CMPE-242
Applied Feedback Control

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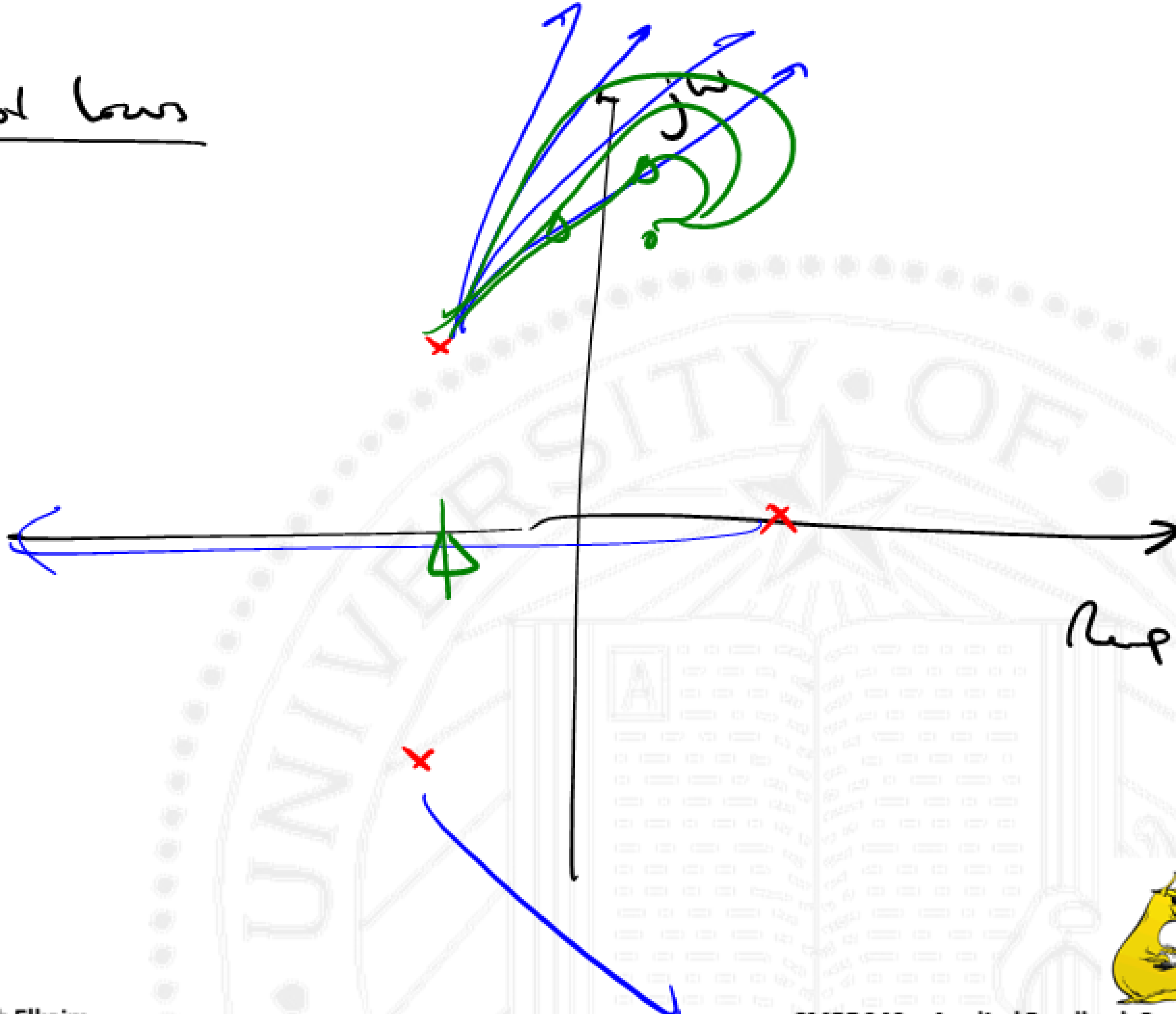
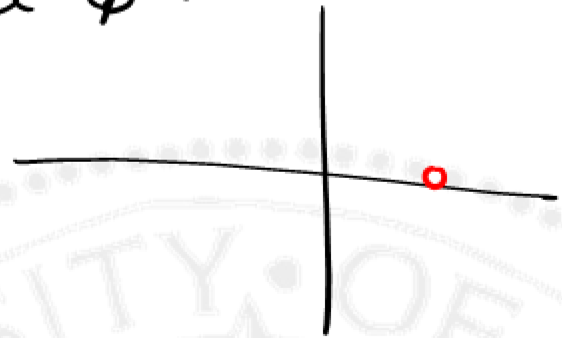
Root Locus

