CMPE-242
Applied Feedback Control

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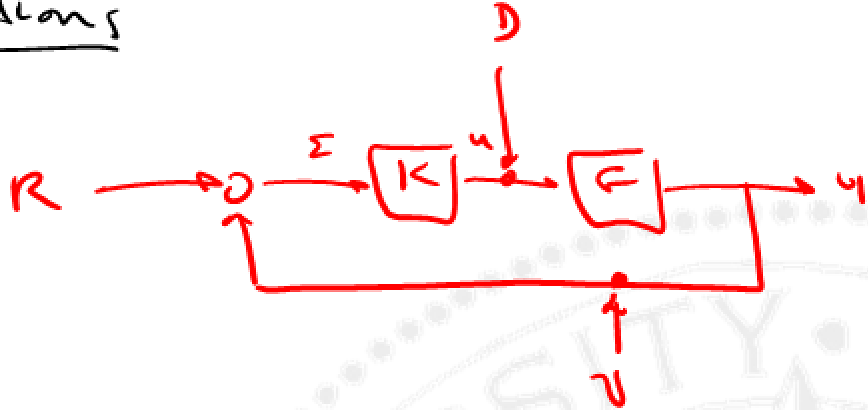
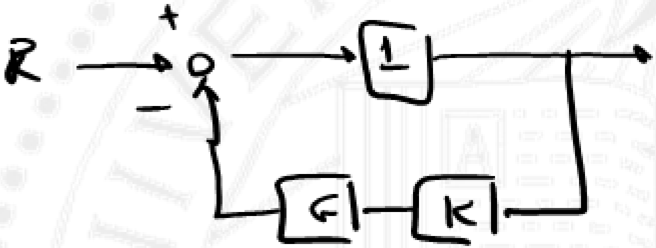
Questions



$$\frac{Y}{R} = \frac{GK}{1+GK}$$

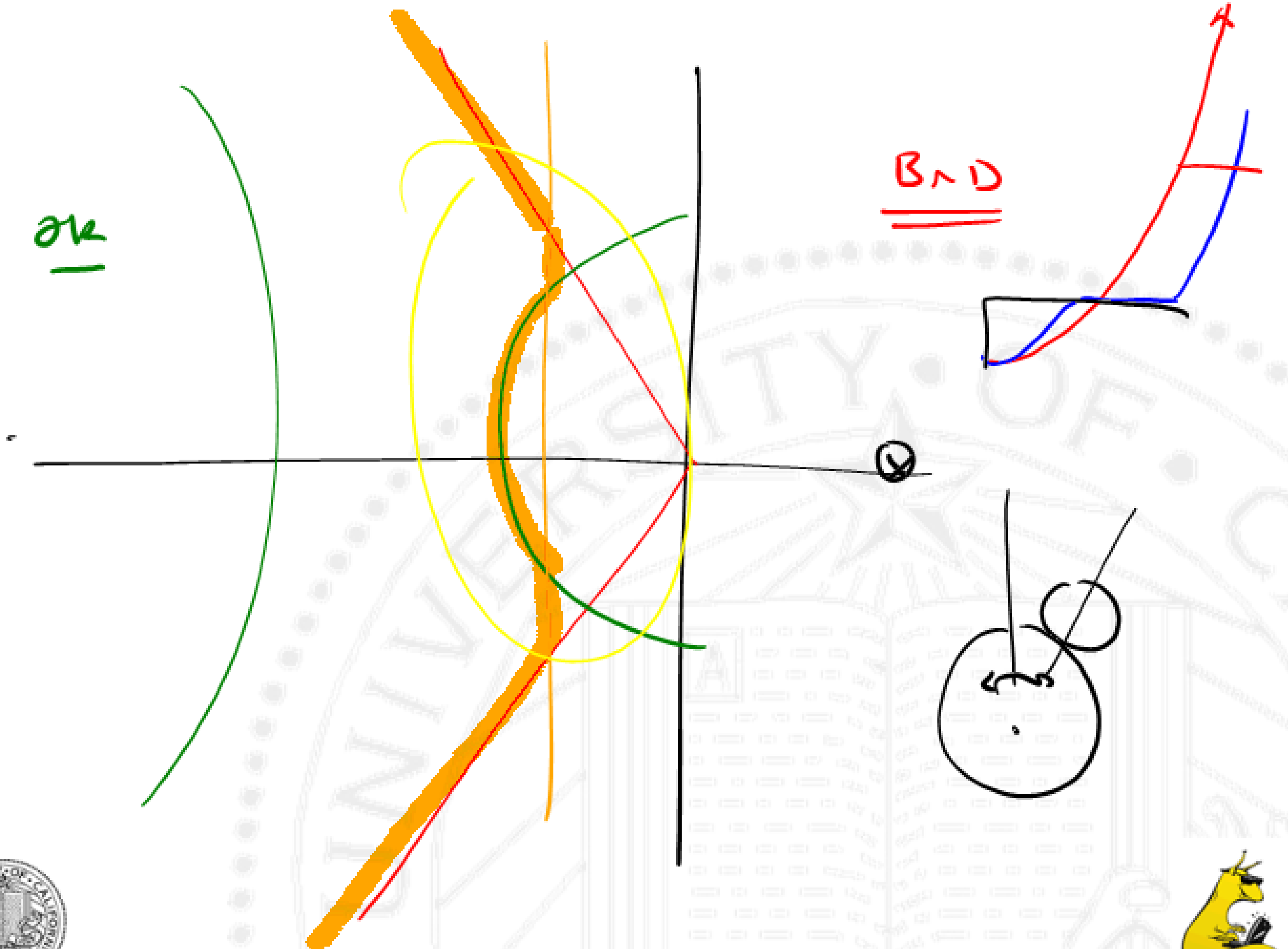
$$\frac{Y}{D_s}$$



$$\frac{E}{R} = \frac{1}{1+GK}$$

$$E_{ss} = \text{FVT} \rightarrow \lim_{s \to 0} s\left[\frac{E}{R}\right] R(s)$$

