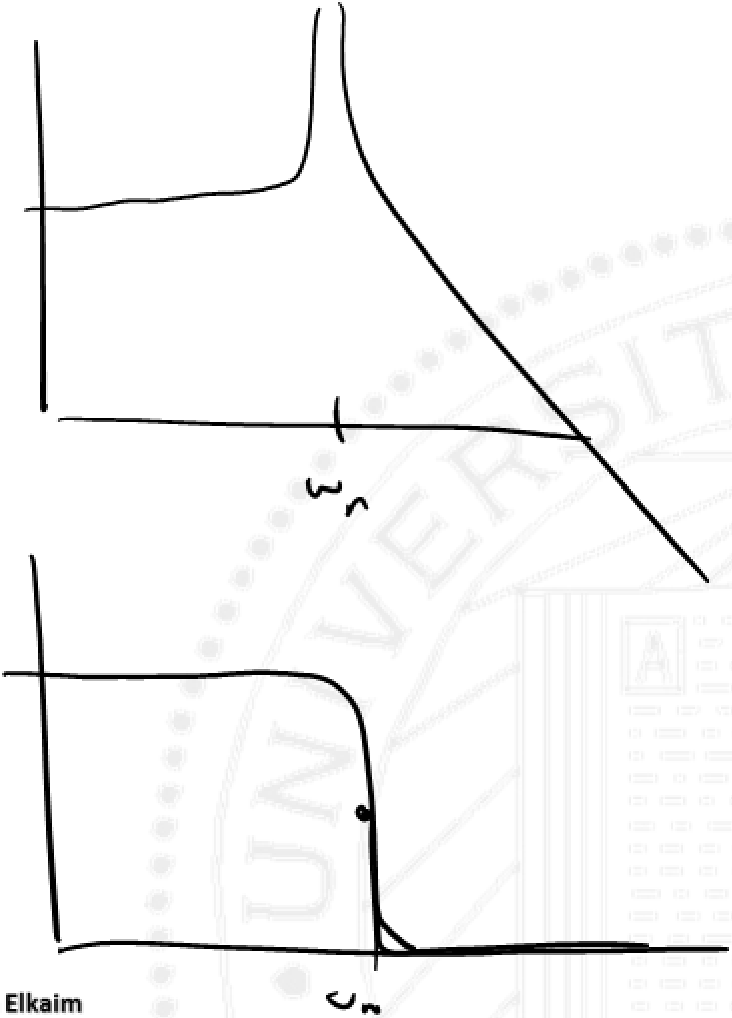
@ $\omega_n \longrightarrow |\cdot| = \frac{1}{2\zeta}$

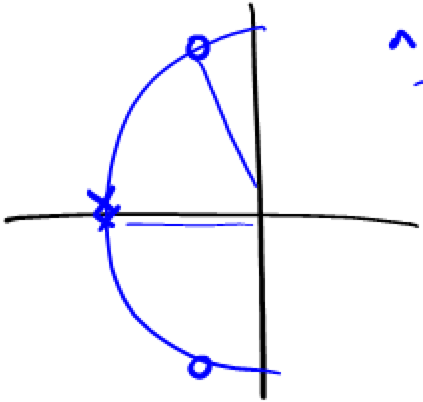
$\measuredangle = -90^\circ$

@ $\omega \rightarrow \infty \rightarrow |\cdot| \cong \frac{1}{\omega^2}$

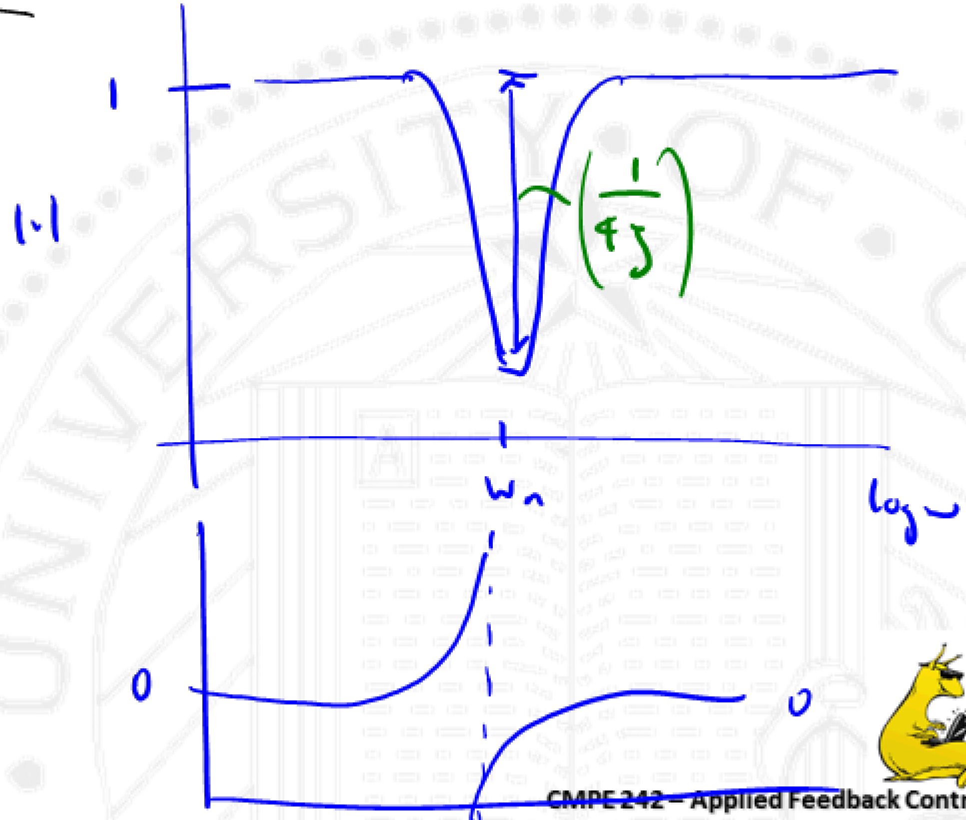
$\measuredangle = -180^\circ$


1
$\frac{1}{2\zeta} = Q$
-2
ω_n
0
-22
-90
-180
+22
ω_n
$e^{-\frac{\pi}{2}\zeta}$ ω_n
$e^{+\frac{\pi}{2}\zeta}$ ω_n