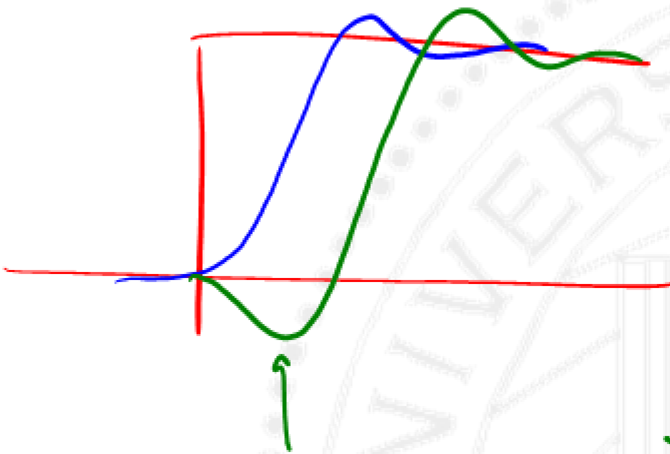


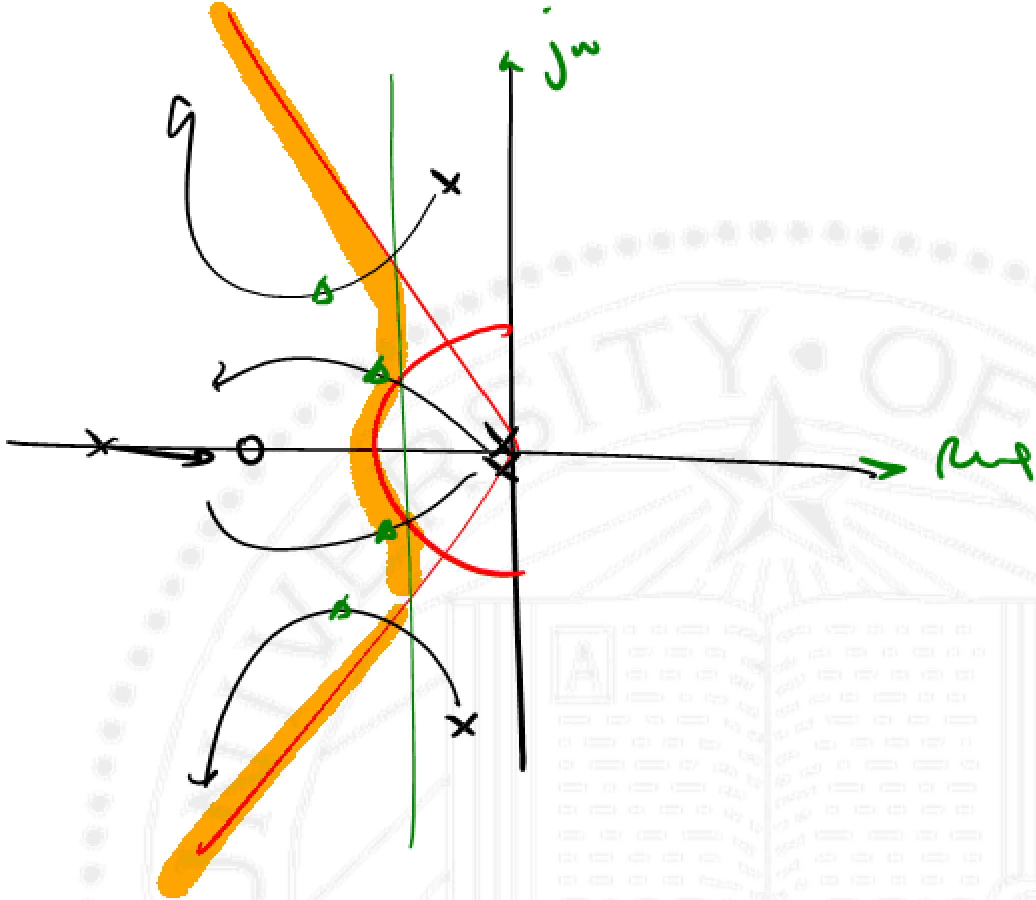
Announcements

HW #2 is out, Root Locus — Sherman's addition

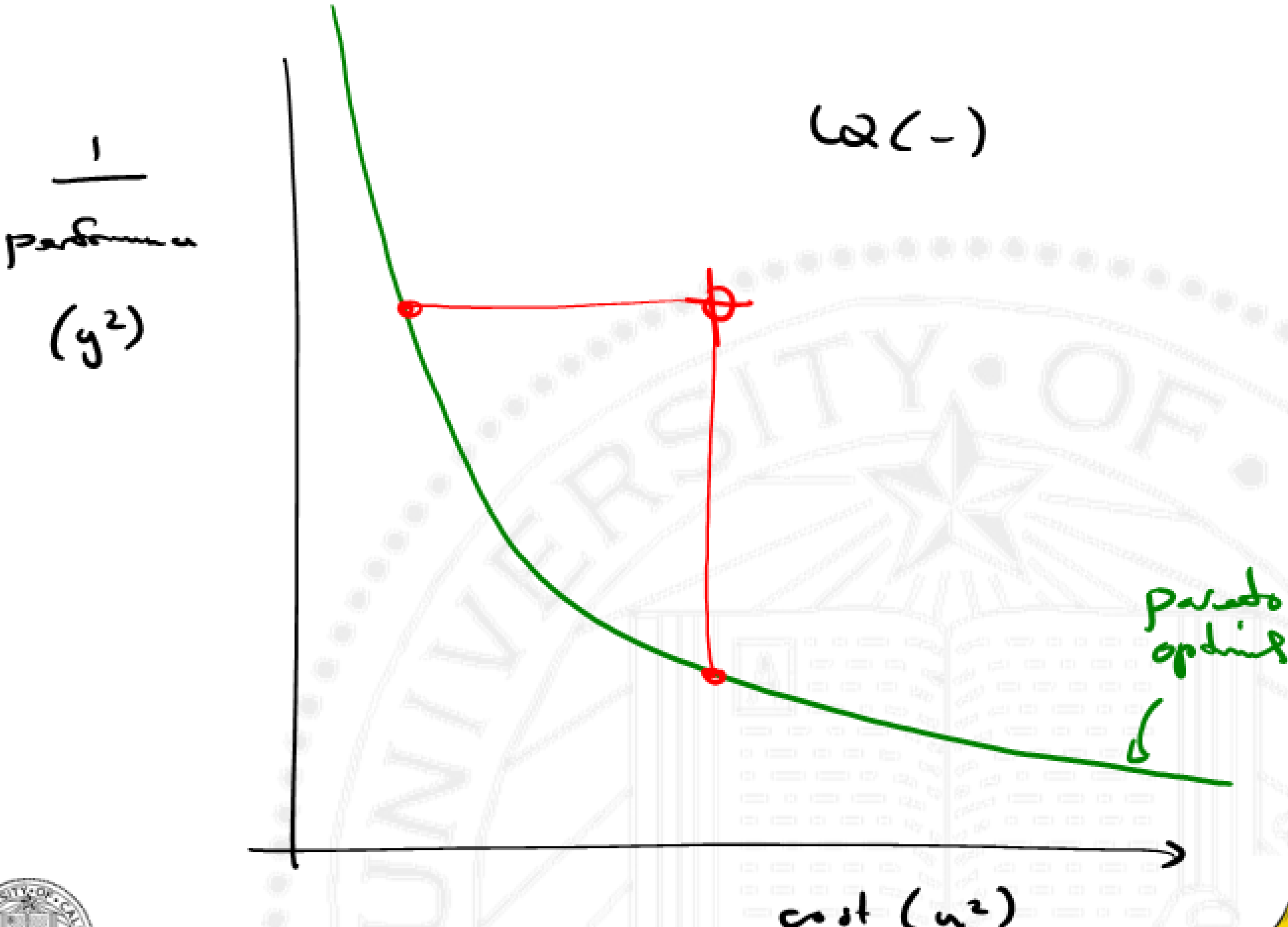
Non-minimum phase ϕ:

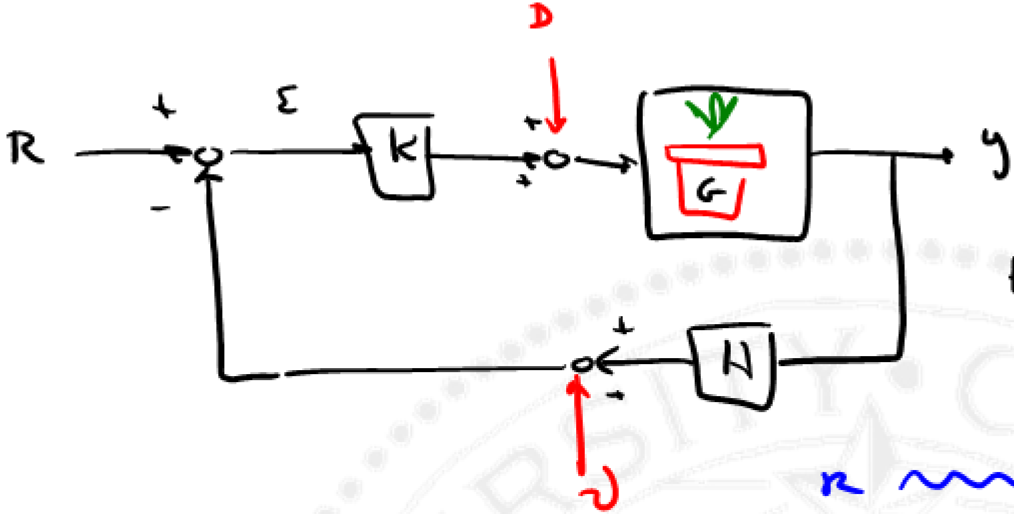






jω
Re


1
performance
(y²)
LQ(-)
pareto optimal
cost (u²)