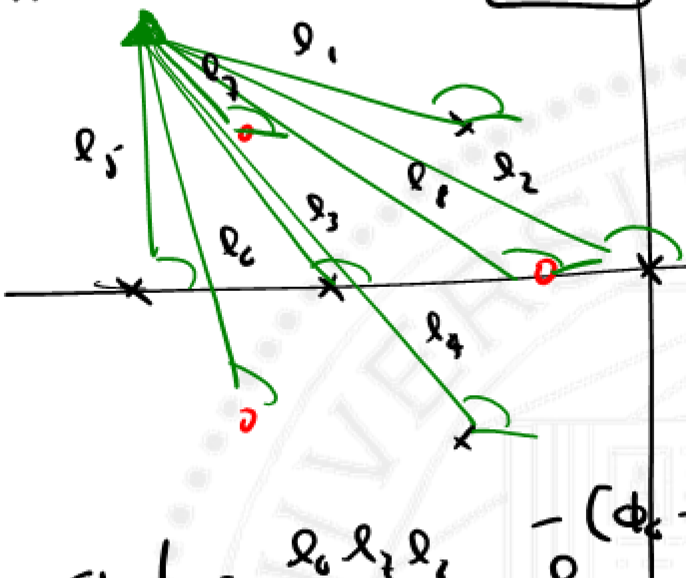
$\uparrow \frac{1}{s}$


ok
BAD

Root Locus

$\Delta_{cl}(s) = 1 + GK = 0 \therefore \boxed{GK = -1}$

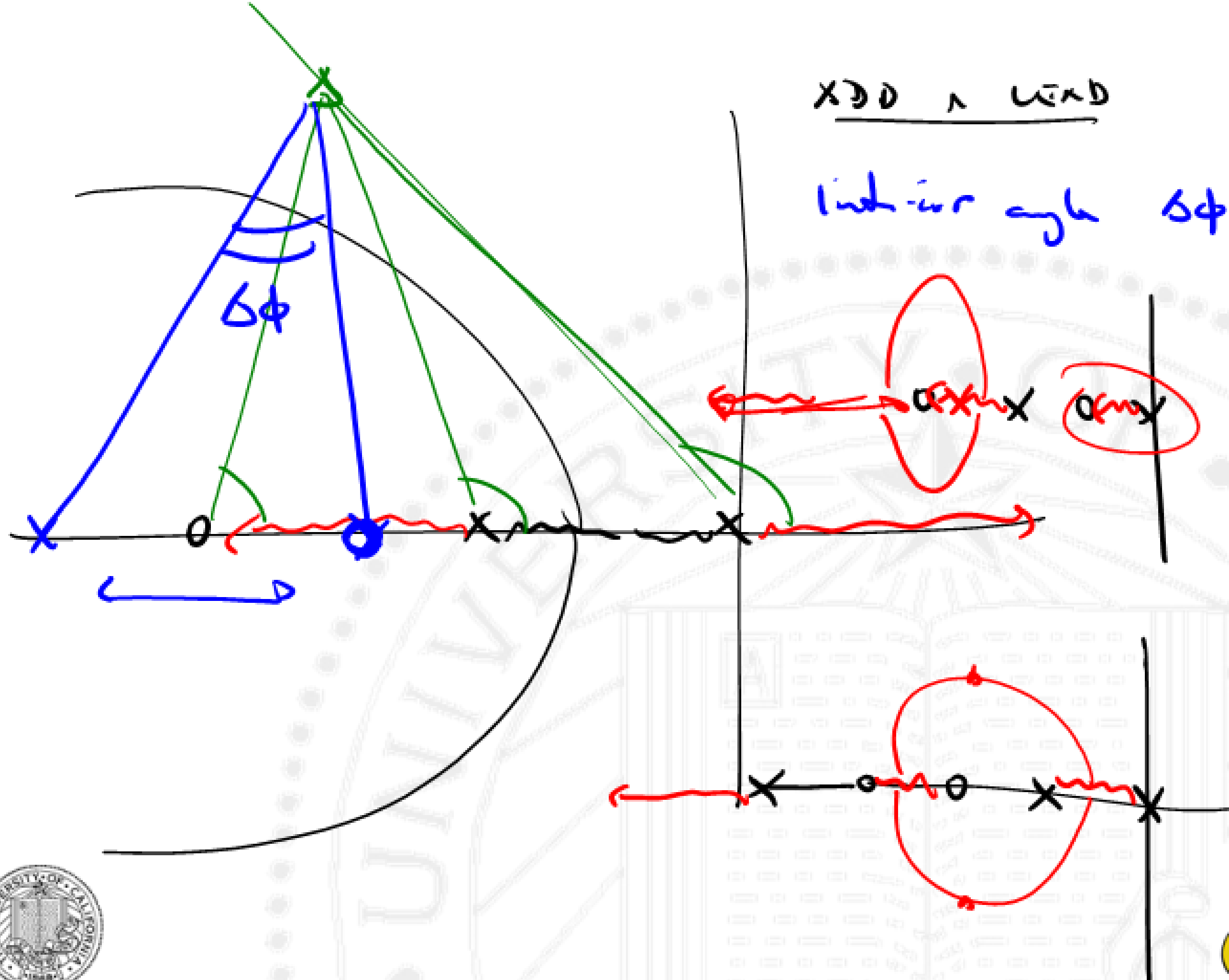
- $\angle \pm 180$
- $|\cdot| = 1$

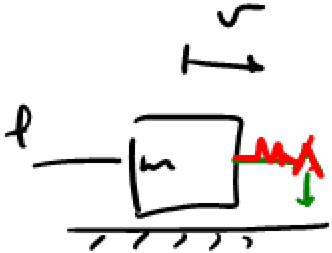


$\angle \sum z_i - \sum p_i$

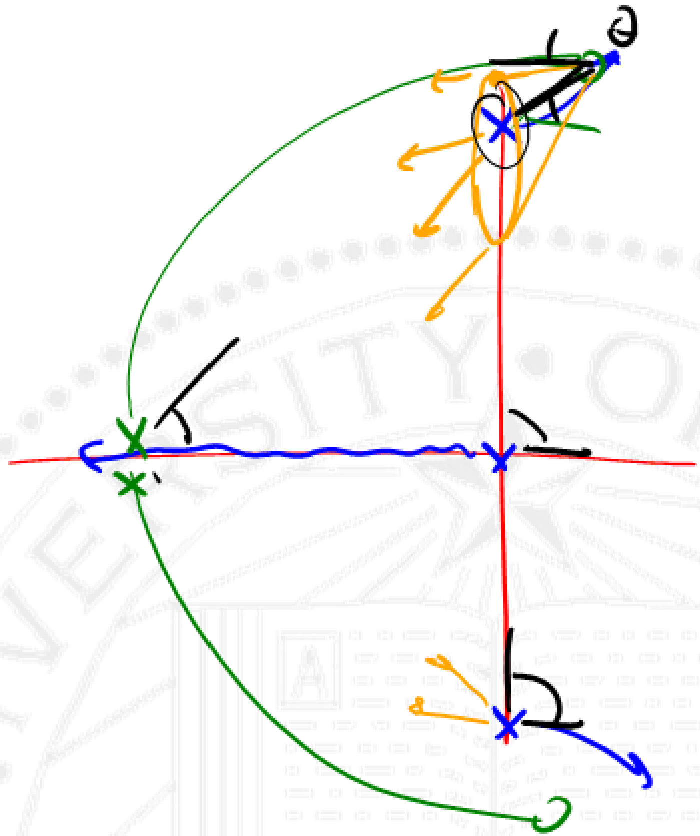
$\| \; \dfrac{\prod z_i}{\prod p_i}$

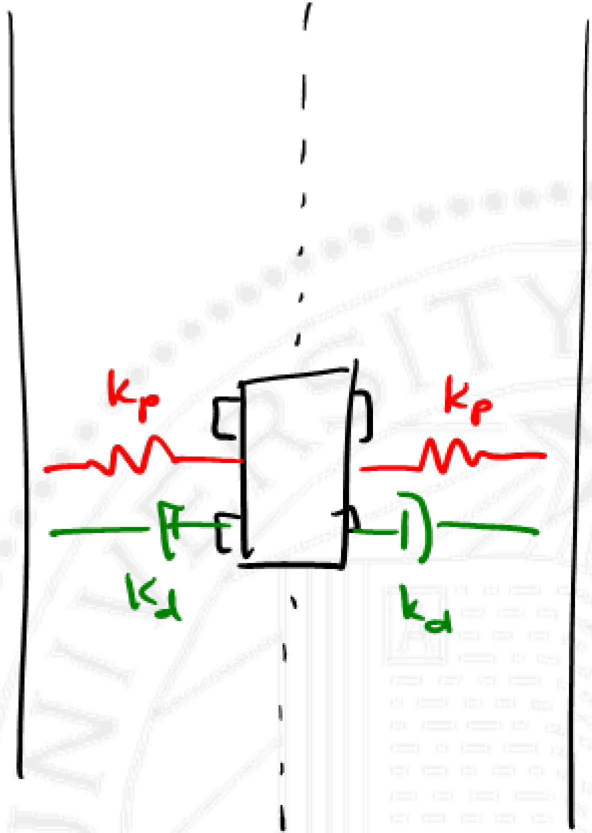
$$GK\Big|_s = \frac{l_6 l_7 l_8}{l_1 l_2 l_3 l_4 l_5} e^{-(\phi_6 + \phi_7 + \phi_8 - \phi_1 - \phi_2 \dots \phi_5)j}$$


Δφ



$$\phi_d \sim f(\theta)$$





K_p "spring"

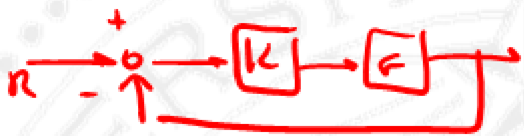
K_b "dampers"

K_i – kill steady state error.

BODE — TRACKING SPEC'S / ROBUSTNESS

— MODEL FREE DESIGN

ROOT LOCUS



$$\frac{Y}{R} = \frac{GK}{1+GK} \leftarrow \Delta_{cl}$$

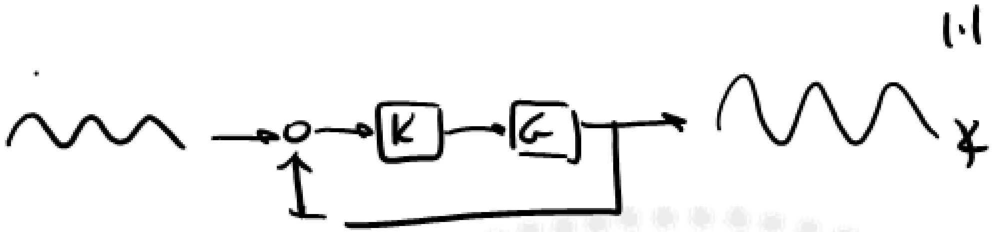
$$1+GK=0 \rightarrow GK = -1$$

∠ 180

|| 1

$$KG = \frac{K(s+a)(s+b)}{(s+c)(s+d)(s+e)} = K\frac{k_a k_b}{k_c k_d k_e} e^{j(\phi_a+\phi_b-\phi_c-\phi_d-\phi_e)}$$

Bode:

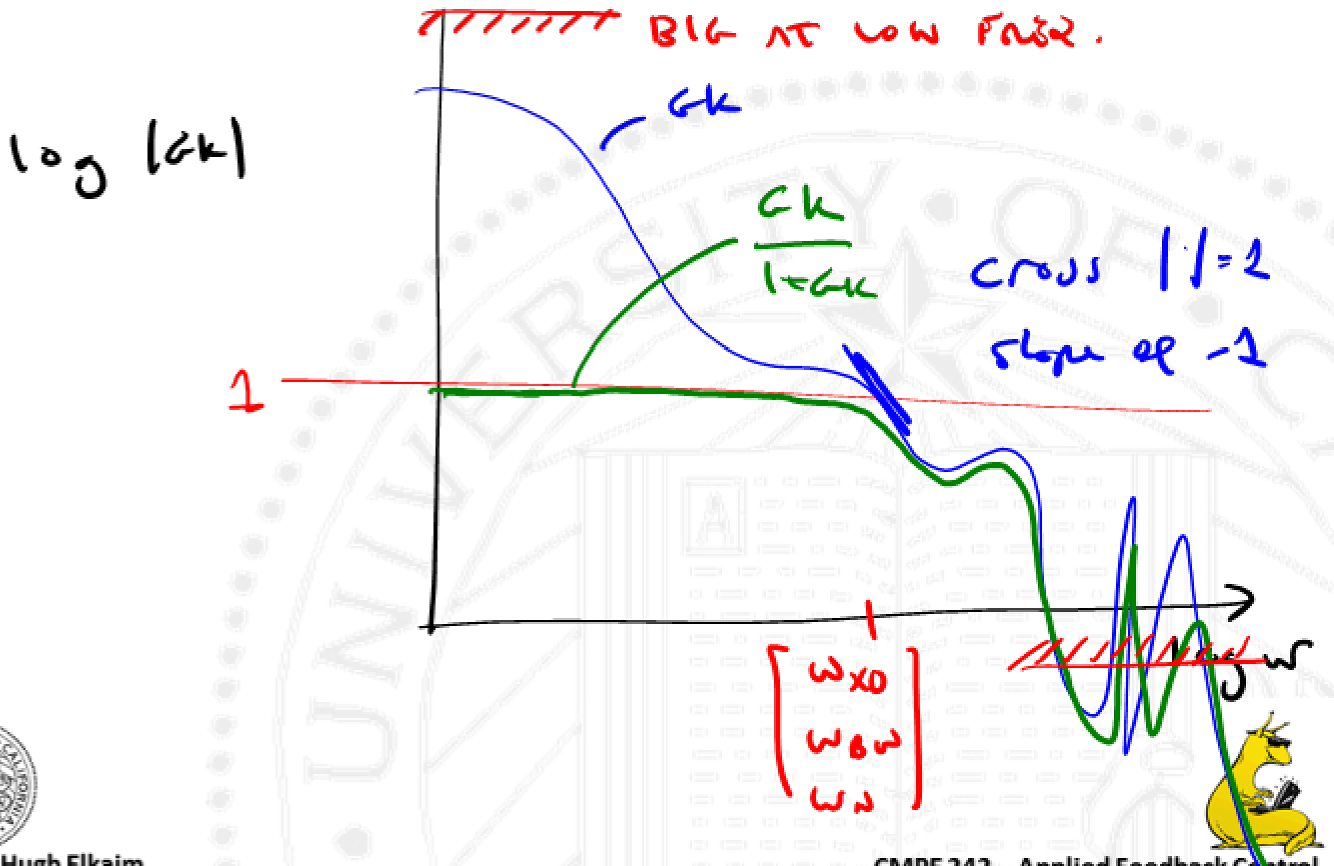


$$\frac{Y}{R} = \frac{GK}{1+GK}$$

if $|GK|$ big

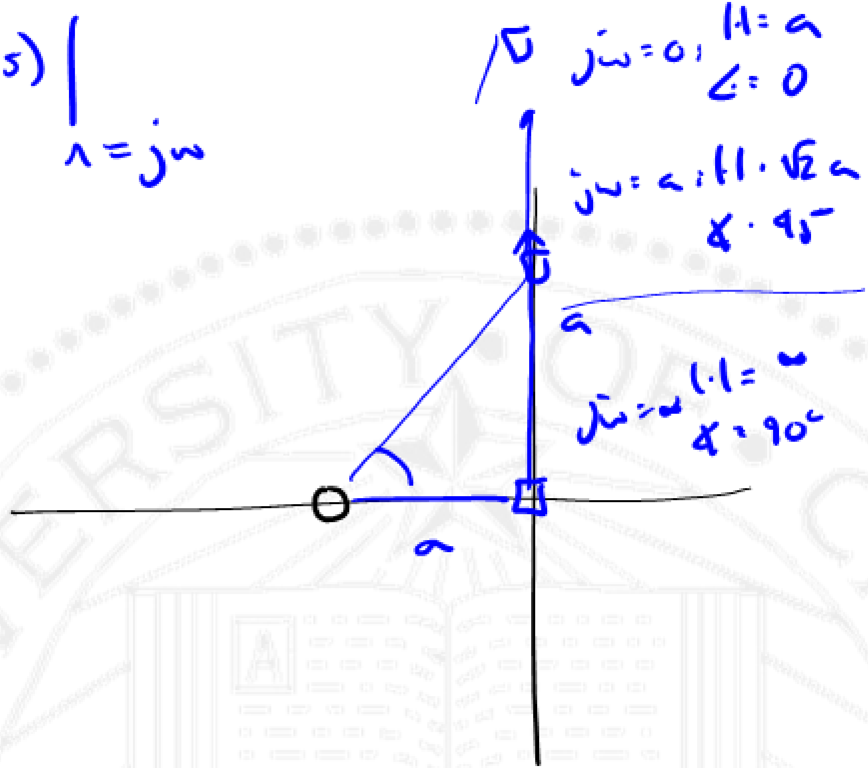
$$\frac{GK}{1+GK} \simeq 1.$$

BODE DESIGN



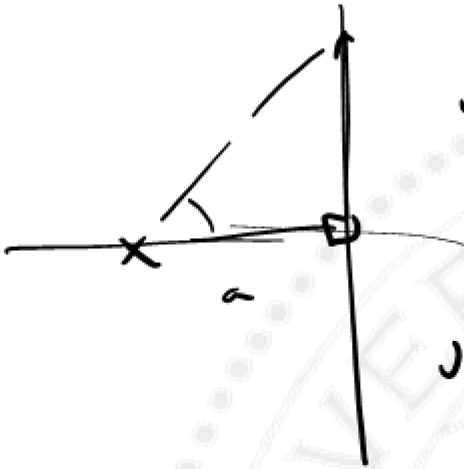
Bode → $GK(s)\big|_{s=j\omega}$

$G(s) = s + a$



$$G(s) = \frac{1}{s+a}$$

$j\omega = 0 \rightarrow |\cdot| = \frac{1}{a}$, $\angle = 0$



$j\omega = a$ $\quad |\cdot| = \frac{1}{\sqrt{2}a}$

$\angle = -45^\circ$

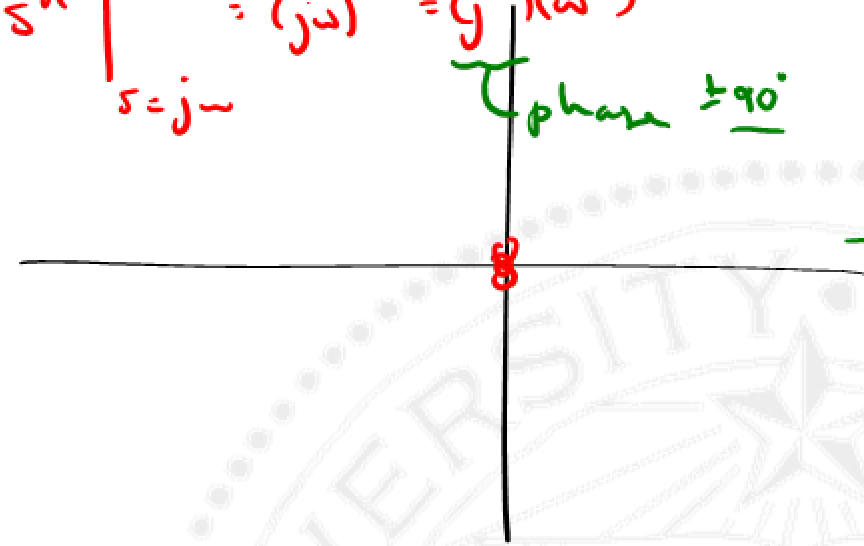
$j\omega = \infty$ $\quad |\cdot| = 0$

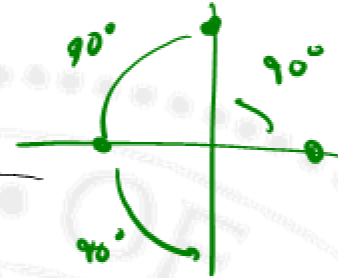
$\angle = -90^\circ$

$$s^n \Big|_{s=j\omega} = (j\omega)^n = (j^n)(\omega^n)$$

(ω^n) → magnitude

(j^n) → phase $\pm 90°$





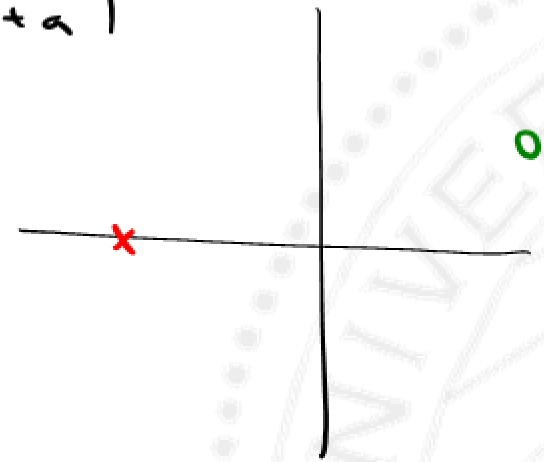
$$\log(\omega^n) = n \underline{\log \omega}$$