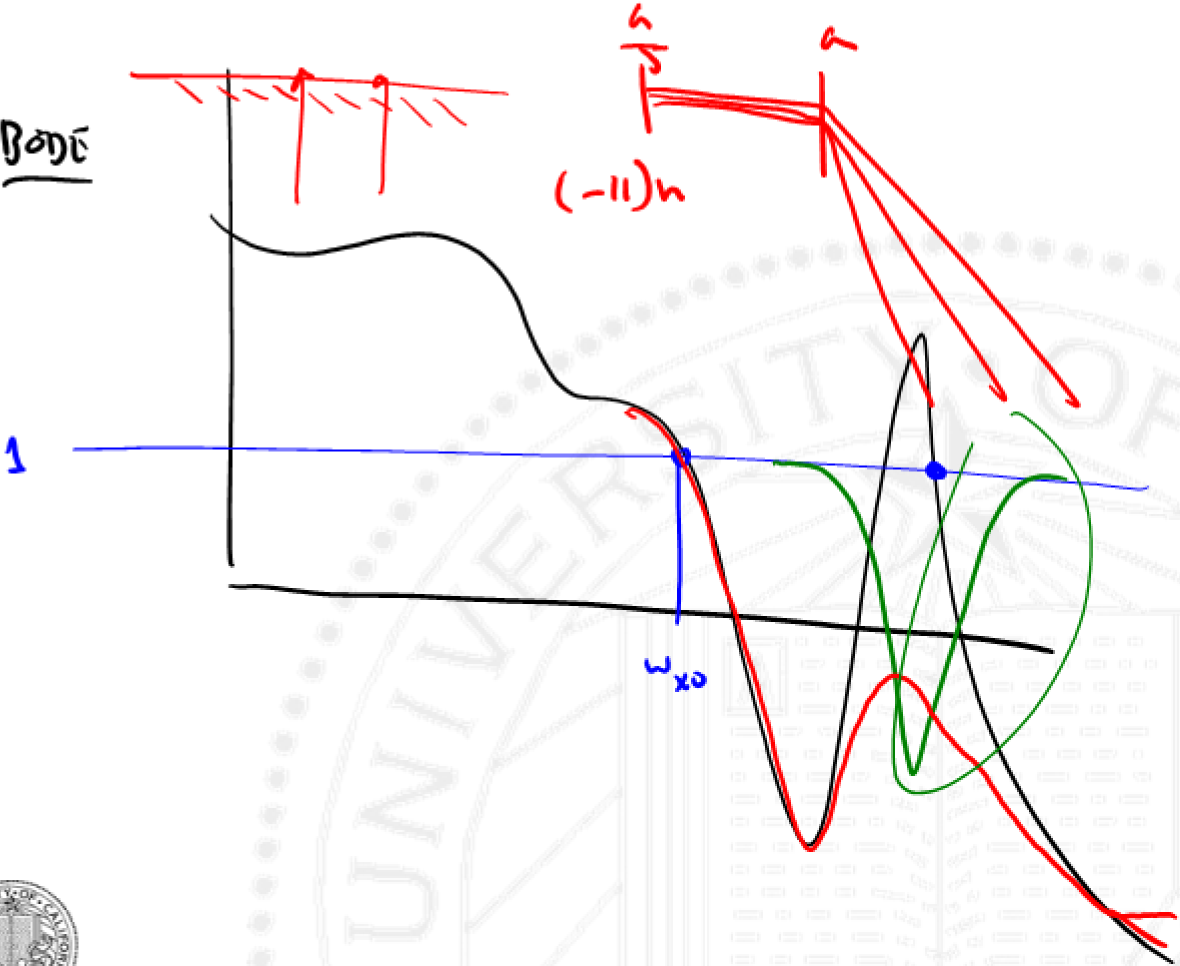

ωr
ωn

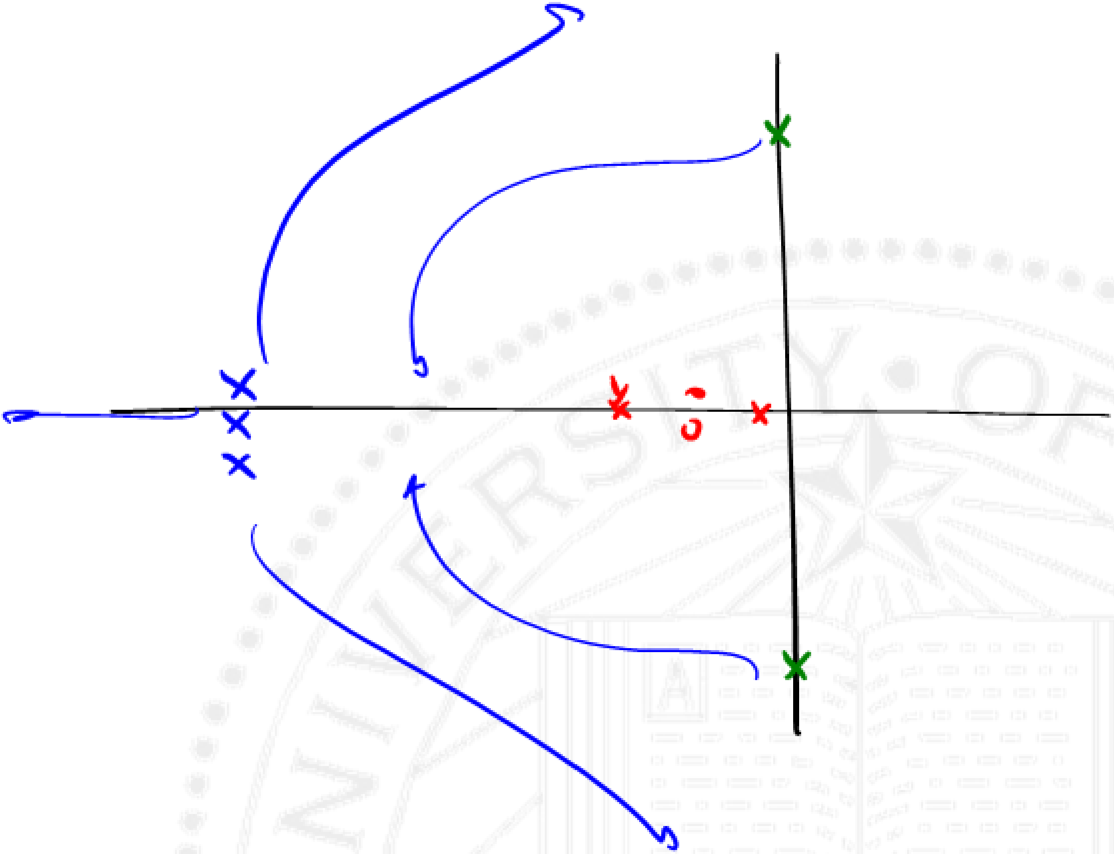


$$\frac{s^2+2\zeta\omega_n s+\omega_n^2}{(s+\omega_n)^2}$$



Bode





BODE

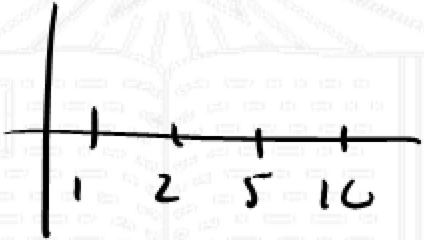
(1) $\frac{1}{\sqrt{2}}$ or $\sqrt{2}$ @ break points

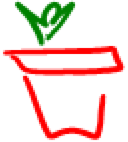
(2) $\frac{1}{5}\omega$ 5ω for ∡ $(e^{-\frac{\pi}{2}j}, e^{+\frac{\pi}{2}j})$

(3) error → ~ 11° per pole/zero

(4) $\log_{10}(2) \simeq 0.3$

$\log_{10}(5) \simeq .7$



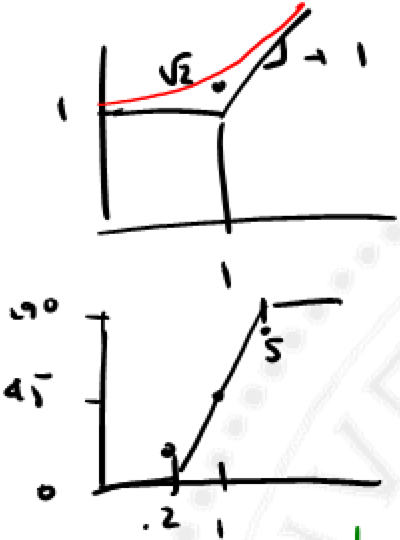


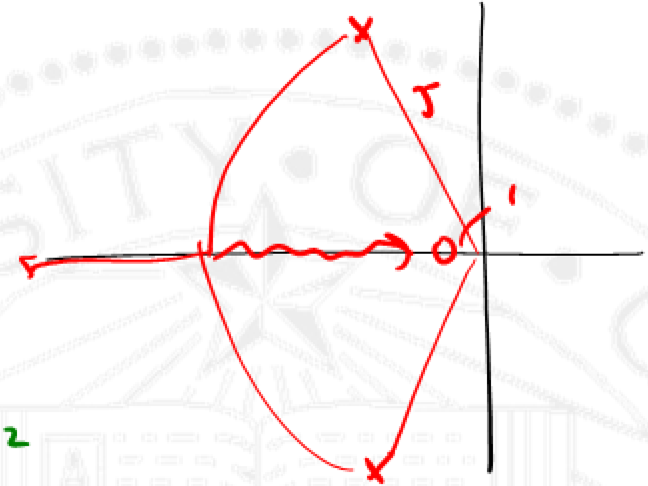
$$G(s) = \frac{(s+1)5^2}{s^2 + 2(0.25)5s + 5^2}$$