$H \simeq 1$

r

y

$$\frac{Y}{R} = \frac{GK}{1+GK} \leftarrow \Delta(s) = 0$$

$$\frac{E}{R} = \frac{1}{1+GK}$$

$|GK|_{s=j\omega}$ $\quad |GK| \uparrow$ big

$E_{ss} < 10\%$ $\quad |1+GK| > 10 \rightarrow |GK| > 11$

$$\frac{Y}{R} = \frac{GK}{1+GK} \qquad GK \uparrow$$

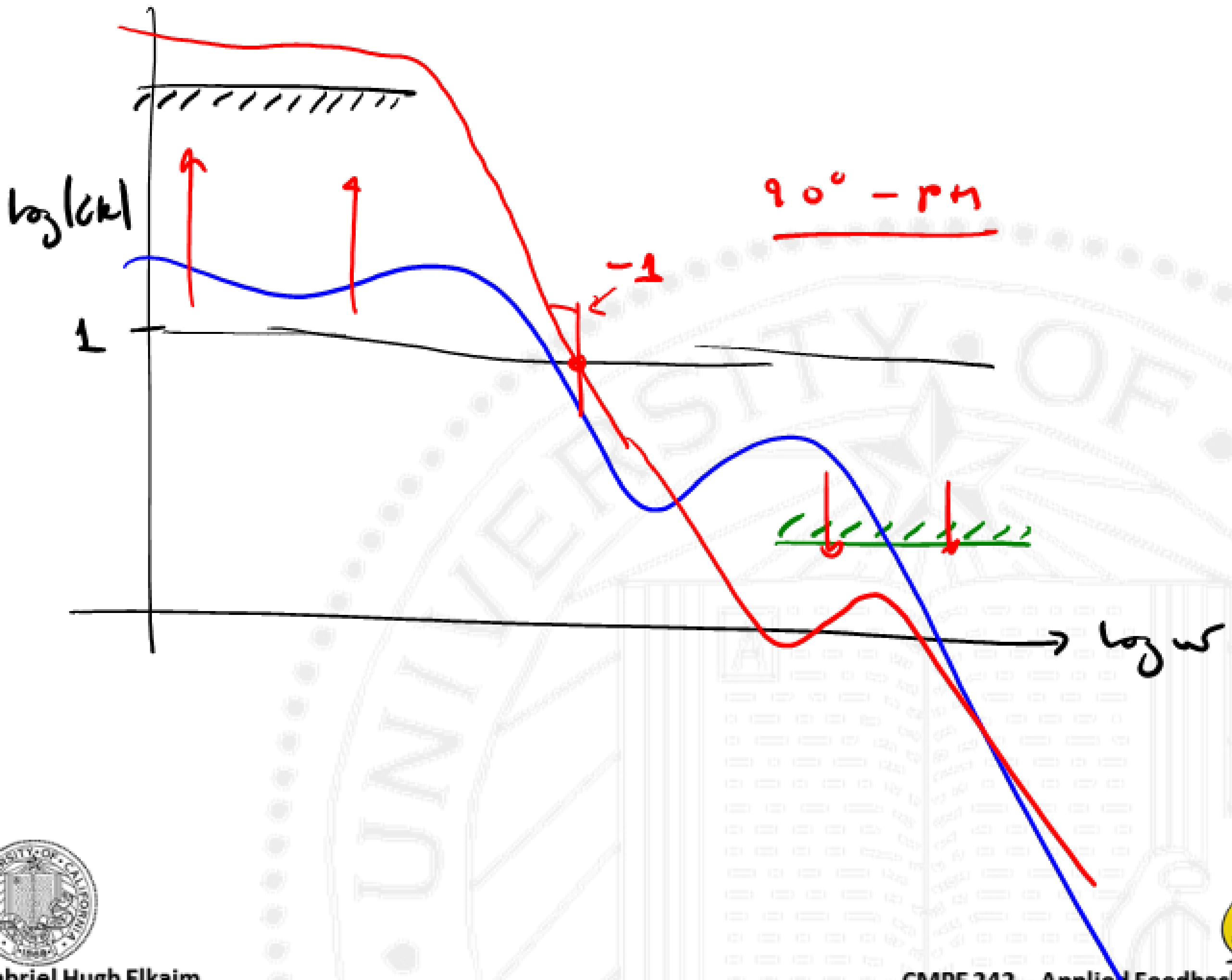
$$\frac{E}{R} = \frac{1}{1+GK} \qquad GK \uparrow$$

$$\frac{Y}{D} = \frac{G}{1+GK} \qquad K \uparrow$$

$$\frac{Y}{\upsilon} = \frac{GK}{1+GK} \qquad GK \downarrow$$

$|GK|$ BIG FOR TRACKING

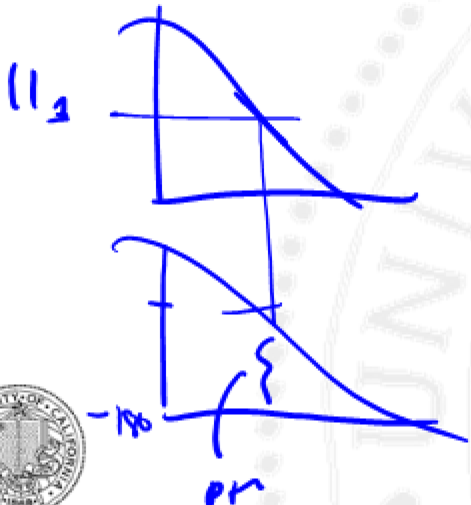
SMALL FOR NOISE


log|ck|
1
-1
90° - PM
log ω

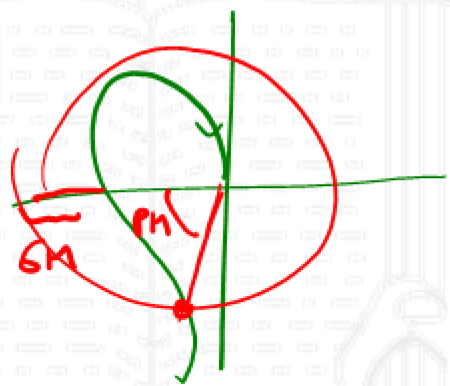
BODE GAIN/PHASE THEOREM

Simple system → slope $|\cdot| \leftrightarrow \measuredangle$

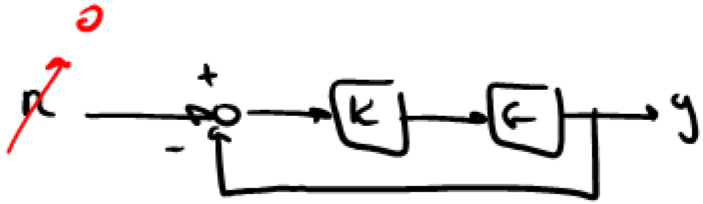
slope of -1 = phase of $-90°$



PM & GM ARE ON NYQUIST



	ROOT LOCUS	BODE
STABILITY	L.H.P	GM / PM
TRANSIENT	POLE LOCATIONS	AD ω_c ~ $\zeta \cong$ PM/100°
TRACKING	—	$\lvert GK \rvert$ including DC
SENSITIVITY / ROBUSTNESS	— (NO LOC)	GM / PM
CONTROL EFFORT	— (~K)	— (~ω_{x_0}) ← LQ