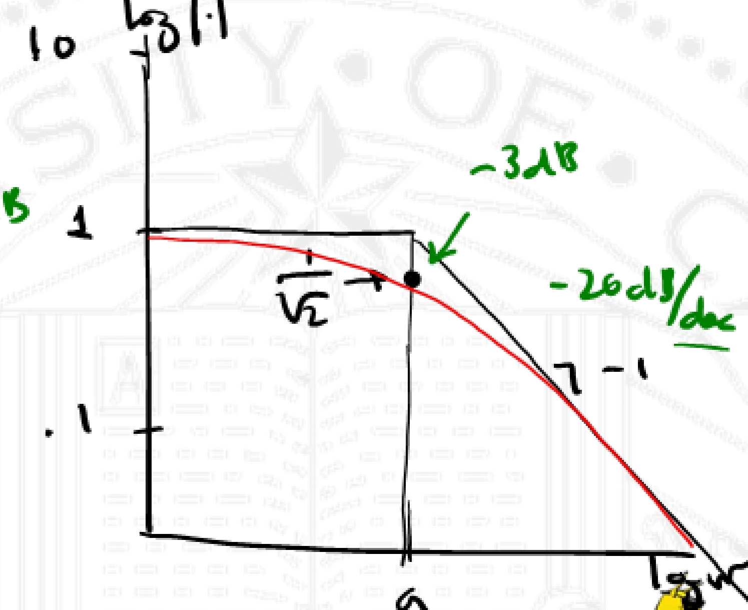
↑ straight lines

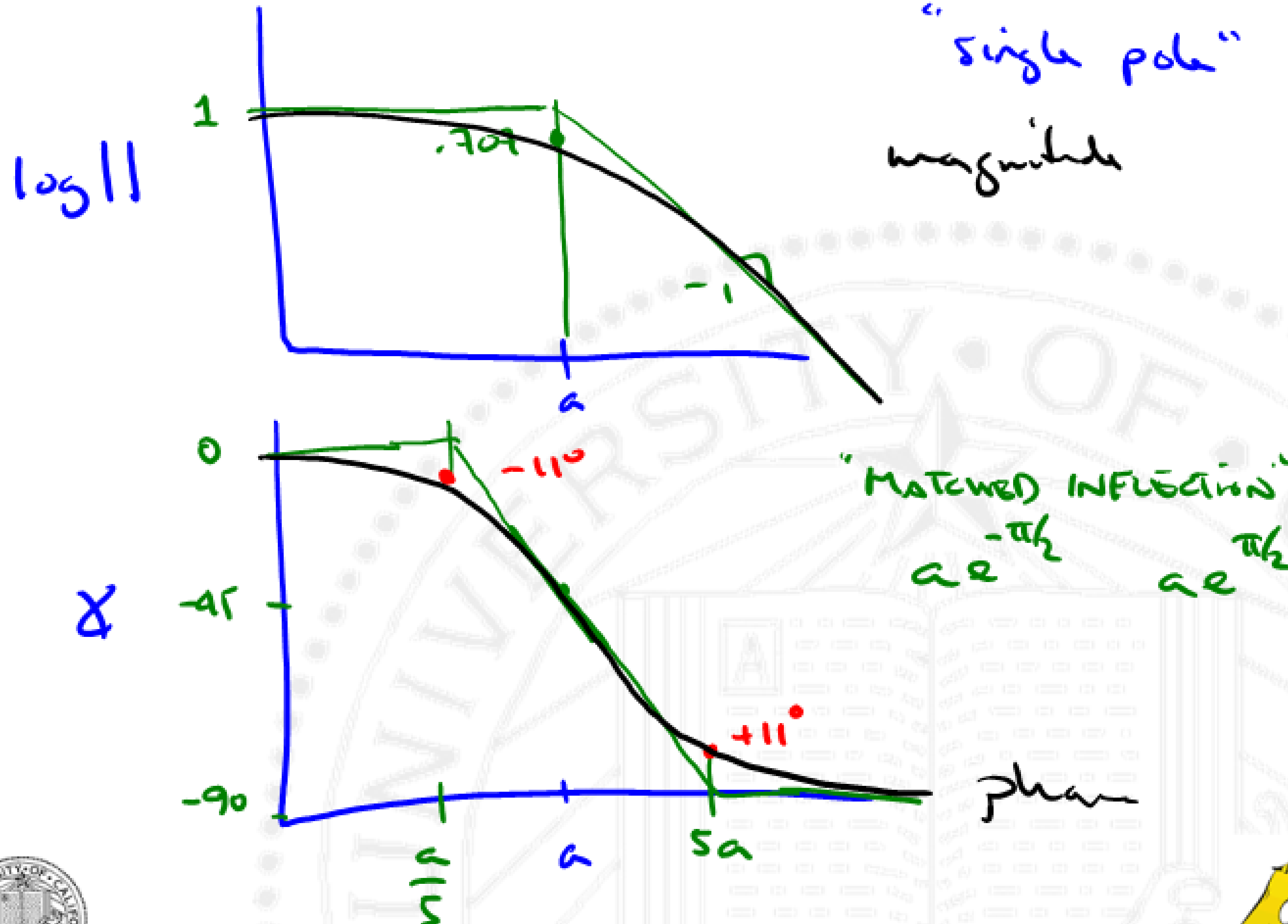
$|\,|$ — $\log|\,|$ vs $\log\omega$

$\angle$ — ϕ vs $\log\omega$

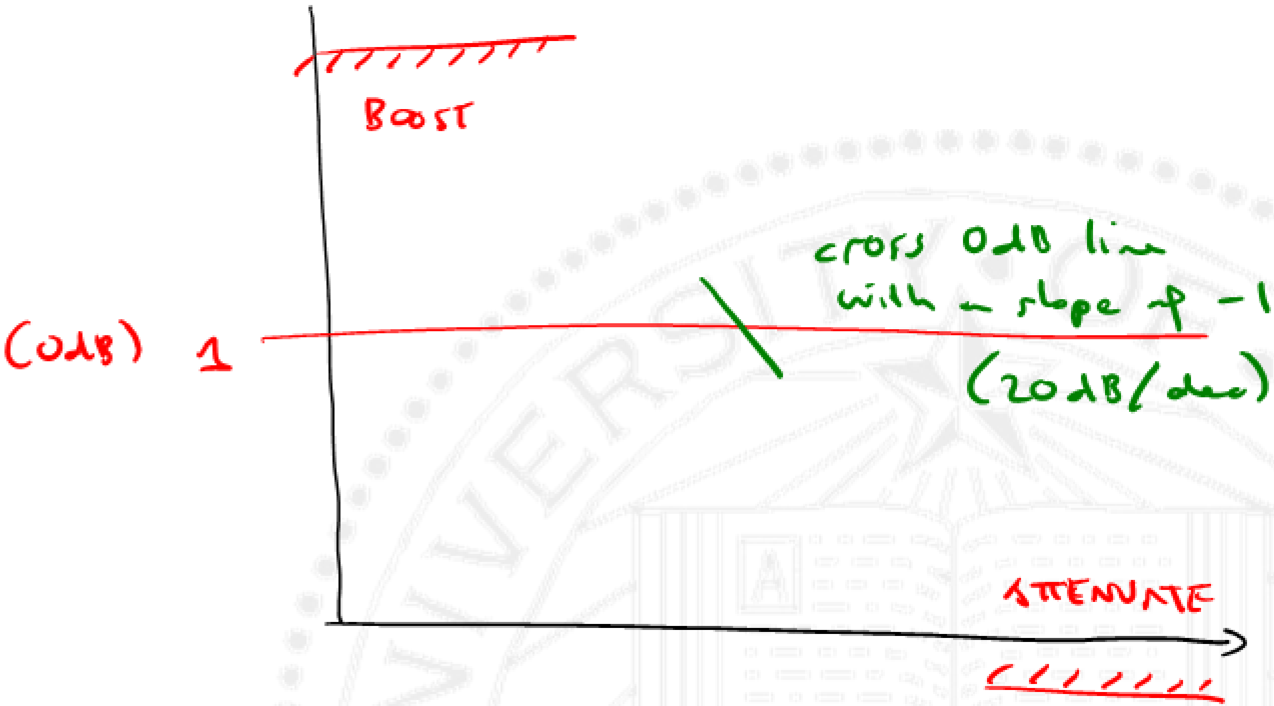
$\left(\frac{a}{s+a}\right)$





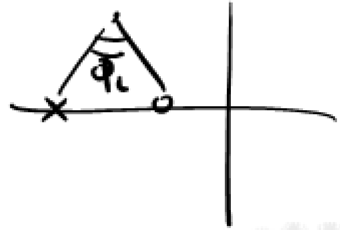

"single pole"
magnitude
log||
1
.707
-1
a
0
-11°
"MATCHED INFLECTION"
$ae^{-\pi/2}$
$ae^{\pi/2}$
-45
-90
+11°
phase
a/5
a
5a