$\zeta = .25$

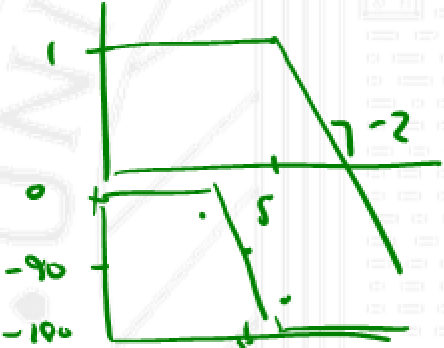
$\omega_n = 5$

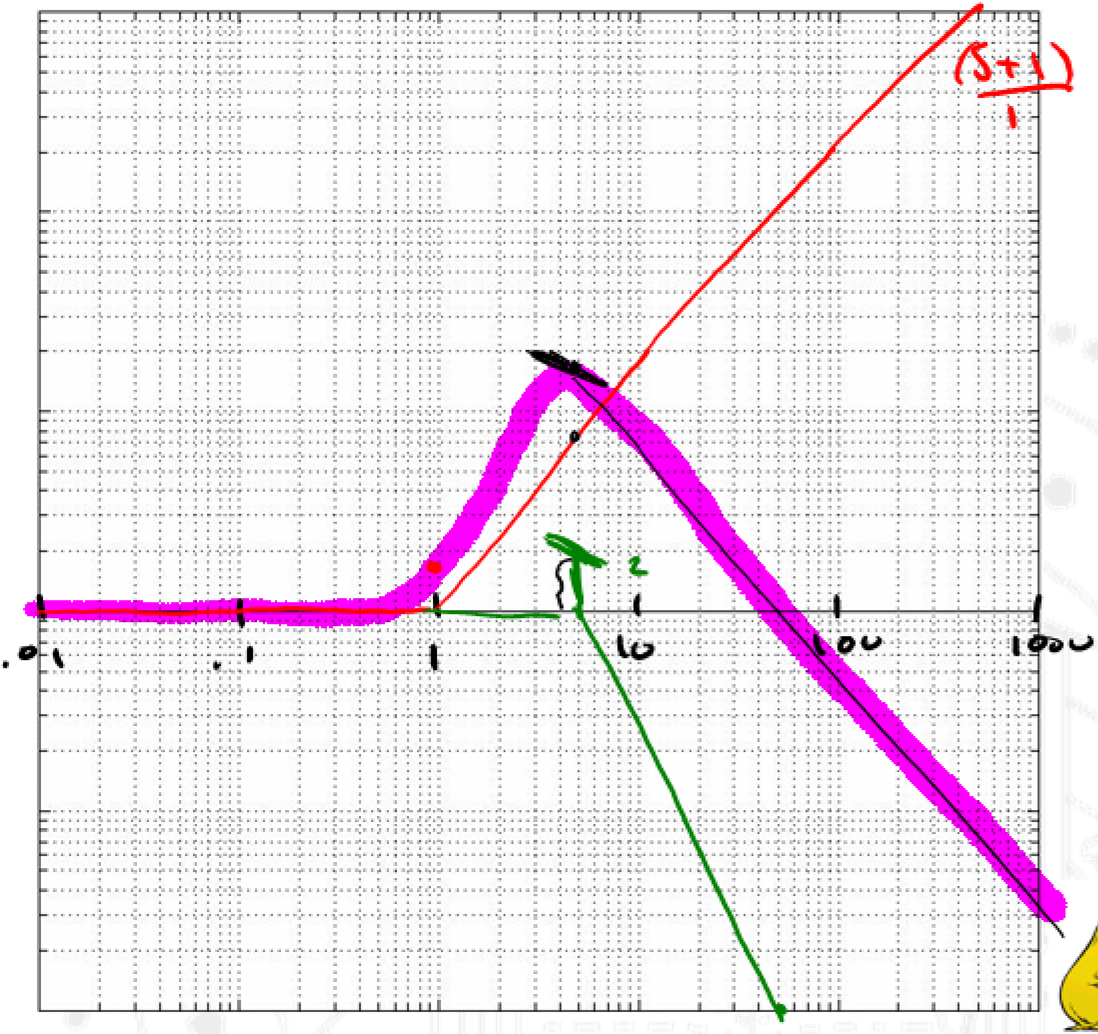
$$\frac{s+1}{1}$$

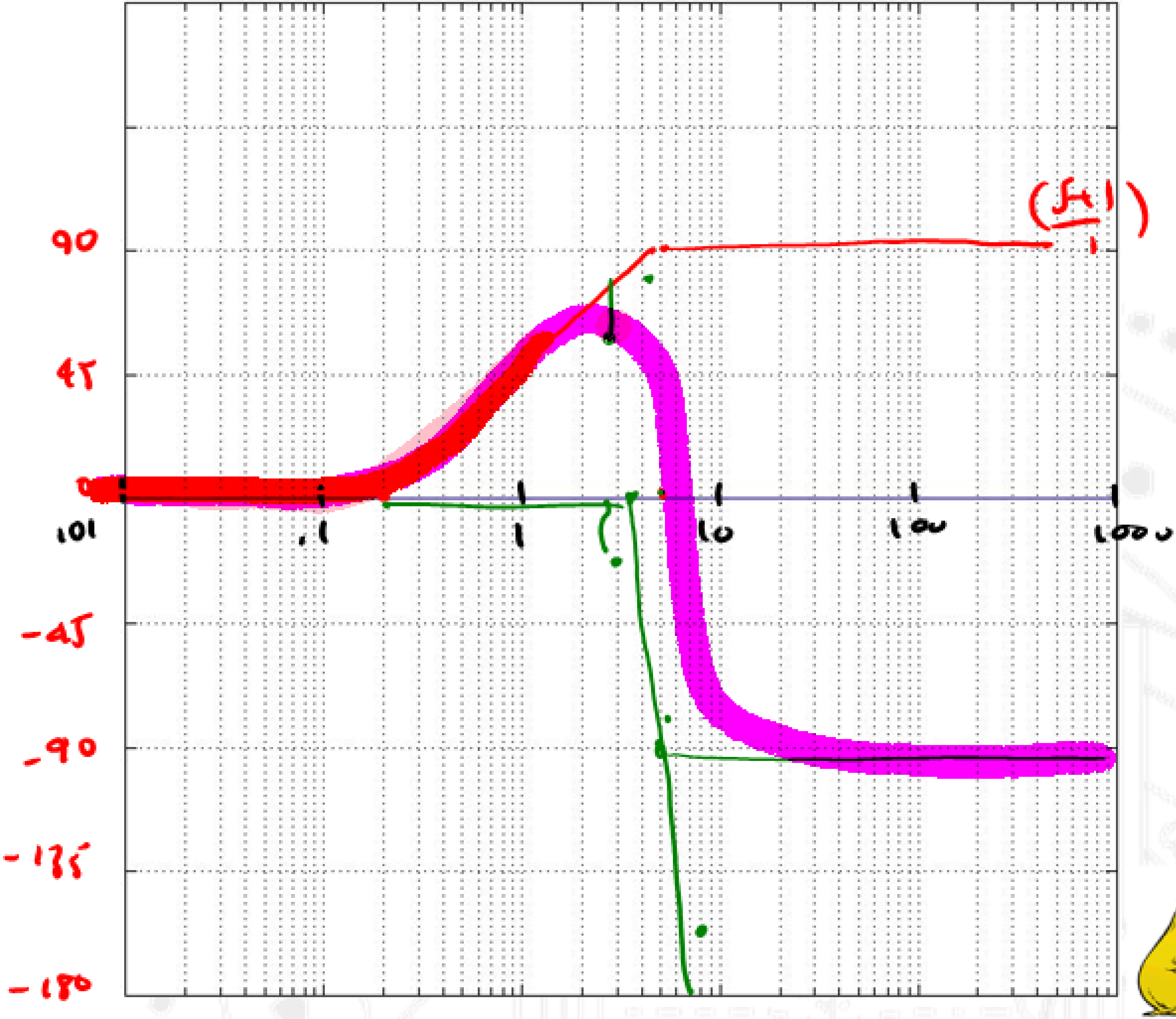




$$\frac{5^2}{s^2+2(.25)5s+5^2}$$




(s+1)
1
2
.01
.1
1
10
100
1000


(s/1 +1)
90
45
0
-45
-90
-135
-180
.01
.1
1
10
100
1000

$$G(s) = \frac{10^2}{s^2 + 2(.2)10s + 10^2}$$

$\zeta = .2$

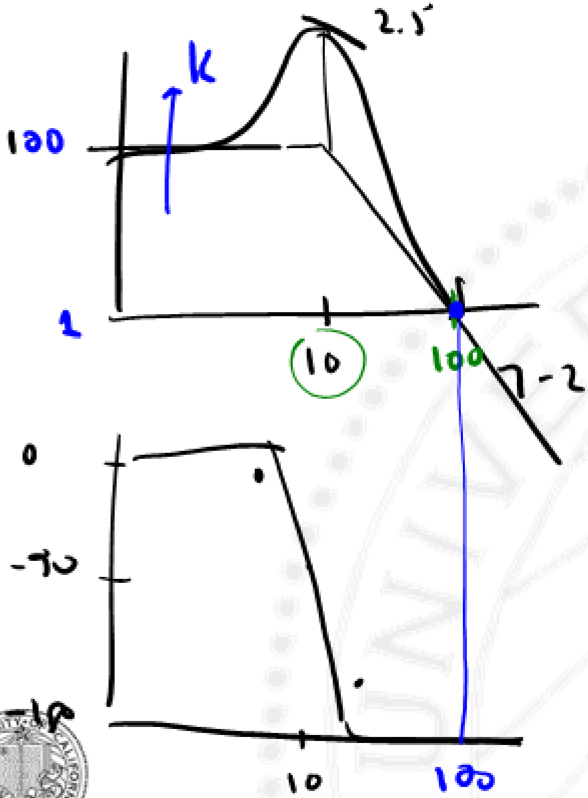
$\omega_n = 10$

$\omega_{xo} = 100$ rad/s

$\phi_{pm} > 50°$

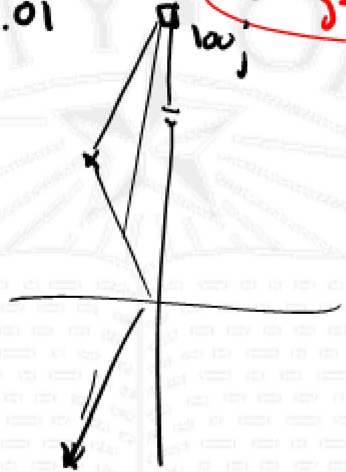
$E_{ss} < .1\%$

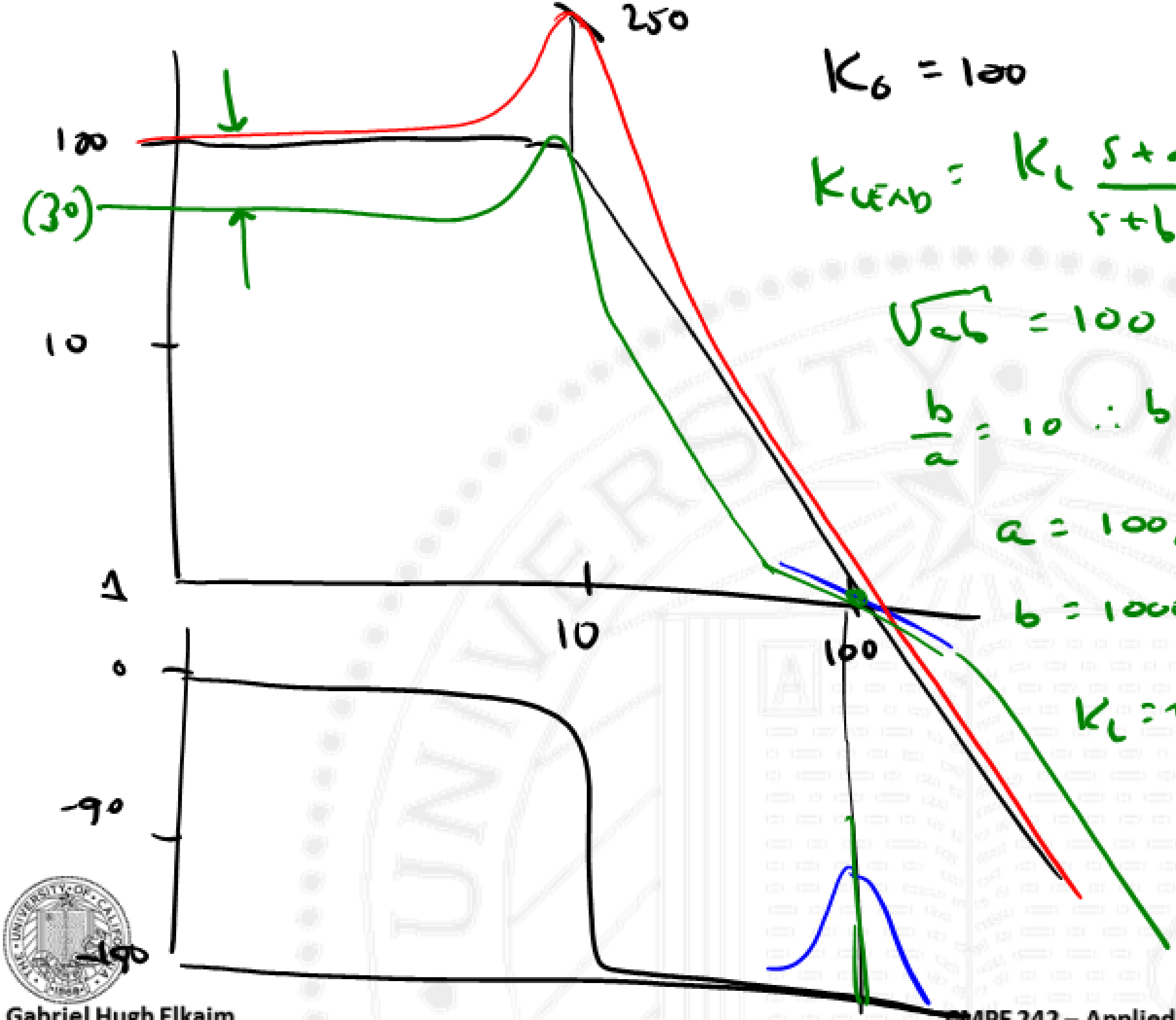
$DC_{gain} > 1000$



$$|GK|_{\omega_{xo}} \approx 0.01$$

$K = 100$





$K_0 = 100$