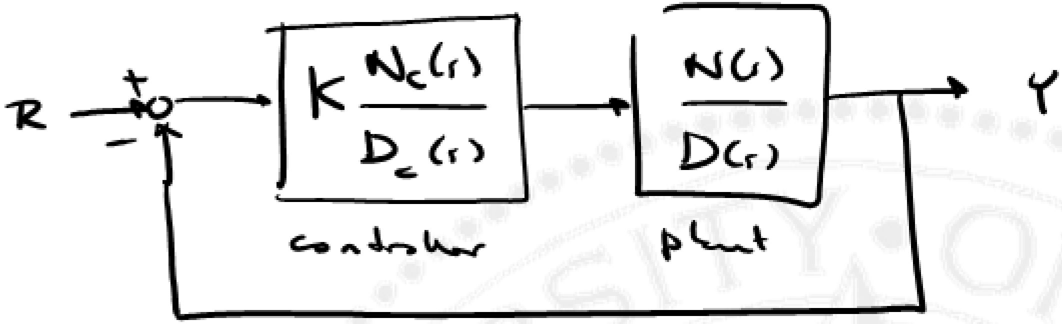
$$\frac{Y}{R} = \frac{GK}{1+GK}$$

ROOT LOCUS $\longrightarrow$ $R = \emptyset$ "REGULATOR"

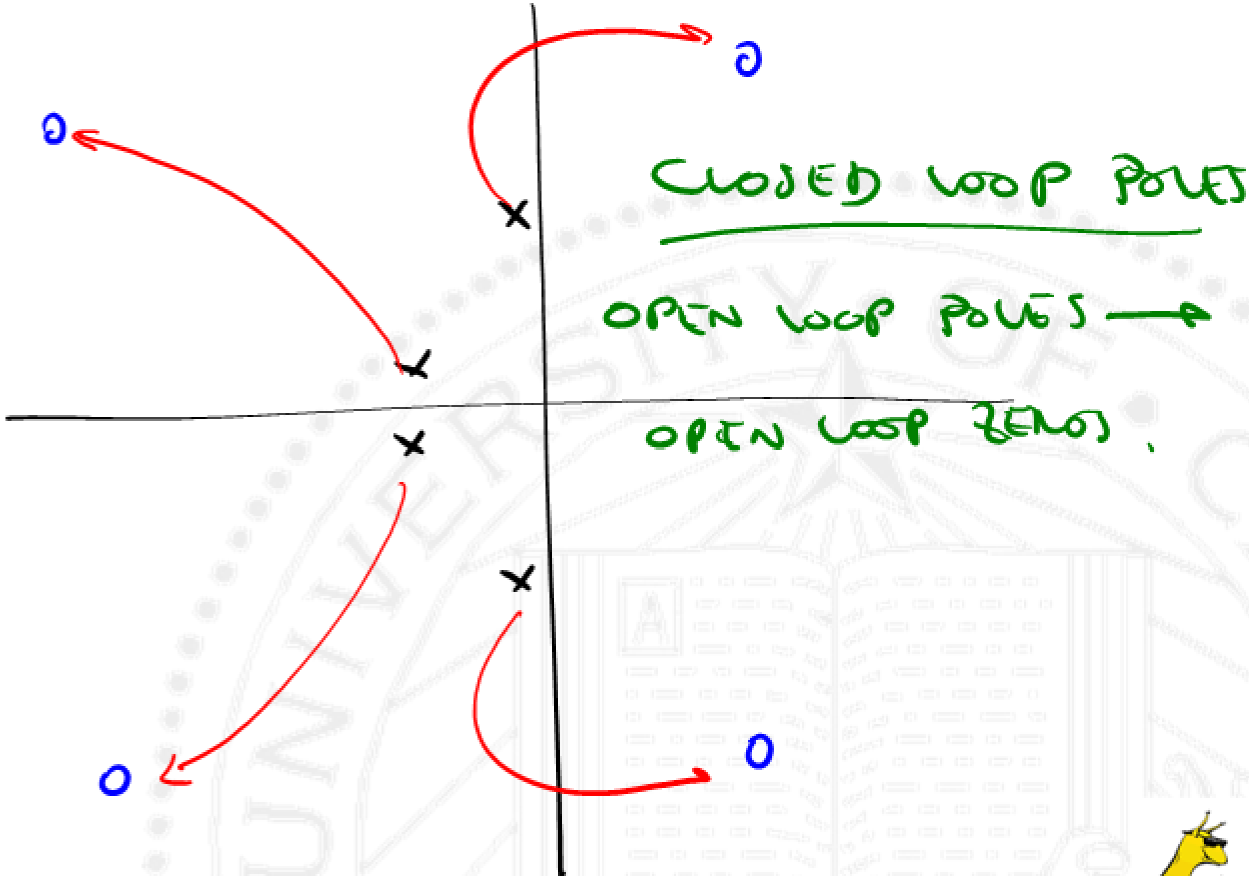
BODE $\longrightarrow$ R "TRACKER"

LQR(−) $\longrightarrow$ $J = \int_0^{\infty} (y^2 + u^2)\,dt$ — "OPTIMAL CONTROL"

Root locus



$$\frac{Y}{R} = \frac{K\frac{N_c N}{D_c D}}{1 + K\frac{N_c N}{D_c D}} = \frac{K N_c N}{\boxed{\underbrace{D_c D}_{\text{poles of open loop system}} + \underbrace{K N_c N}_{\text{zeros of open loop system}}}} \leftarrow \Delta_{cl}$$

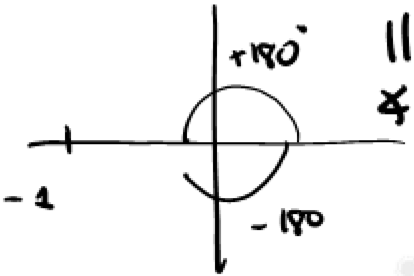

CLOSED LOOP POLES
OPEN LOOP POLES →
OPEN LOOP ZEROS.