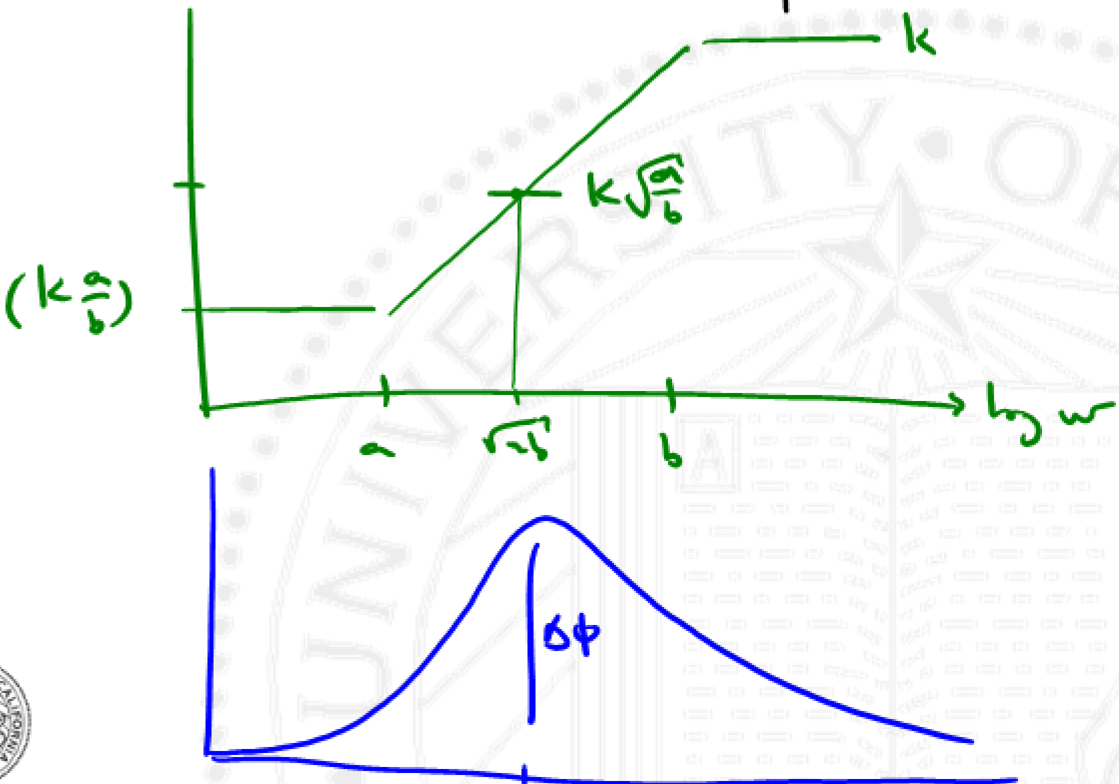
BODE DESIGN

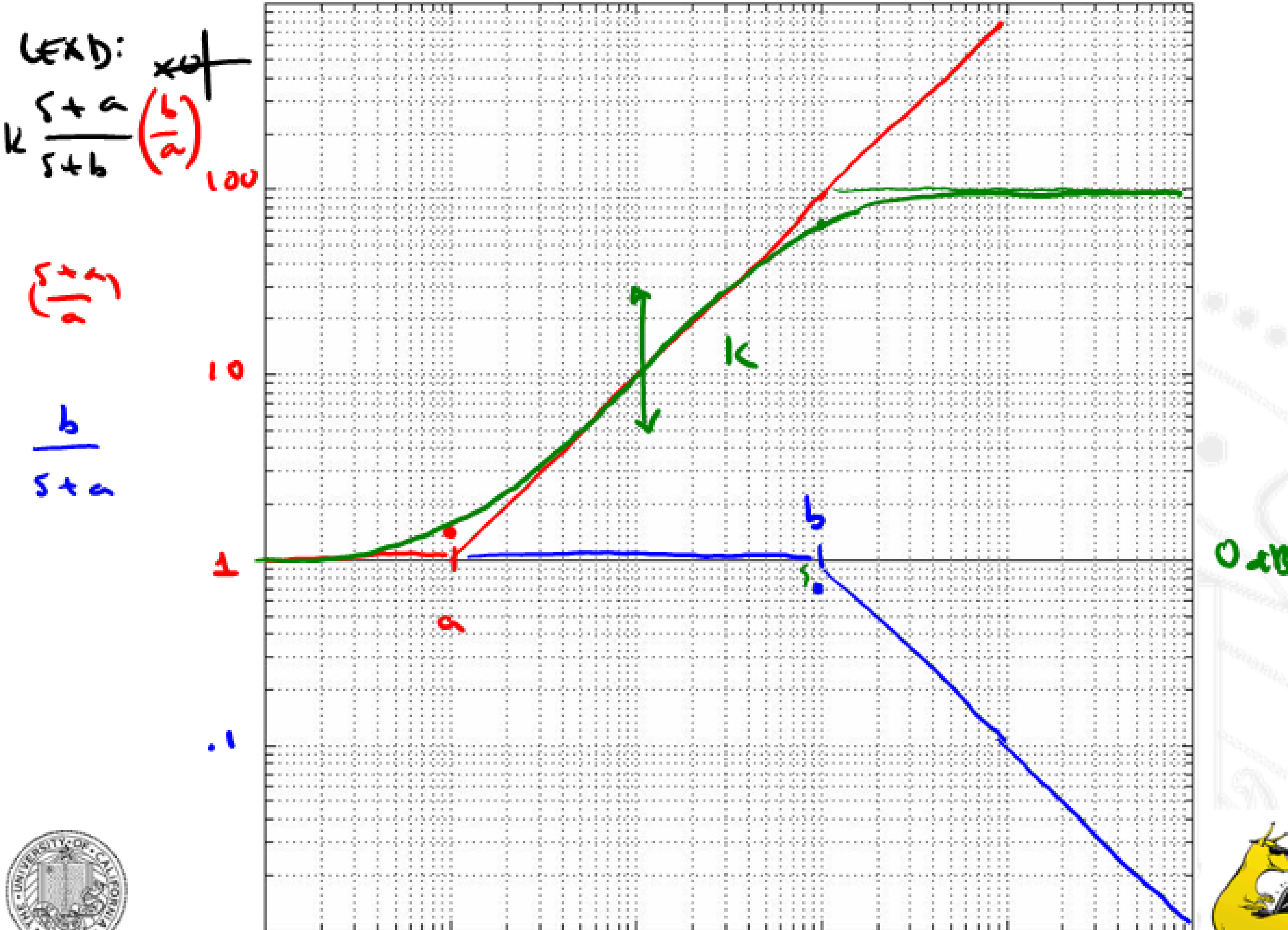

BOOST
cross 0dB line
with a slope of -1
(20dB/dec)
(0dB)
1
ATTENUATE

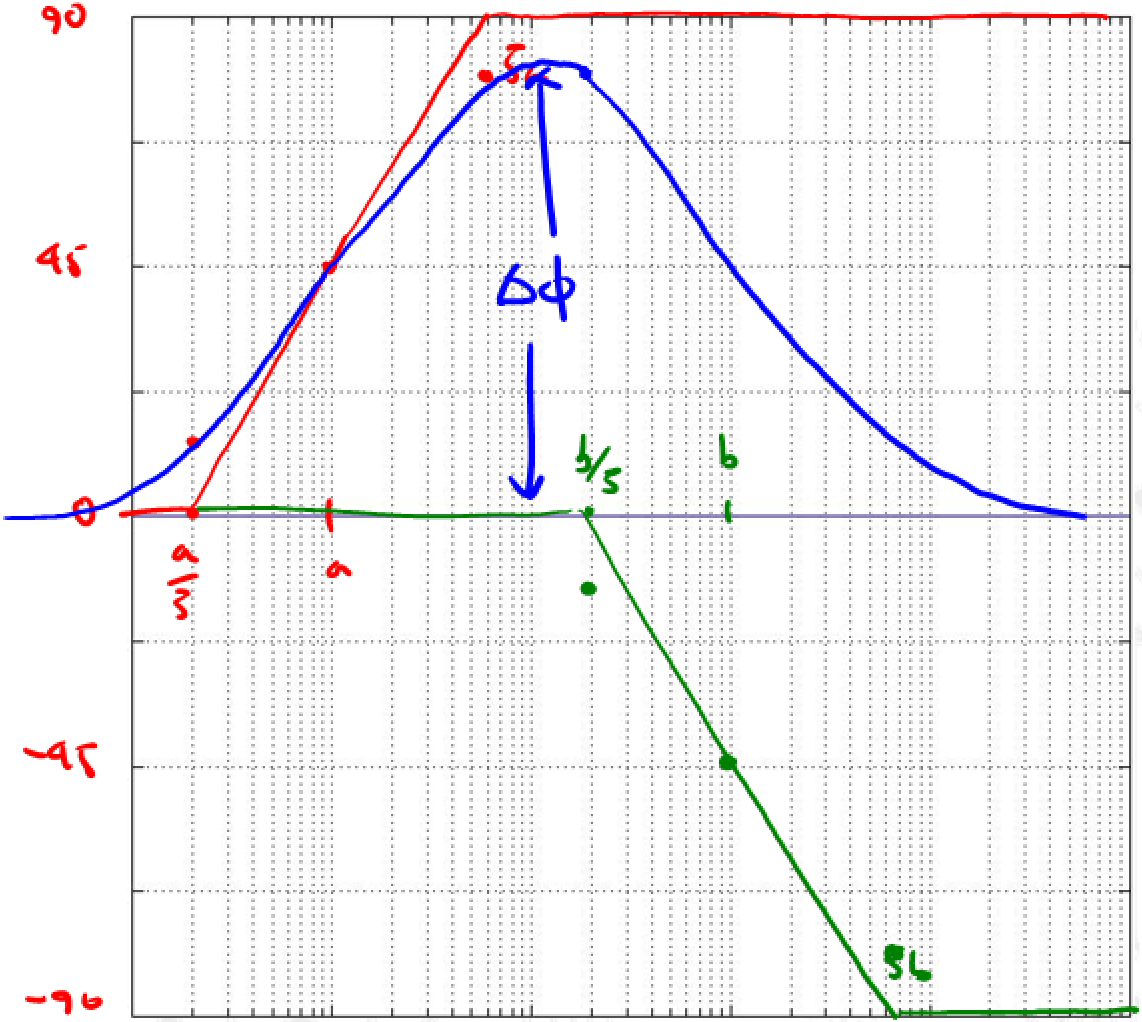
LEAD

$K \frac{s+a}{s+b}$ $\quad b > a$






100
10
1
.1
K
a
b
0 dB



90
0
Δφ
a
b

LEAD

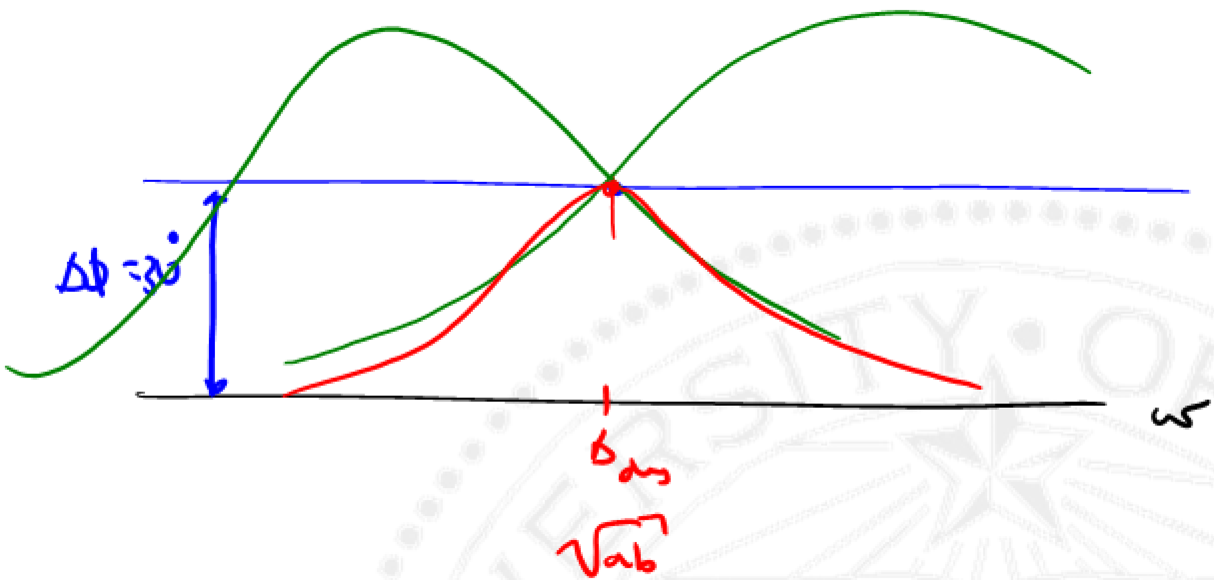