$K_{LEAD} = K_L \frac{s+a}{s+b}$

$\sqrt{ab} = 100$

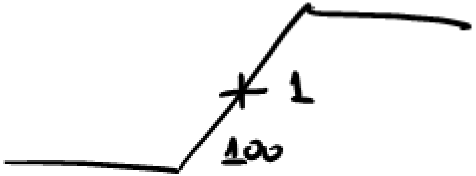
$\frac{b}{a} = 10 \therefore b = 10a$

$a = 100/\sqrt{10}$

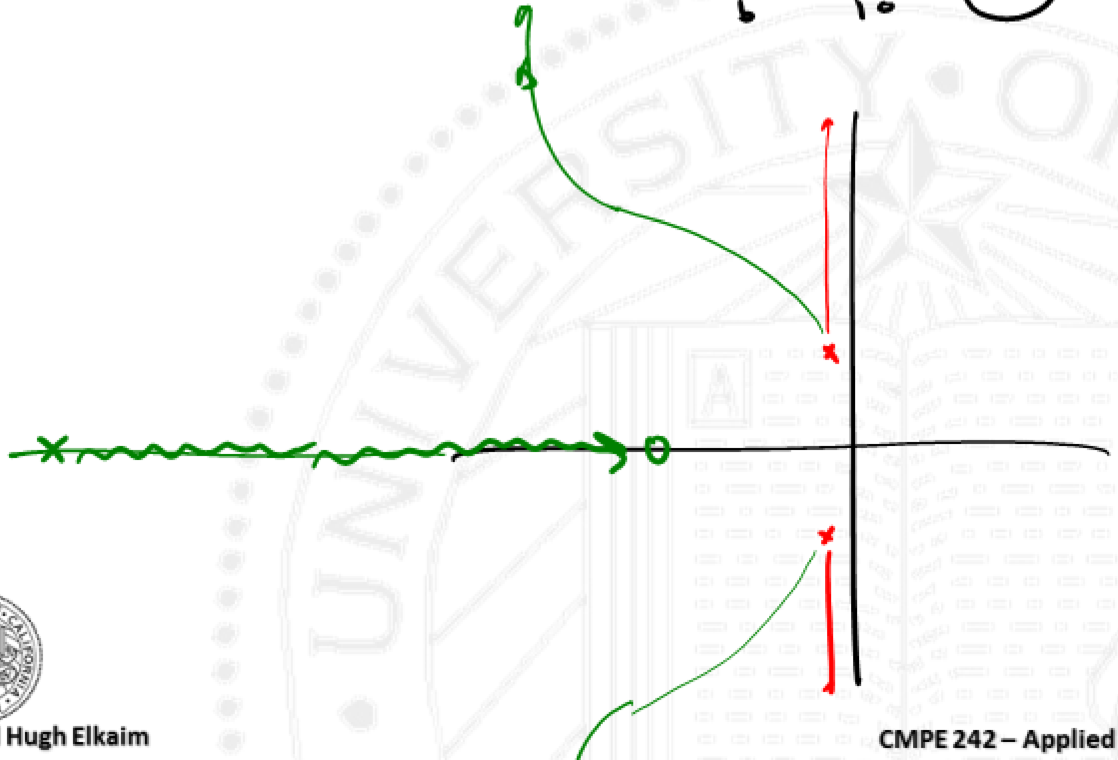
$b = 1000/\sqrt{10}$

$K_L = \sqrt{\frac{b}{a}} = \sqrt{10}$

$$K(s) = \underset{\overset{\uparrow}{100}}{K_0} \cdot \sqrt{10}\,\frac{s+31.6}{s+316}$$



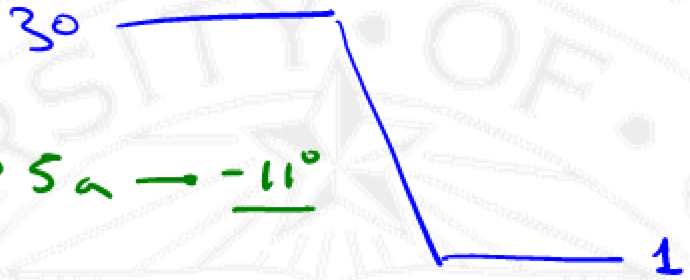
$$K \underset{6}{\le} \to \frac{\sqrt{10}}{10} \approx \boxed{.3}$$



DC gain ~ 1 → K → 100 → LEAD → $\underline{33}$ need) $\underline{1000}$

×100 ÷3

$\underline{\text{LAG}}$ $\frac{b}{a} = \frac{1000}{33} \rightarrow \frac{b}{a} \sim \underline{30}$