$$D_c D + K N_c N = \phi \longrightarrow \boxed{K \frac{N_c N}{D_c D} = -1}$$

evan's form

$$-1 = 1 e^{\pm 180 j}$$

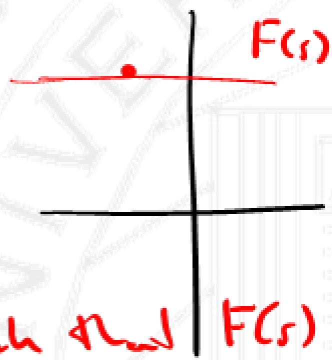
$$-1 + 0j$$



$| \, | = 1$

$\angle = \pm 180°$

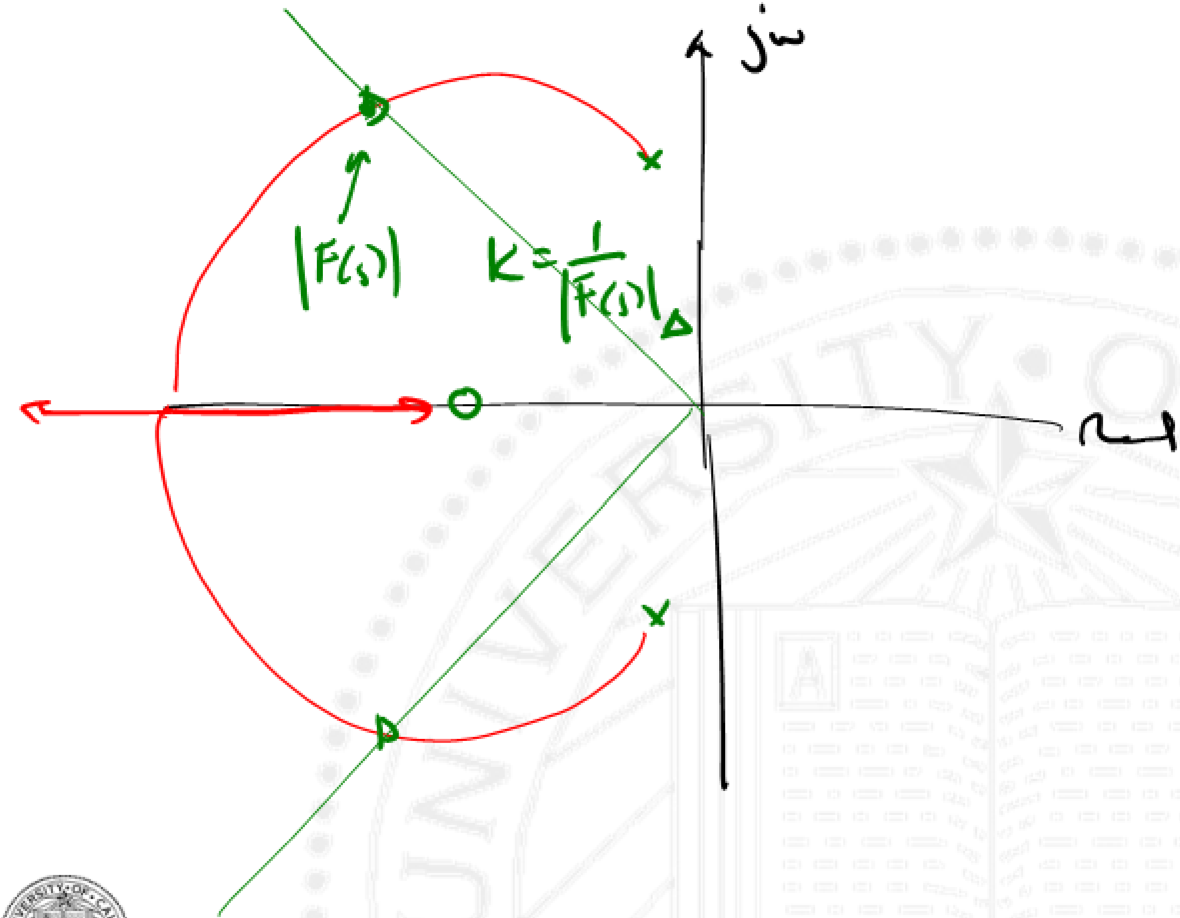
$$F(s) = \frac{N_c N}{D_c D}$$



$F(s)|$. $| \, |$ $\angle$

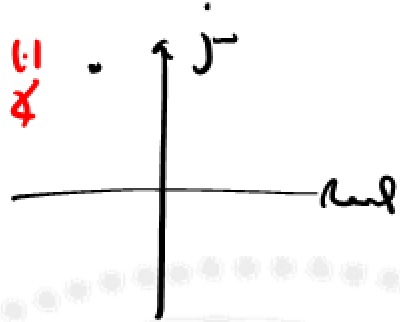
Find s such that $\angle F(s)|_s = \pm 180°$

$$K = \frac{1}{|F(s)|_s}$$



jω
σ
|F(s)|
$K = \frac{1}{|F(s)|_{\Delta}}$

$$F(s) = K \frac{N_c N}{D_c D}\Bigg|_{s=\bullet}$$



Find all s, such that $\angle |F(s)|_{s=\sigma+j\omega} = \pm 180^\circ$

choose $K = \dfrac{1}{|F(s)|_{s_{des}}}$

<u>ROOT LOCUS</u>

Root Locus Rules

(1) locus starts @ open loop poles $\times$ → open loop zeros $\circ$

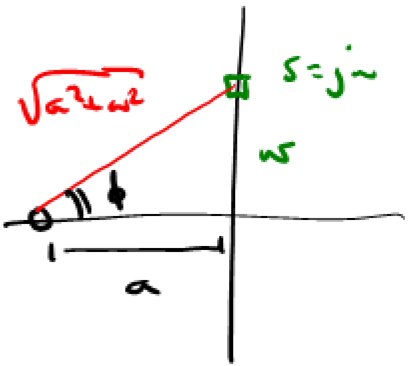
$\times \longrightarrow \circ$

(2) Real Axis: locus is on the real axis to the LEFT of an odd # of x's & o's

(3) Asymptotes: $\alpha = \dfrac{\Sigma p - \Sigma z}{n-m}$ $\quad \measuredangle = \dfrac{180}{n-m} + \dfrac{360(l-1)}{n-m}$

$l = 1, 2, \ldots, n-m$

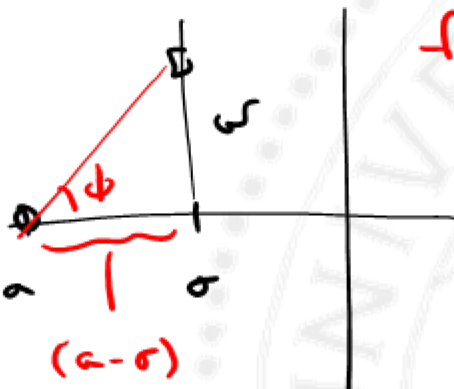
(4) Departure/Arrival Angles



$$f(s) = s+a\Big|_{s=j\omega}$$

$$|\cdot| = \sqrt{a^2+\omega^2}$$

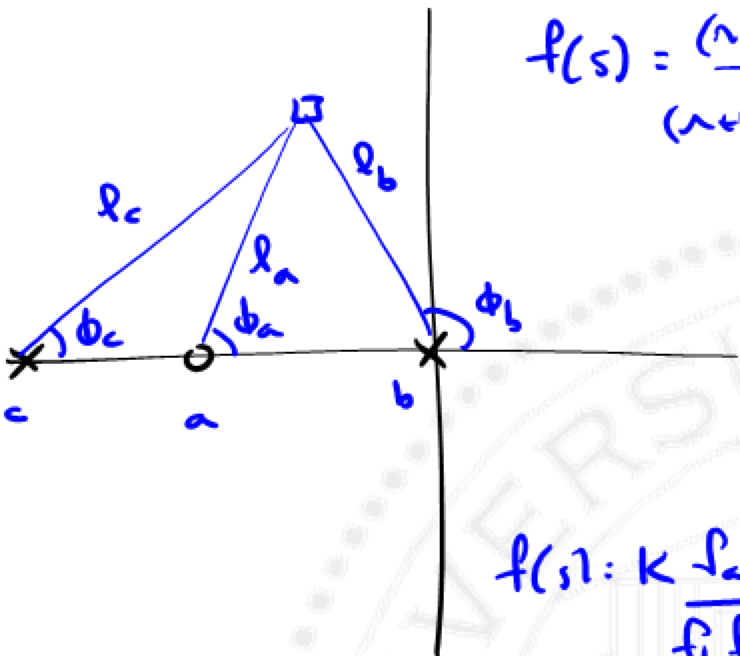
$$\measuredangle = \tan^{-1}\frac{\omega}{a}$$



