CMPE-242
Applied Feedback Control

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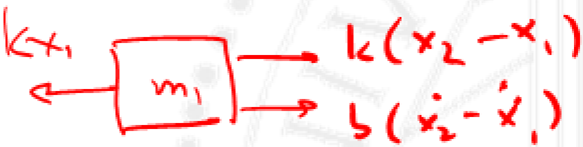
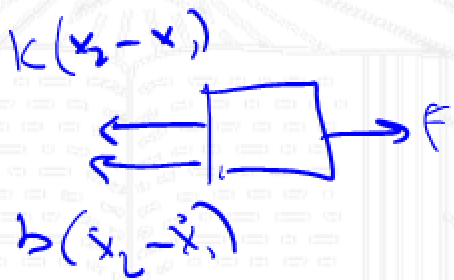
Questions

$\cancel{\frac{Y}{U}} \longrightarrow \frac{Y}{R}$



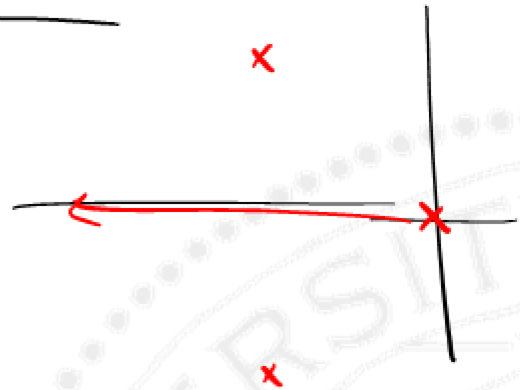


$\Sigma F_{x_1} = m_1\ddot{x}_1$

$\Sigma F_{x_2} = m_2\ddot{x}_2$

6

$$\frac{e^{\pi j} - e^{-\pi j}}{2}$$

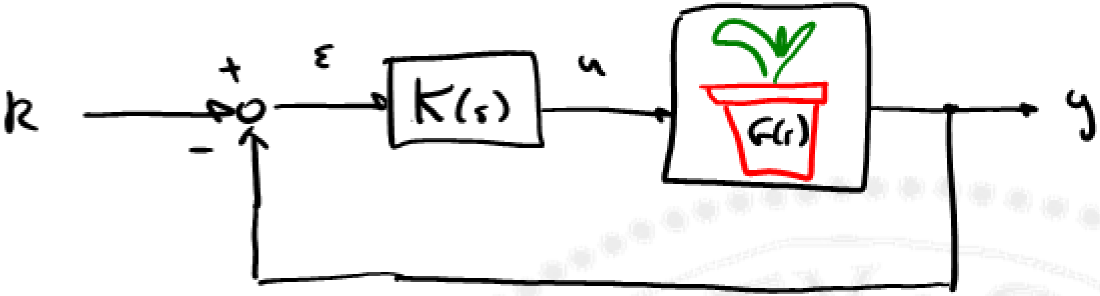


$\alpha \longrightarrow \infty$

$\alpha \longrightarrow 0$

Announcements

1. Homework submission links are now correct
2. HW #2 will go out on webpage ~ 12 noon

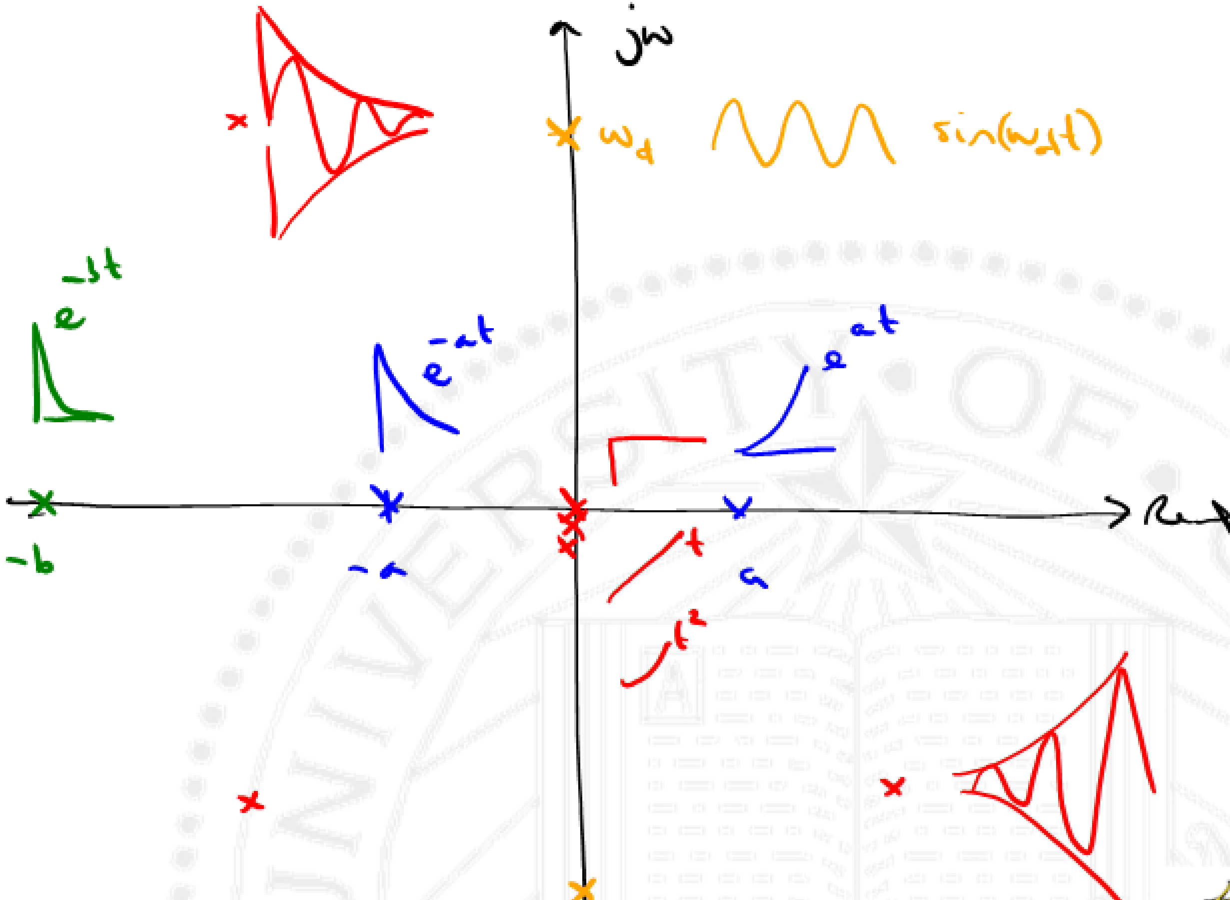