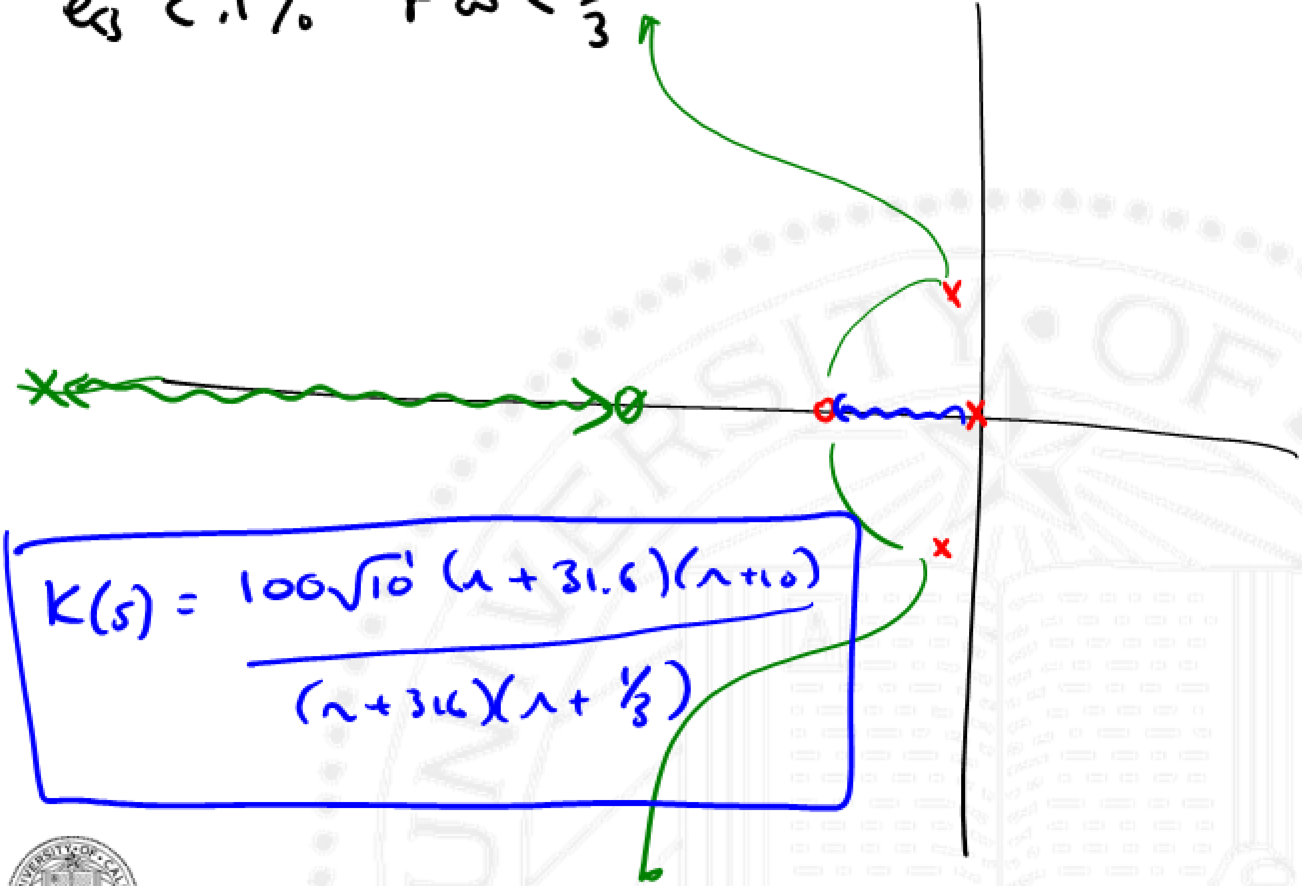


$K_{LAG} = \frac{s+30a}{s+a}$ @ 5a → $\underline{-11°}$

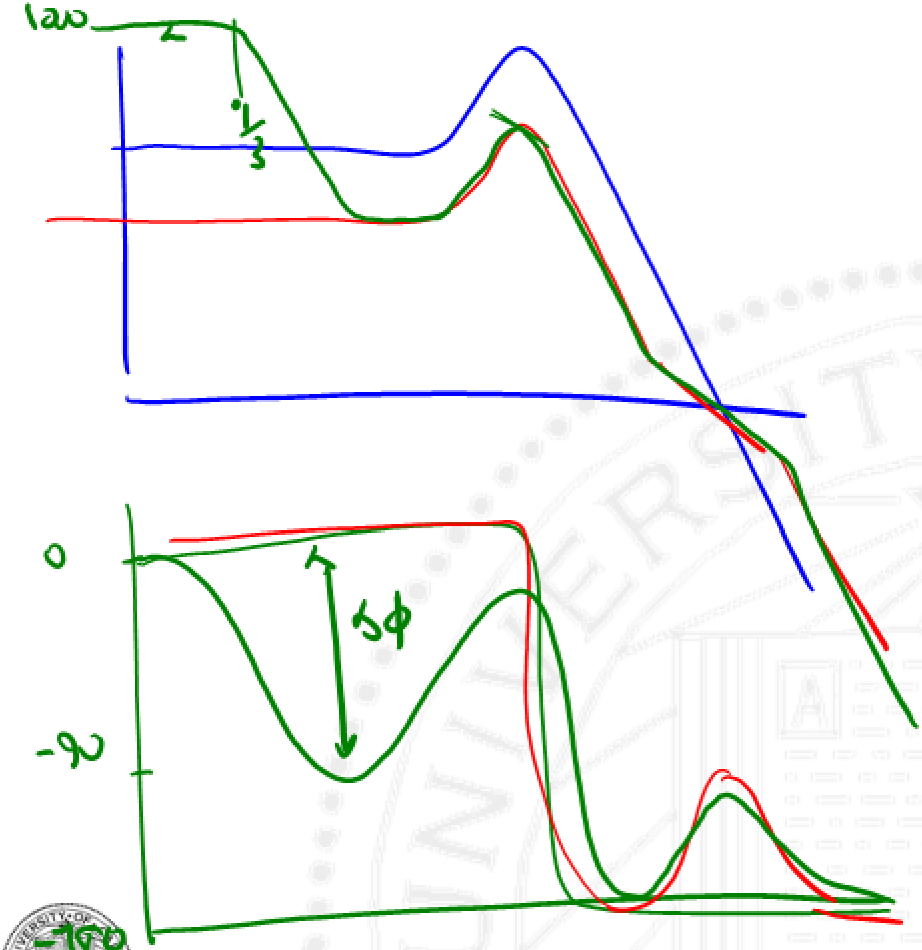
→ $\underline{-11°}$ 150a → $\underline{-11°}$

$300a = 100 \rightarrow a = \frac{1}{3}$ $b = 10.$

$e_{ss} < .1\%$ + $\omega < \frac{1}{3}$



$$\boxed{K(s) = \frac{100\sqrt{10}\,(s+31.6)(s+10)}{(s+316)(s+\frac{1}{3})}}$$


100
2
1/3
0
50
-90
-180

$$G(s) = \frac{10^4}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$\omega_n = 10$