$$f(s) = s+a\Big|_{s=-\sigma+j\omega}$$

$$|\cdot| = \sqrt{(a-\sigma)^2+\omega^2}$$

$$\measuredangle = \tan^{-1}\left(\frac{\omega}{a-\sigma}\right)$$



$$f(s) = \frac{(s+a)}{(s+b)(s+c)}\Bigg|_{s=-\sigma+j\omega}$$

$$\frac{l_a e^{-\phi_a j}}{l_b e^{+\phi_b j}\, l_c e^{+\phi_c j}}$$

$$f(s) = K\frac{l_a}{l_b l_c}$$

$$f(s) = \frac{l_a}{l_b l_c} e^{-(\phi_a-\phi_b-\phi_c)j}$$

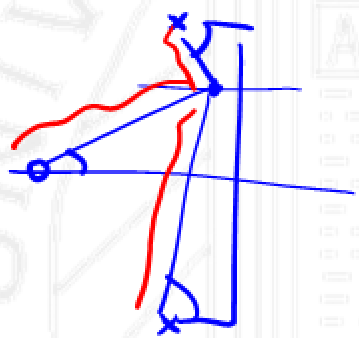
$$1 \cdot | = \frac{l_a}{l_b l_c} \qquad \angle\ \phi_a - \phi_b - \phi_c$$

$$F(s) = \frac{\prod (s - z_i)}{\prod (s - p_i)}$$

$$|F(s)| = \frac{\prod (l_{z_i})}{\prod (l_{p_i})}$$

choose $K = \dfrac{\prod (l_{p_i})}{\prod (l_{z_i})}$

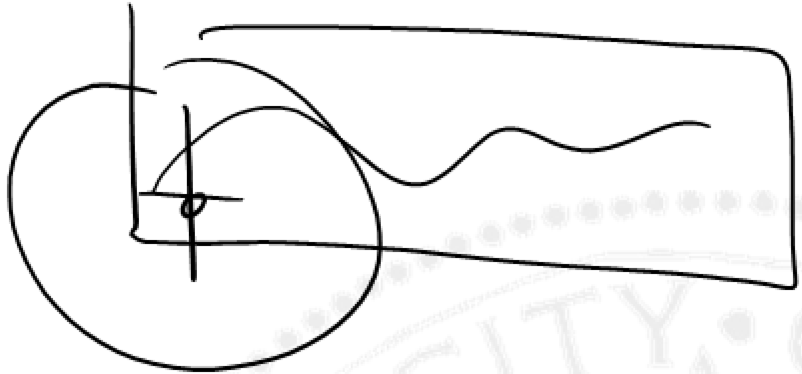
$$\measuredangle F(s) = \sum \phi_o - \sum \phi_x$$

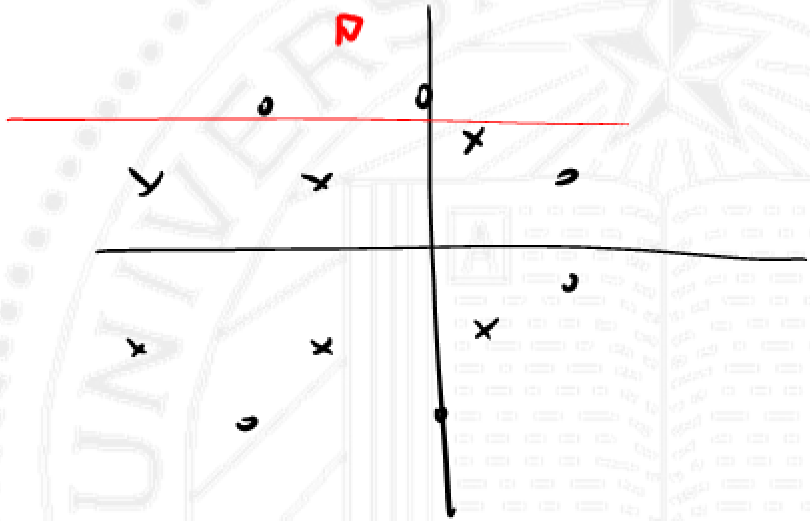


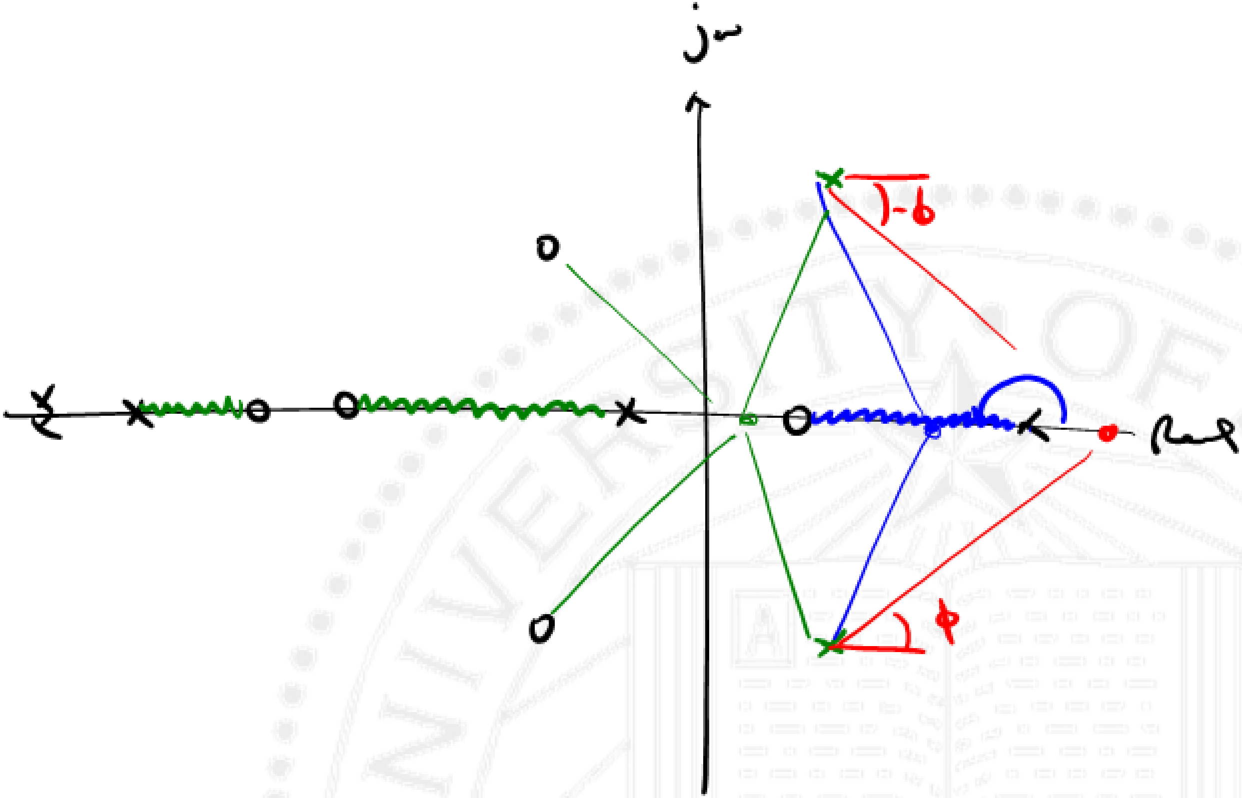
$\measuredangle\ \phi_1 - \phi_2 - \phi_3$