$$\frac{Y}{R} = \frac{GK}{1+GK}$$ closed loop response

$1+GK \leftarrow \Delta(s)$ "characteristic polynomial"

$\Delta(s) = 0 \leftrightarrow$ poles of closed loop system.

<u>Root locus</u> $1 + GK = \phi \longrightarrow \boxed{GK = -1}$


jw
wd
sin(wdt)
-b
-a
a
t

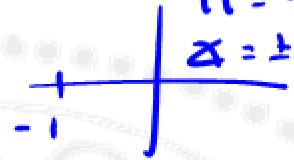
$\Delta(s) = 0$ closed loop poles

$1 + GK = 0 \longrightarrow \boxed{GK = -1}$

Evan's Form.

$|\cdot| = 1$

$\measuredangle = \pm 180^\circ$

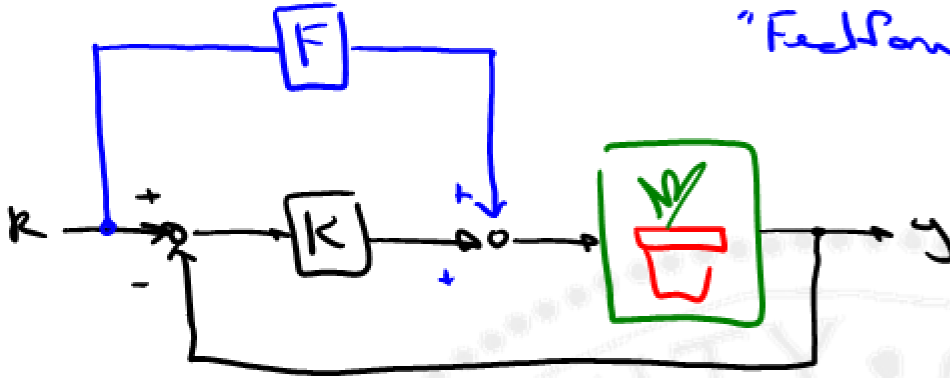


$$K(s) = K_0 \frac{N(s)}{D(s)}$$

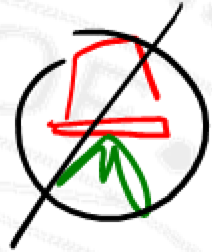
Root locus — find $\measuredangle GK = \pm \underline{180^\circ}$