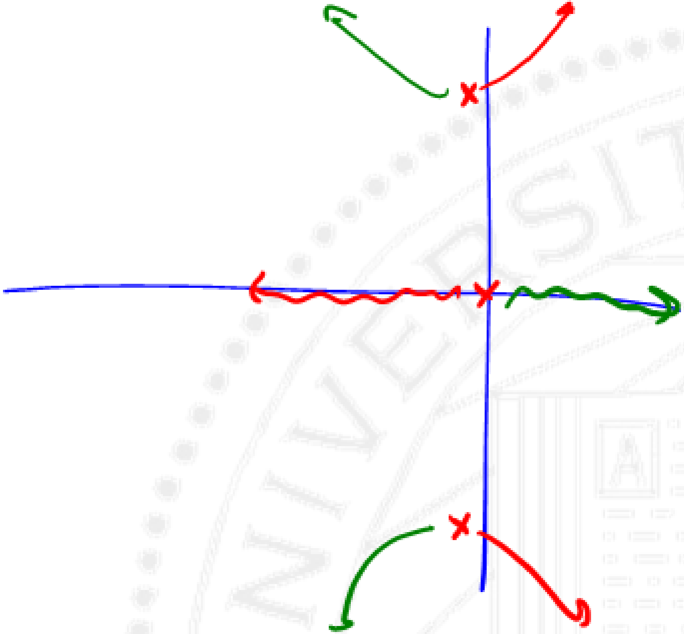
$\zeta = 0.05$

$\frac{1}{2\zeta} = 10$

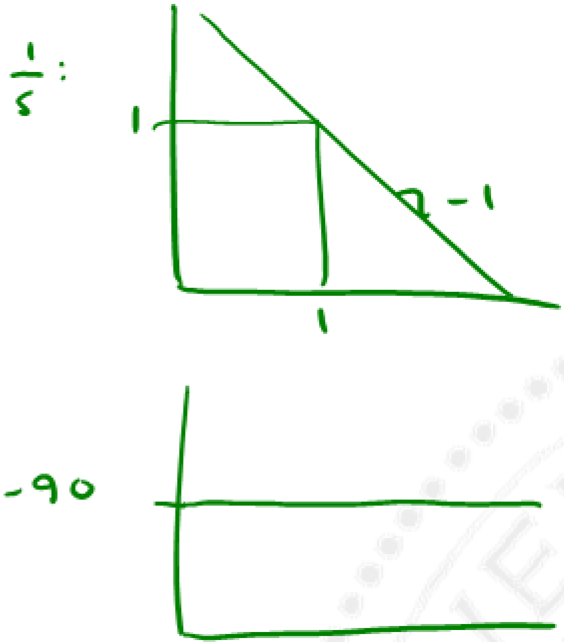
$\left|\frac{e}{r}\right| < 0.01$

$0 < \omega < 100$

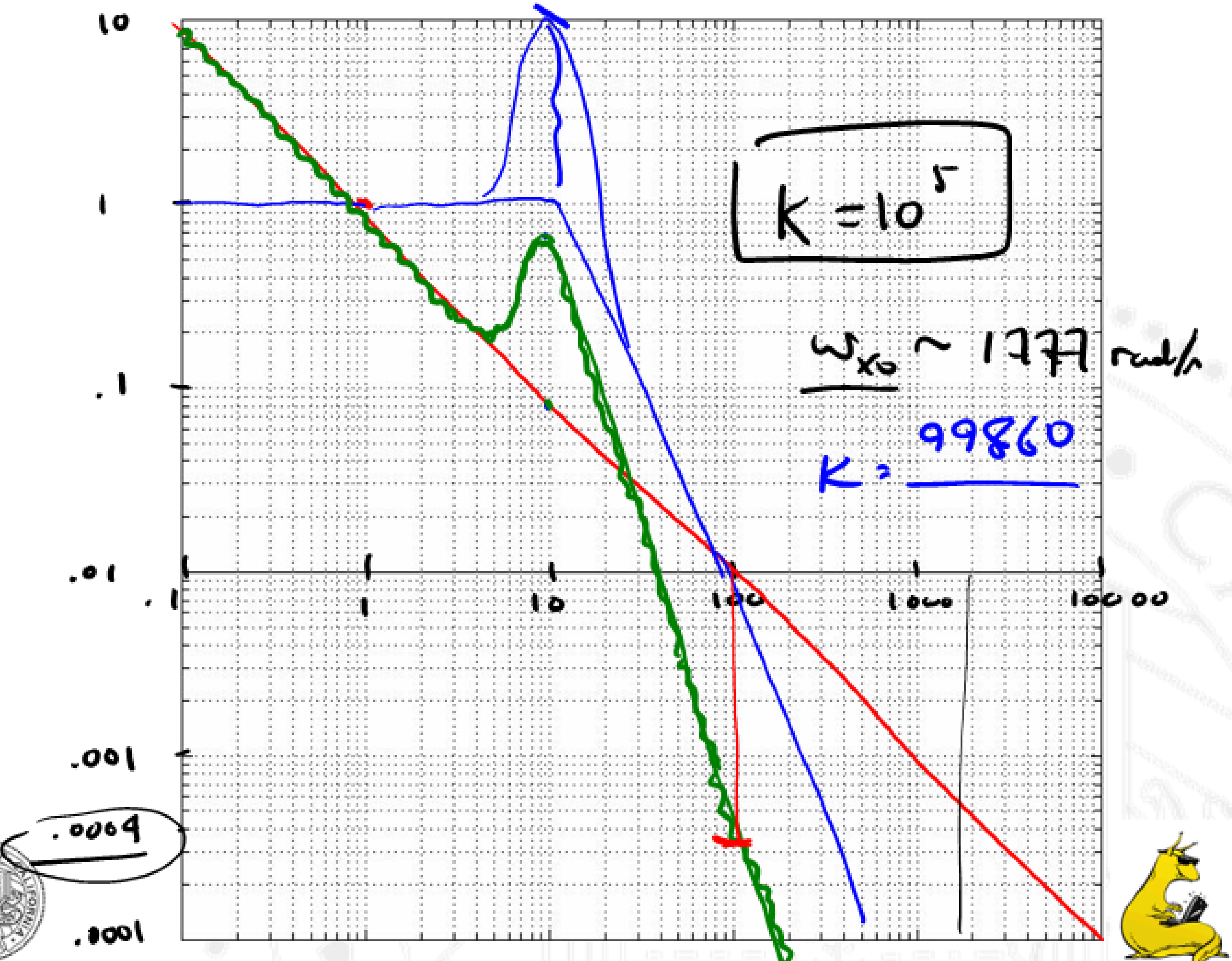
$PM > 50°$



$\frac{1}{s}$:



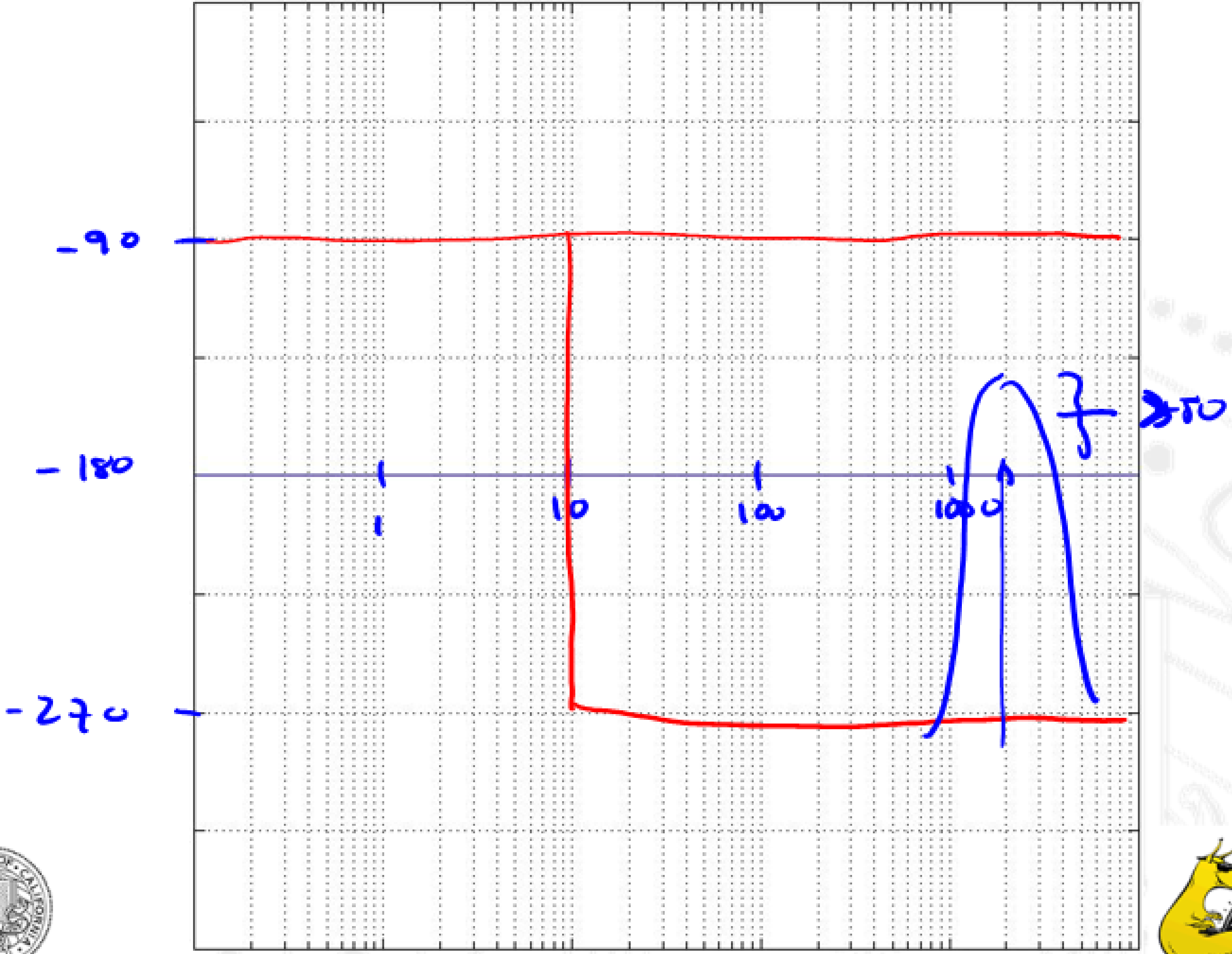
$$\frac{10^2}{s^2+2(0.05)10s+10^2}$$

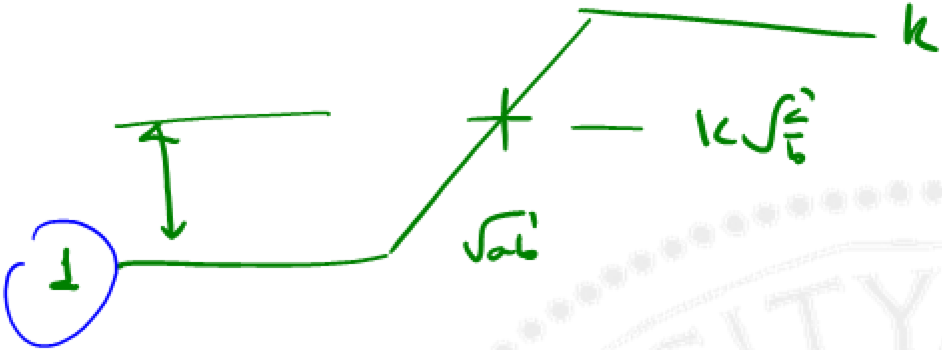


$K = 10^5$

$\underline{\omega_{xo}} \sim 1777$ rad/s

$K = \frac{99860}{\quad}$

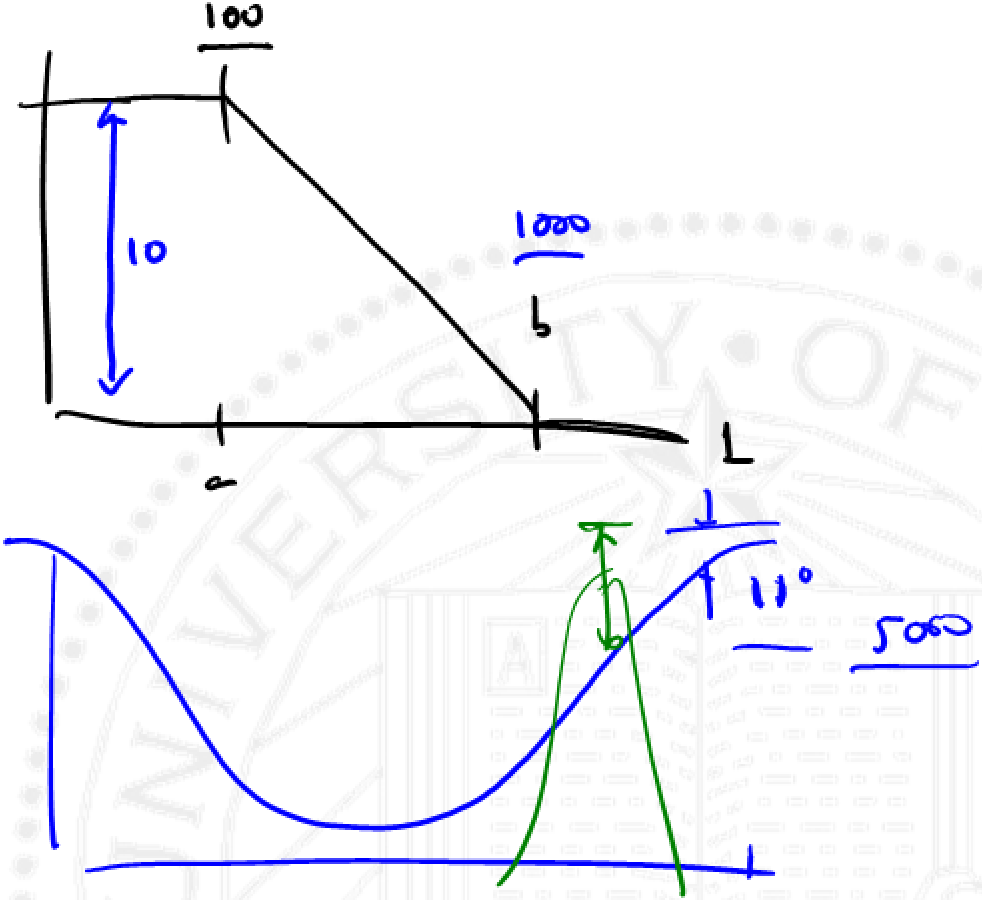

-90
-180
-270
1
10
100
1000
ζ → 0

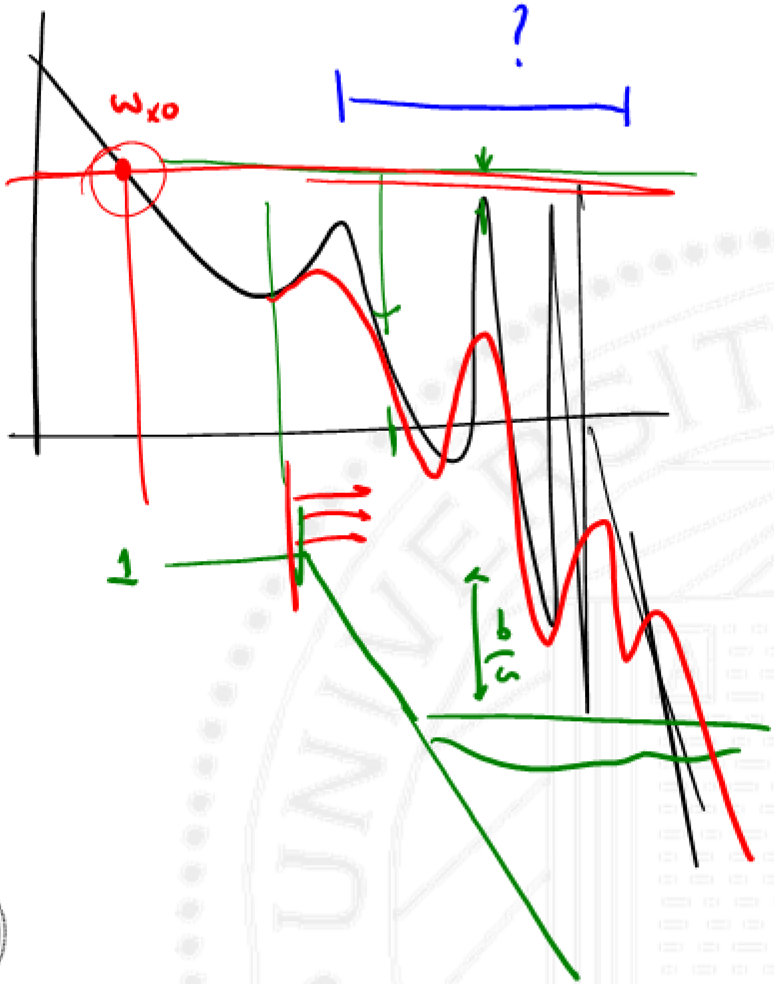


$$K(s) = 10^5 \cdot \left(\frac{s+562}{s+5620}\right)^2$$

Lag

$\frac{L}{a}$





$$K(s) = \left(\frac{a}{s+a}\right)^n$$

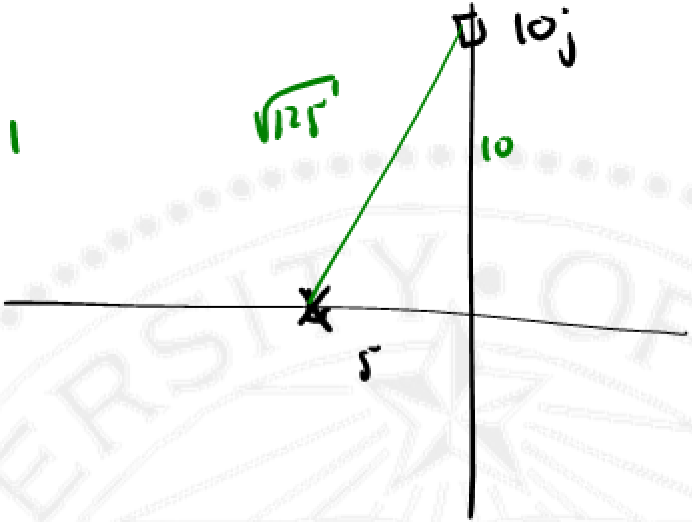