$K \frac{s+a}{s+b}$

choose $K = \sqrt{\frac{b}{a}}$

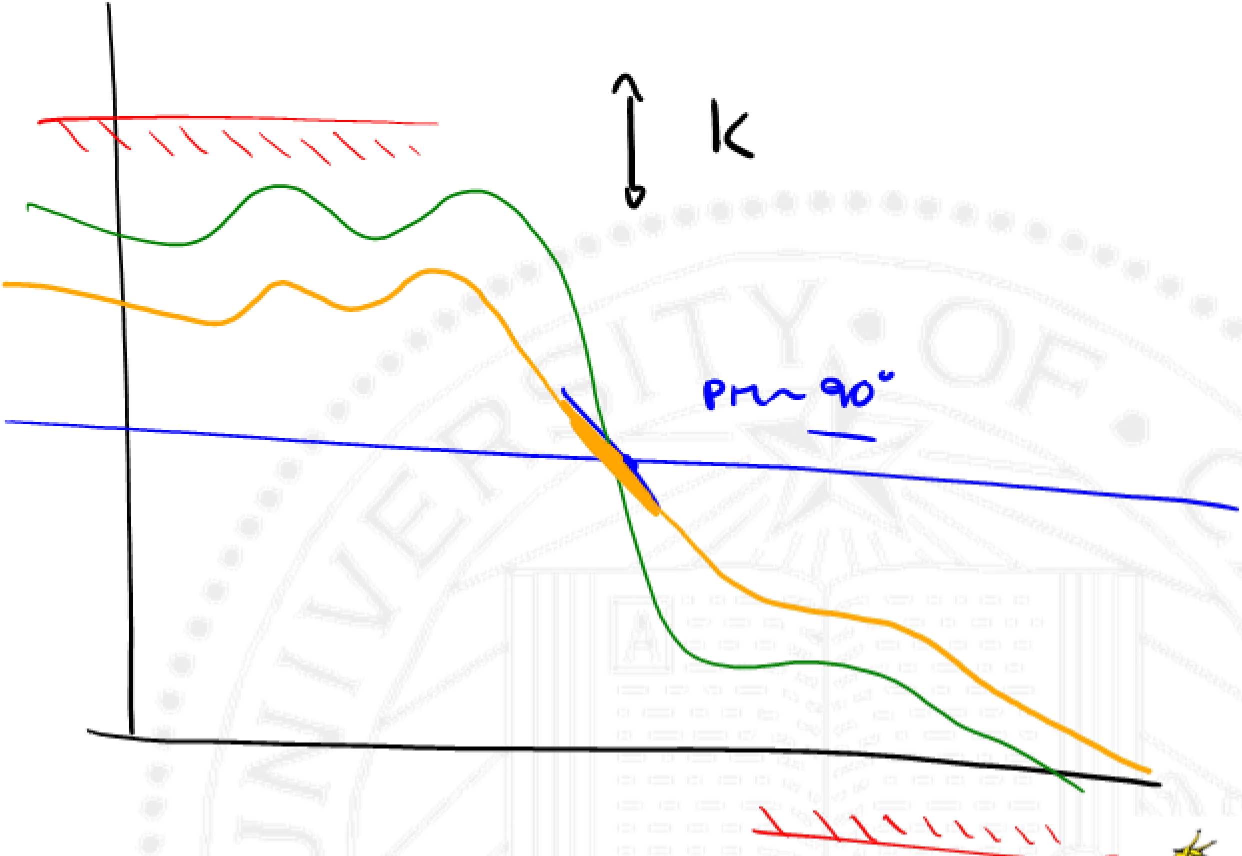
UNITY GAIN

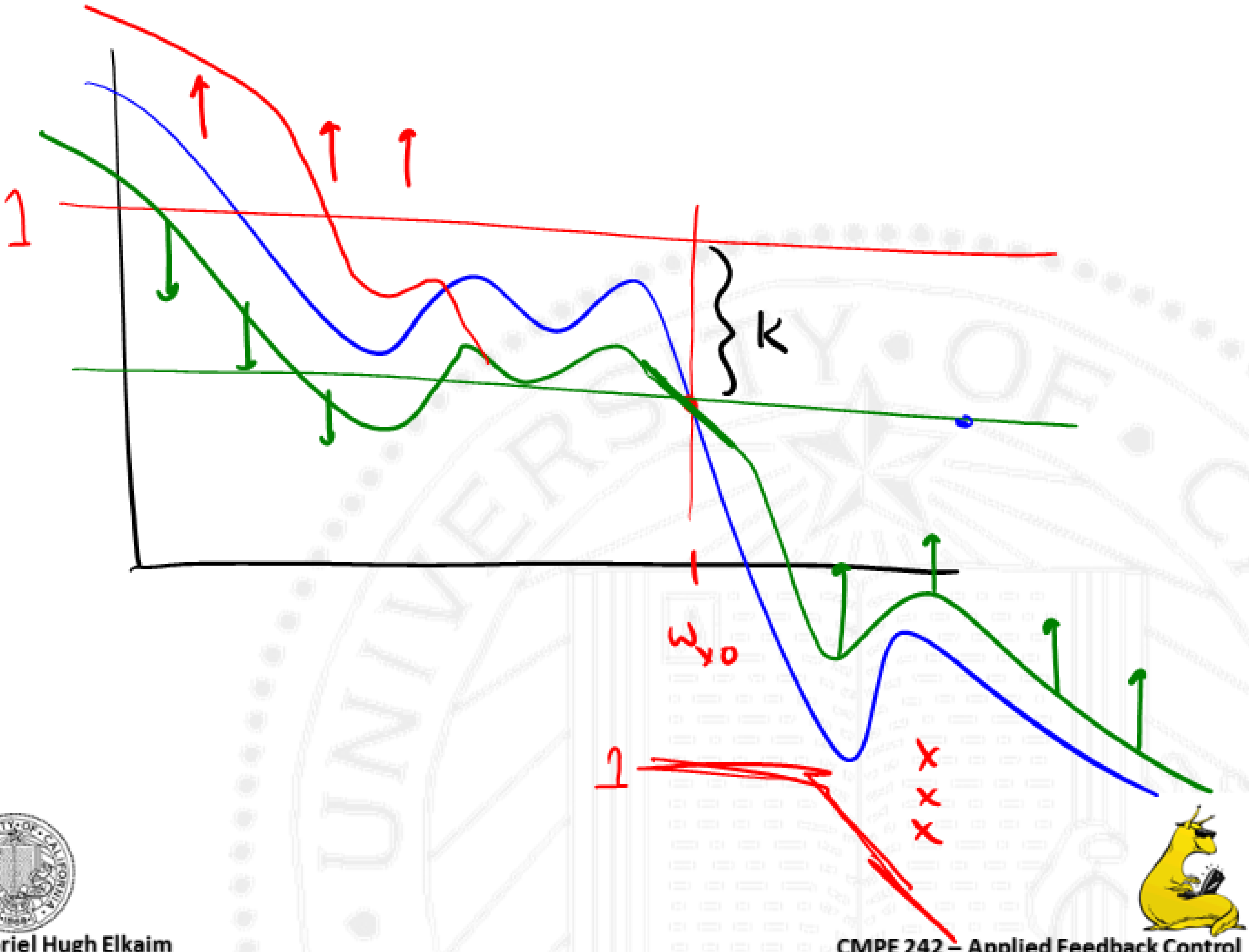
@ $\sqrt{ab}$

(b/a)	Δ PM
4	36°
10	55°
25	68°
50	72°

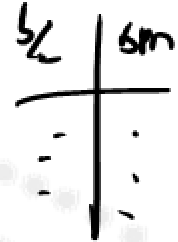


ω
$\sqrt{ab}$


K
PM ~ 90°


1
k
ω_{yo}
1

BODE DESIGN



(1) GAIN FOR ω_{xo} (K)