Nothing about residuals

"Feedforward"



$F = G^{-1}$

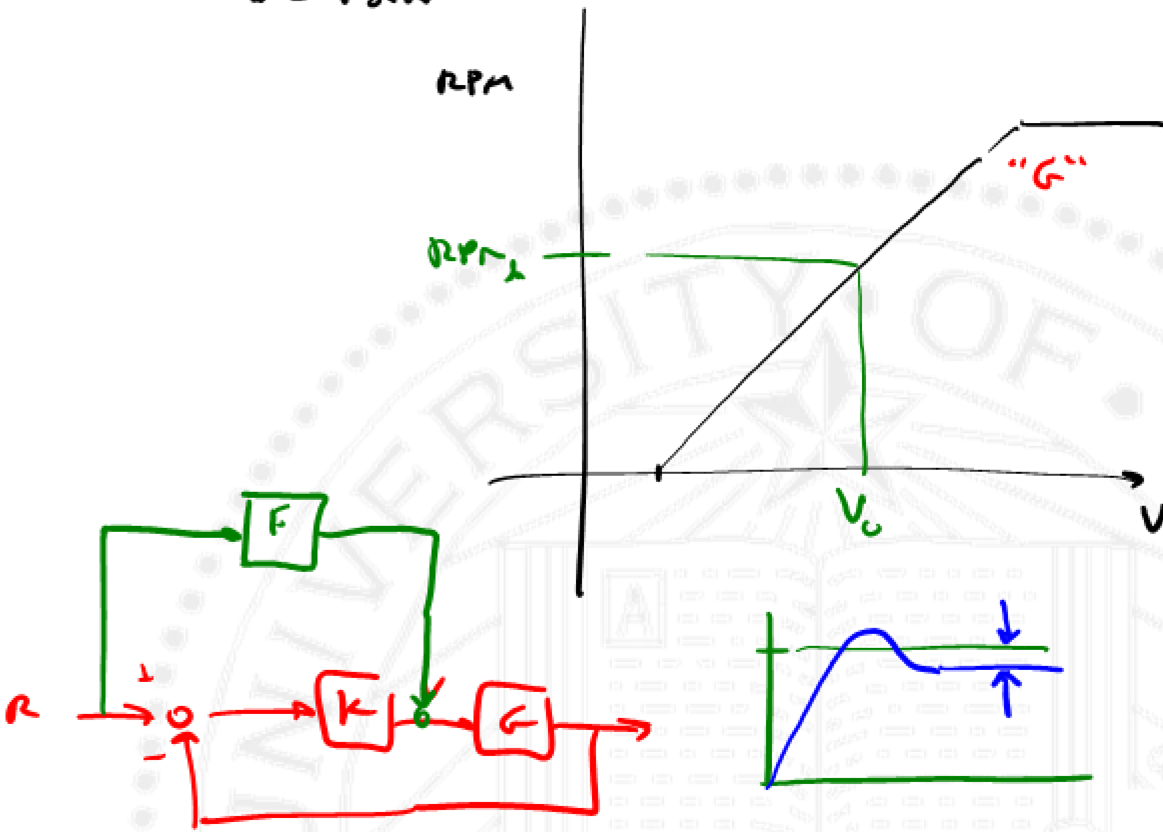


$$K(s) = \frac{N_K}{D_K} \qquad G(s) = \frac{N_G}{D_G}$$

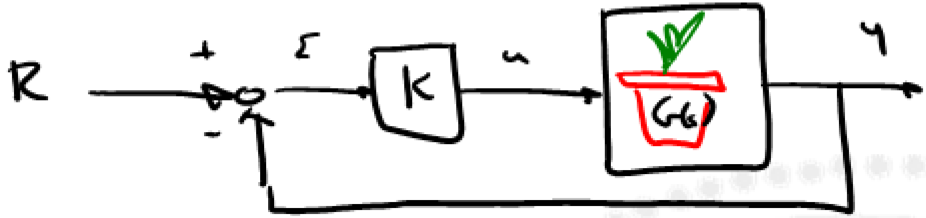
$$\frac{Y}{R} = \frac{GK}{1+GK} = \frac{N_G N_K}{N_G N_K + D_G D_K} \longleftarrow \Delta(s)$$

$$\frac{Y}{R} = \frac{N_G N_K + F N_G D_K}{N_G N_K + D_G D_K} \longleftarrow \Delta(s)$$