$|\cdot| = \dfrac{l_1}{l_2 l_3}$

Spirule






jω
-φ
φ
Real

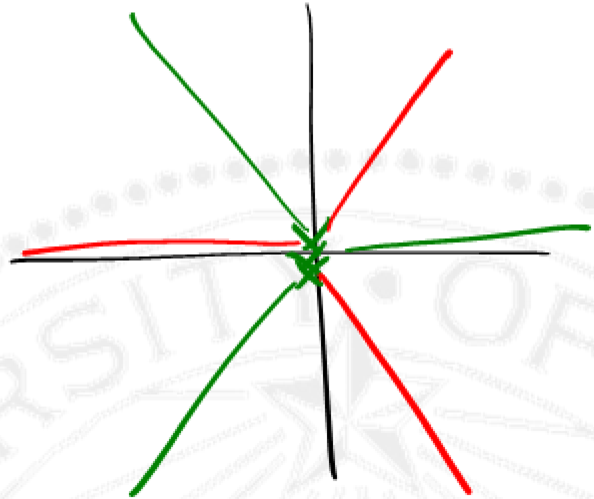
$$(s-a_1)(s-a_2)(s-a_3) = s^3 + (a_1+a_2+a_3)s^2 + \ldots + (a_1 a_2 a_3)$$

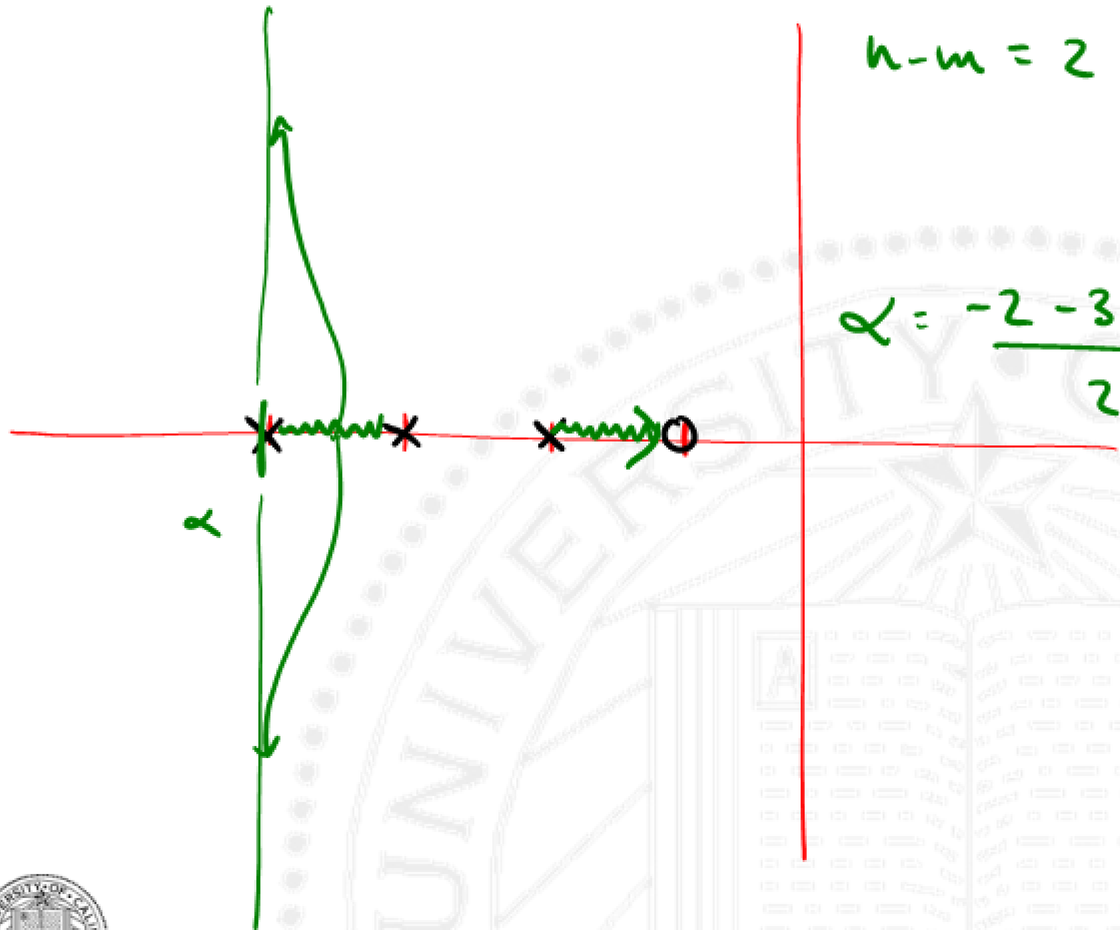
$$\frac{1}{(s-\alpha)^{n-m}} = \frac{1}{s^{(n-m)} + (n-m)\alpha\, s^{(n-m-1)} + \ldots}$$

$$\Delta = 1 + \frac{s^m + (\Sigma z_i)\, s^{m-1} + \ldots + \Pi(z_i)}{s^n + (\Sigma p_i)\, s^{n-1} + \ldots + \Pi(p_i)}$$

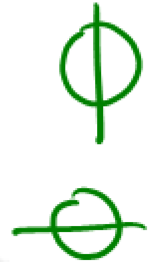
$$\Delta = 1 + \frac{K}{s^{(n-m)} + (\Sigma p_i - \Sigma z_i)\, s^{(n-m-1)} + \ldots}$$