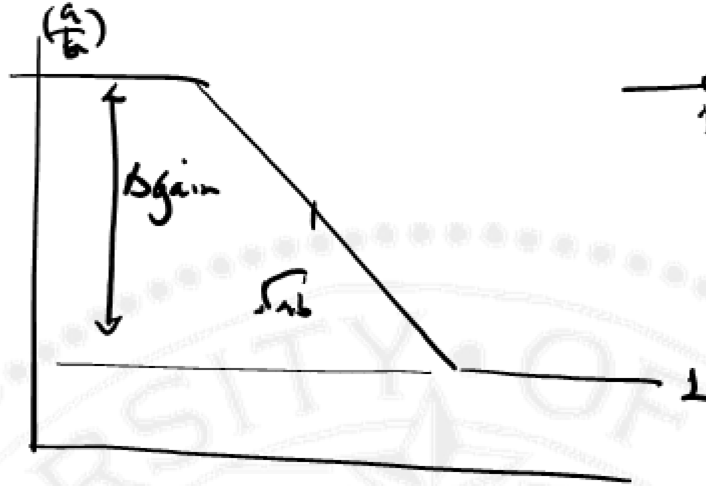
(2) LEAD for PM $\longrightarrow$ $K_{LEAD} = \sqrt{\frac{b}{a}}$ $\omega_{xo} = \sqrt{ab}$

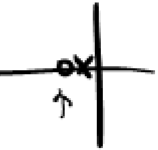
(3) LAG FOR LOW FREQ. GAIN

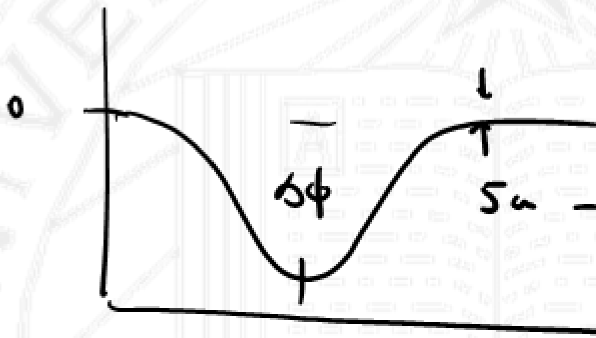
@ 5a $\longrightarrow$ lose 11° of phase