DC motor

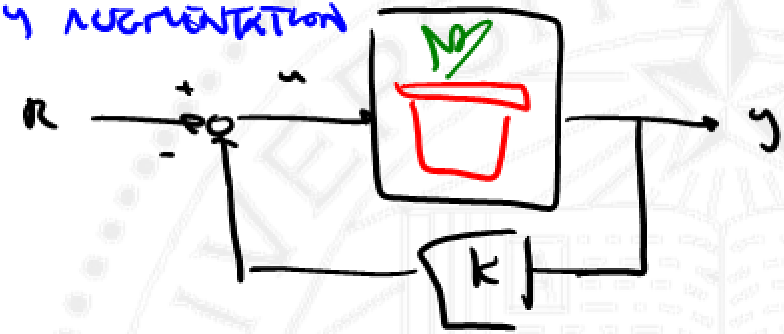


"Full Autopilot"



$$\frac{GK}{1+GK} \leftarrow \Delta(s)$$

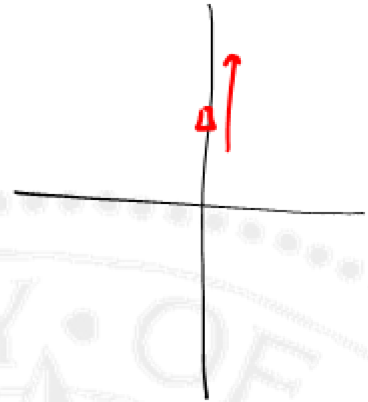
"Stability Augmentation"



$$\frac{G}{1+GK} \leftarrow \Delta(s)$$

Root locus — moving poles of closed loop system — transient response.

$H(s) \leftarrow$ transfer function



$H(j\omega) \longrightarrow$ complex number $|\cdot|$, $\angle$

<u>Bode</u>

$\log\left|\frac{B}{A}\right|$

$\angle\ \phi$

Bode

$\frac{Y}{R} \simeq 1$ for some range of $(j\omega)$

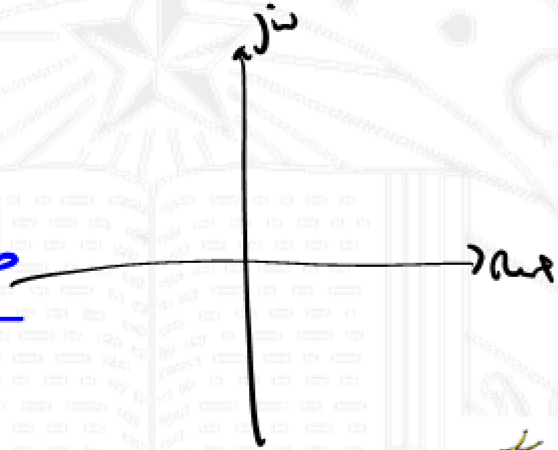
TRACKING RESPONSE

STABILITY

BIBO — "Bounded Input / Bounded Output" Stability

$$|u(t)| \le m < \infty \longrightarrow \boxed{H} \longrightarrow |y(t)| \le N < \infty$$