$$@\ \frac{a}{s} \rightarrow \delta\phi = (-11)n$$

$\underline{na}$

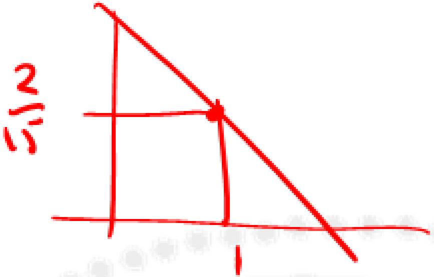
$K_{ua}(s) = \left(\frac{5}{s+5}\right)$

$\omega \overset{?}{=} 10$

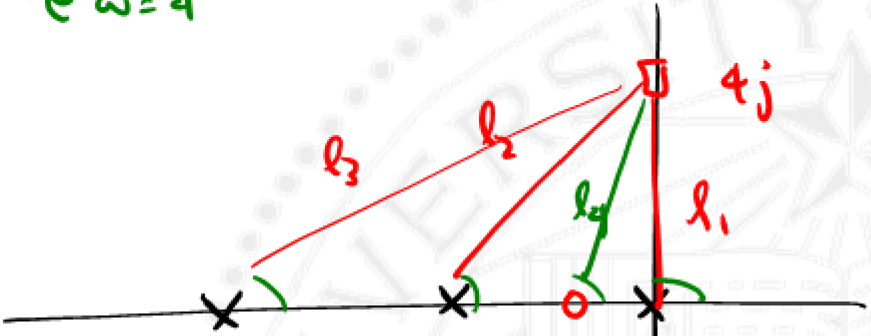
$\left|K_{ua}(s)\right|_{s=10j} = \frac{1}{\sqrt{125}} \simeq .1$



$$G(s) = \frac{s+2}{s(s+3)(s+5)}$$



$|GK|_{@\,\omega=4}$



$$\measuredangle = -90 + \tan^{-1}\left(\frac{4}{2}\right) - \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{4}{5}\right)$$