(4) LAG or POLES @ HIGH FREQUENCIES

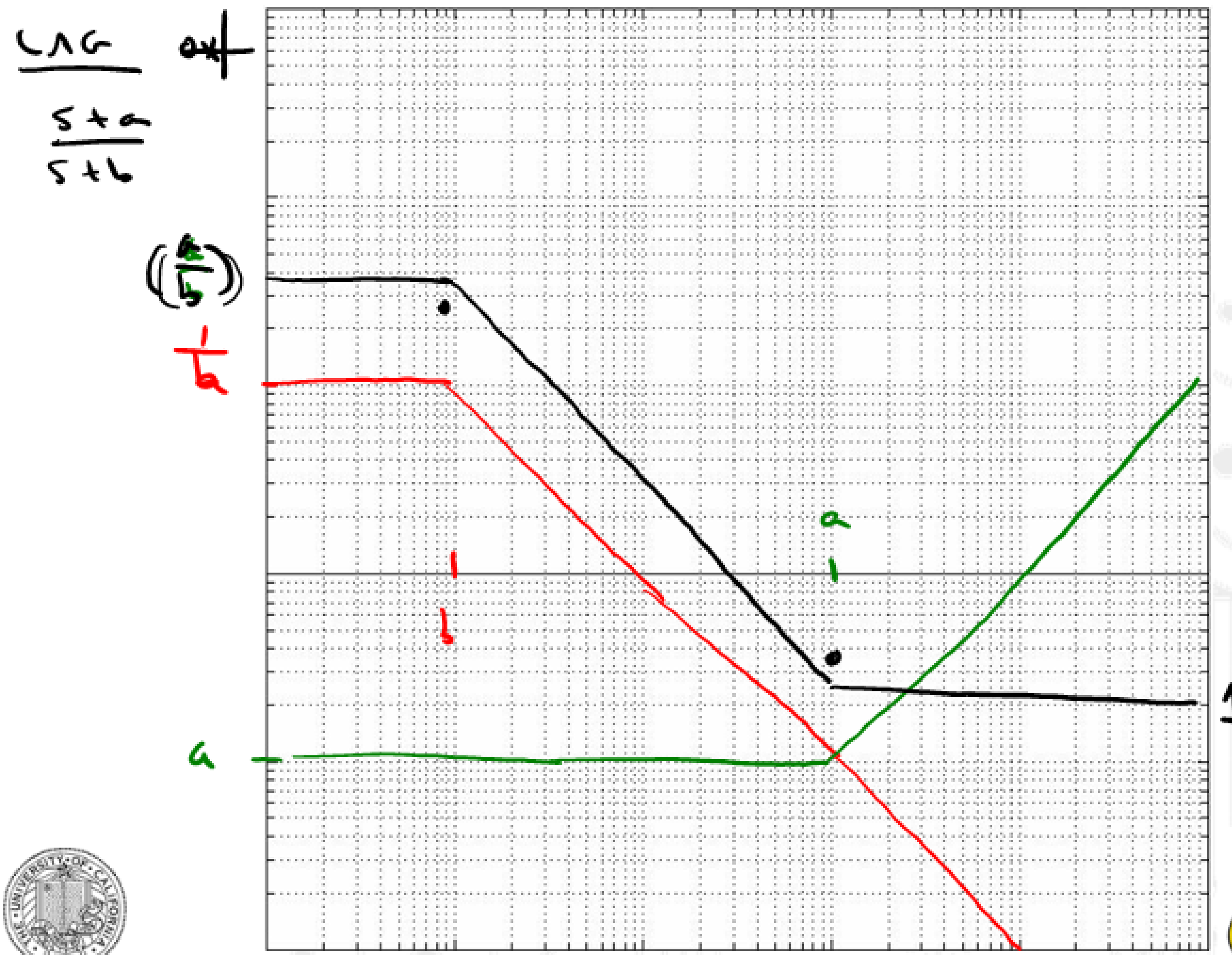
Lag $\left(\frac{s+a}{s+b}\right)$



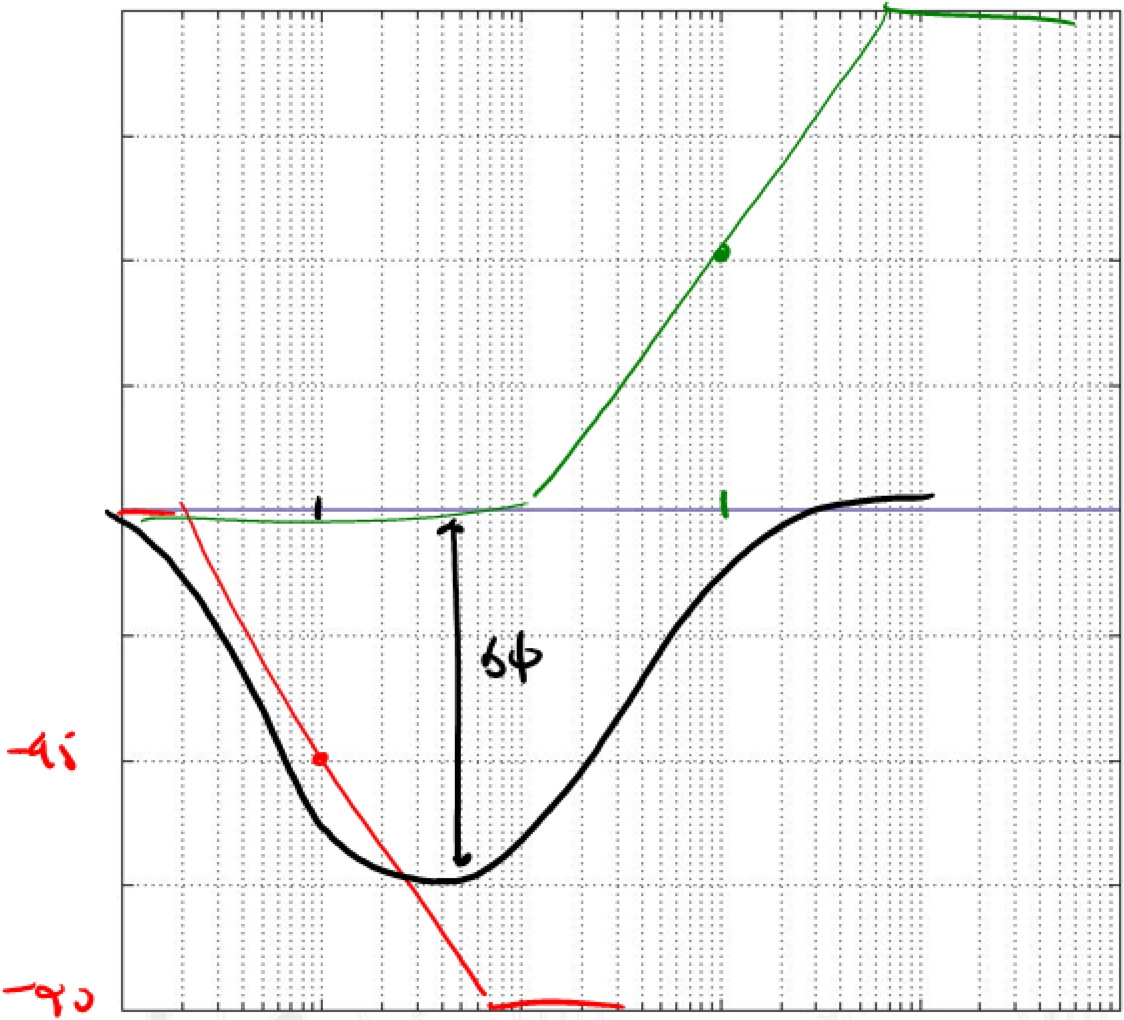




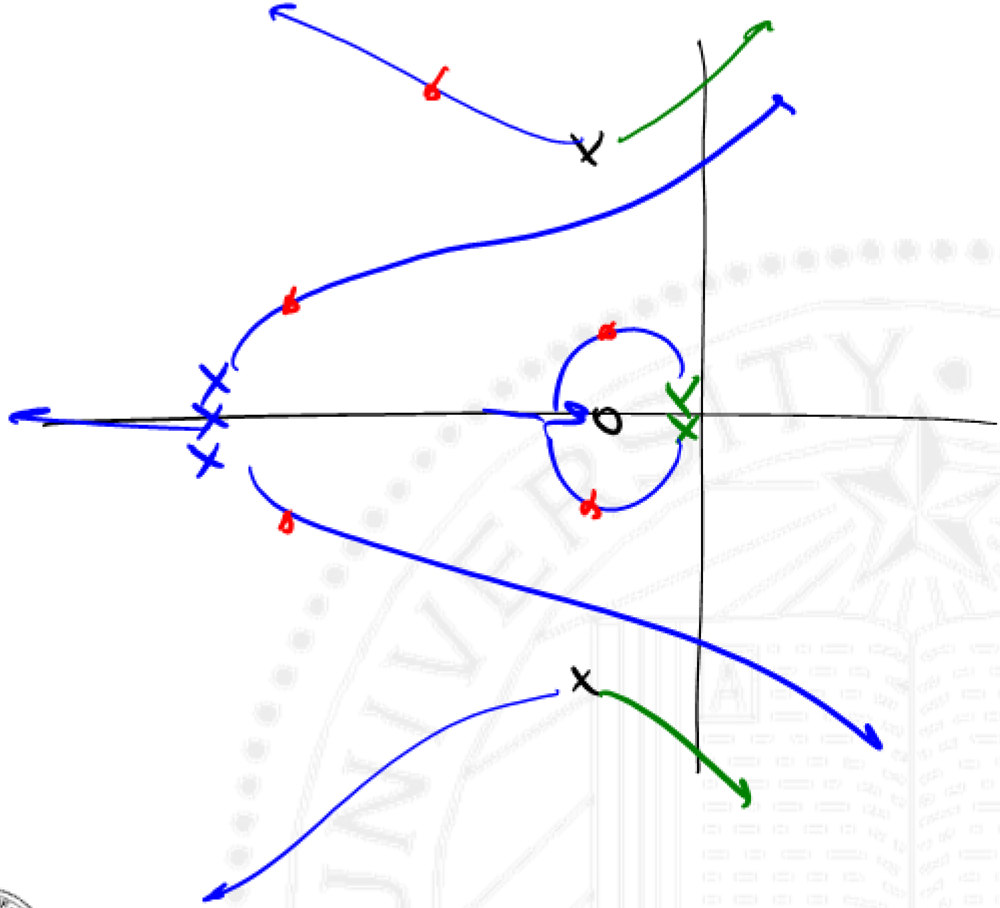
5a – 10sin 11°



1
a

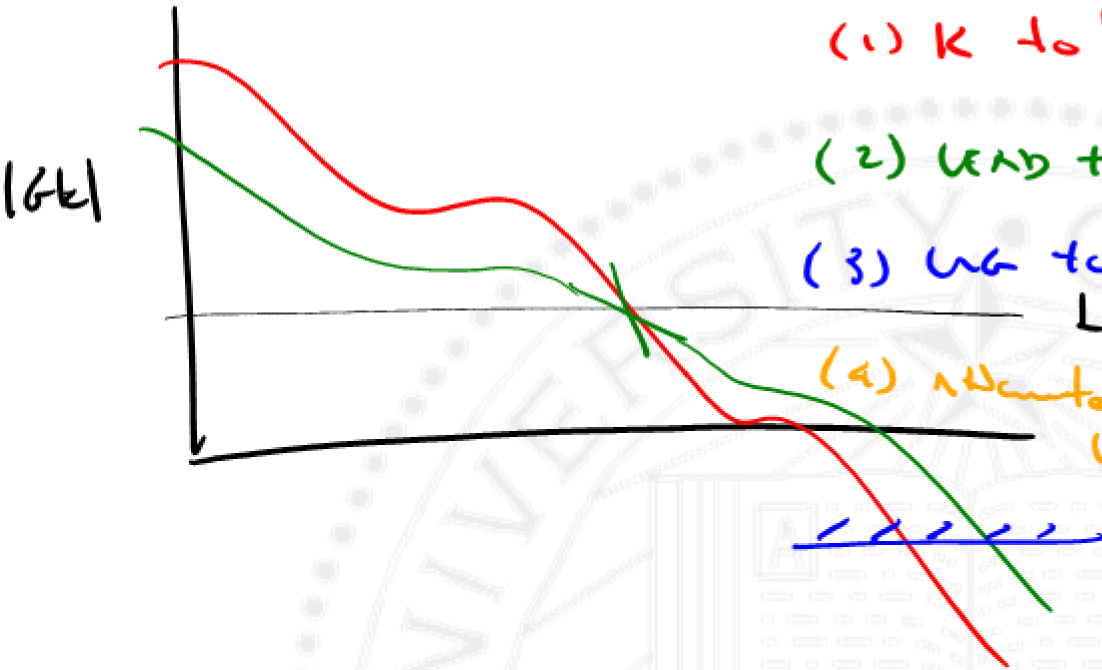


-45
-90



OPEN LOOP BODE

|GK|

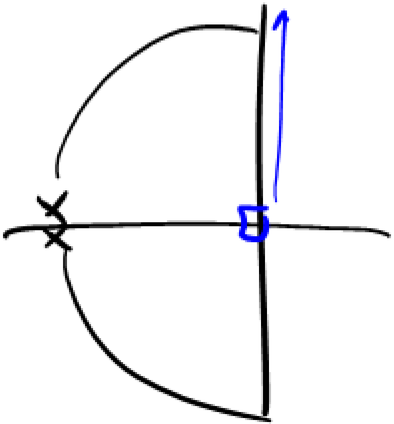


(1) K to hit $\omega_{xo} \approx BW$

(2) LEAD to hit PM

(3) LAG to hit low freq. gain

(4) attenuate high freq. LAG or poles

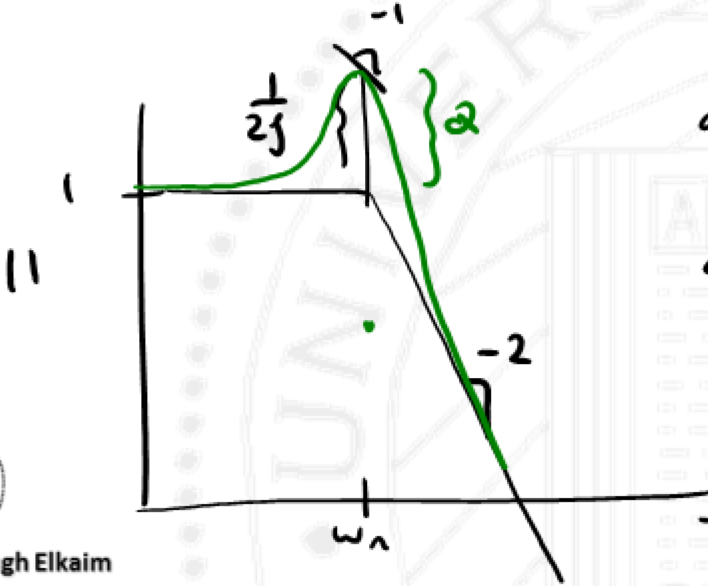


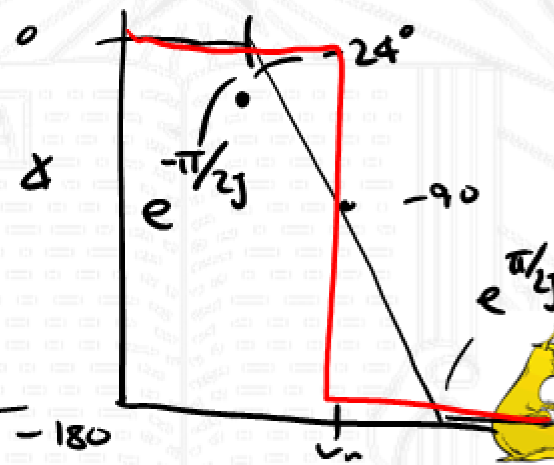
$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$j\omega = 0$: $|\cdot| = 1$, $\measuredangle = 0^\circ$

$j\omega = \omega_n$: $|\cdot| = \frac{1}{2\zeta}$ (Q), $\measuredangle = -90^\circ$

$j\omega = \infty$: $|\cdot| = 0$, $\measuredangle = 180^\circ$





$$G(s) = \frac{(s+1)\cdot 5^2}{s^2 + 2(.25)5s + 5^2}$$