$$H(s) = k \frac{\prod(s - z_i)}{\prod(s - p_i)}$$



All poles must be in L.H.P

$$\mathrm{Re}(p_i) \le 0 \quad \forall i$$

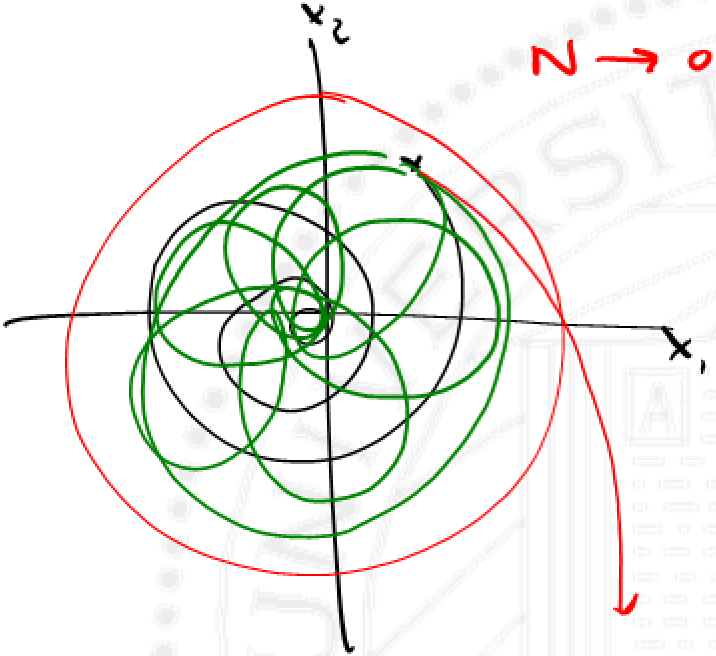
↳ non-repeated roots on jω-axis

Lyaponov

$|\underline{x}_{ic}| < M$

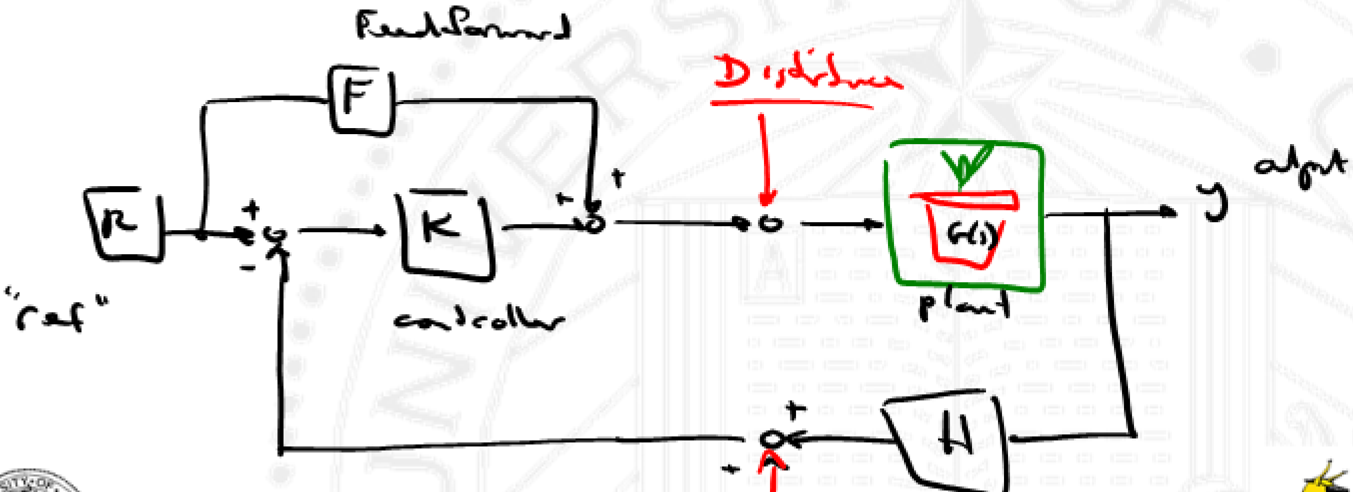
$|\underline{x}(t)| < N \quad \forall t$ then it is stable

$N \to 0$ asymptotically stable.



Poles – Root locus, modes of the system, TRANSIENT
"kick the system"

$H(j\omega)$ – complex gain, Bode !! & TRACKING



BE CAREFUL of LARGE GAINS

(1) Stability — all poles in LHP, BIBO, Lyapunov

(2) Overshoot
Rise Time
Steady State
Damping

} TRANSIENT SPEC'S — ROOT LOCUS

(3) Bandwidth
Disturbance Rejection
Steady State Error

} TRACKING SPEC'S — BODE

(4) Gain Margin
Phase Margin

} Robustness Spec's — NYQUIST

(5) Control effort vs. Performance — TRADE off — LQ(-) $J = \int_0^\infty (y^2 + u^2)\,dt$