$$\alpha = \frac{\sum p_i - \sum z_i}{(n-m)}$$

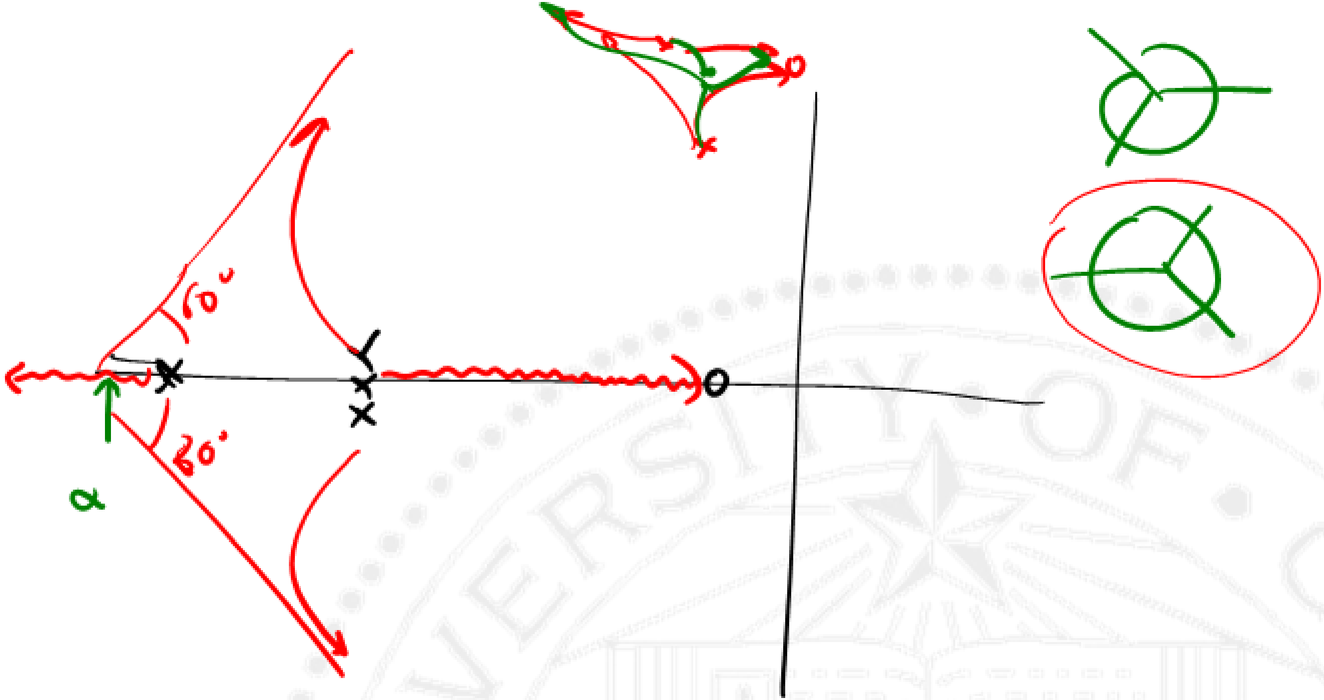




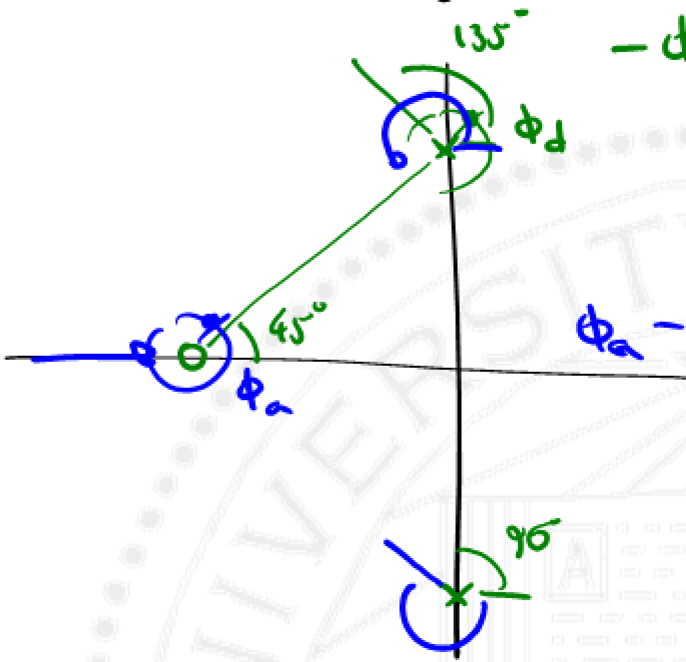
$n - m = 2$



$$\alpha = \frac{-2-3-4-(-1)}{2} = -4$$


60°
60°
α

Departure/Arrival Angles



$$-\phi_d - 45 = \pm 180$$

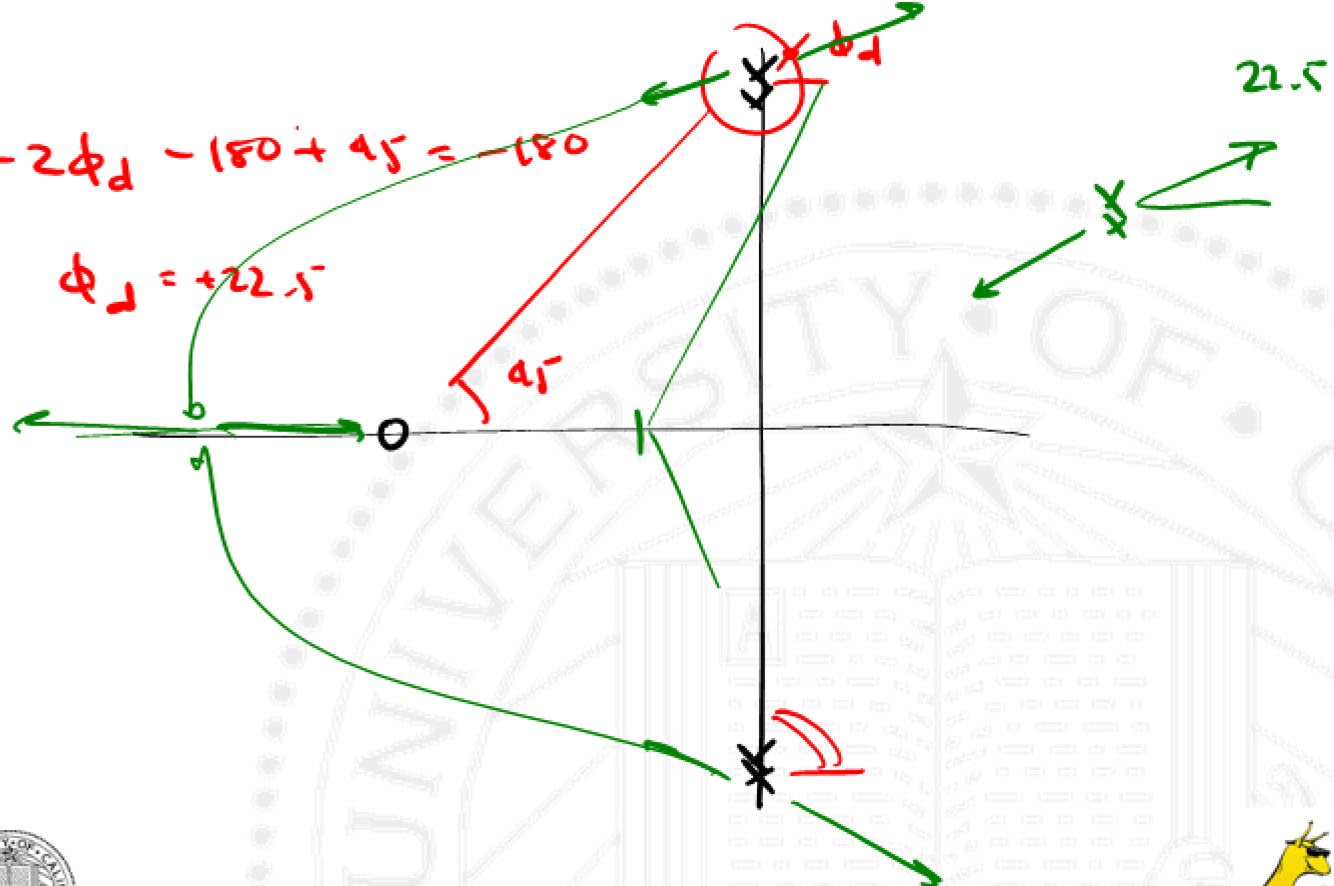
$$\phi_d = 135°$$

$$\phi_a - 0 = \pm 180$$

$$\phi_a = \pm 180$$

$$-2\phi_d - 180 + 45 = -180$$

$$\phi_d = +22.5$$



Rule #5

When does the system cross the

$j\omega$ - axis

$$\Delta_{cl}(s=j\omega)=0$$

$$D_cD+KN_cN=0+0j$$

$$s=j\omega$$

$$\downarrow$$

solve real part $=0$, imag $=0$

$$K,\ \omega$$