$\underline{x}$ — position

$\underline{x}^5$

$\frac{dx}{dt}$ — velocity

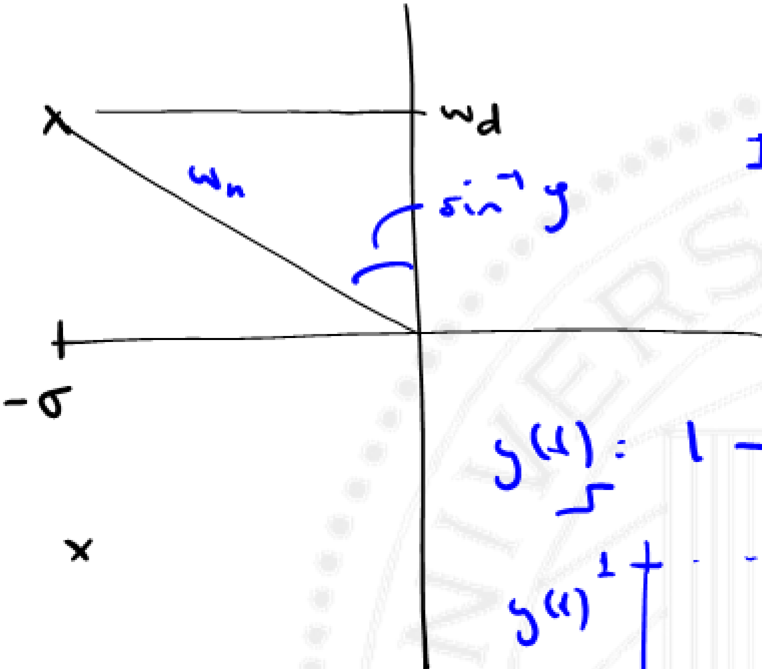
$\frac{d^2x}{dt^2}$ — acceleration

$\frac{d^3x}{dt^3}$ — jerk "ride quality"

$\frac{d^4x}{dt^4}$ — snap "comfort / fatigue"

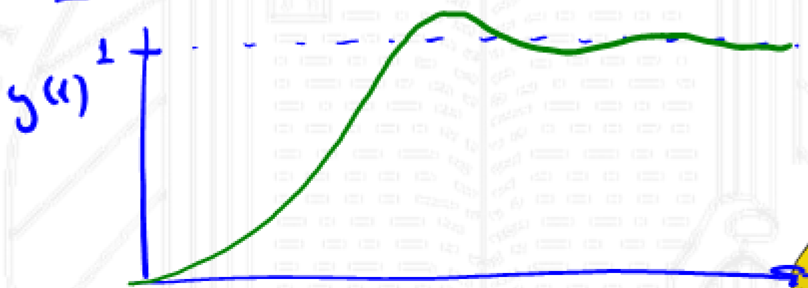
Performance Spec's

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

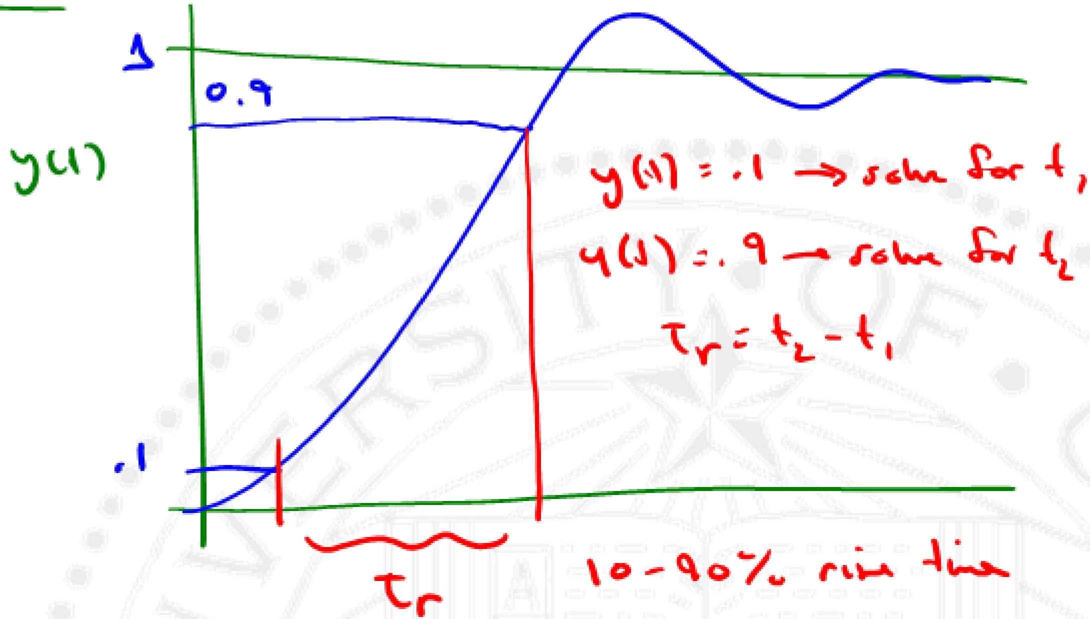


DC gain $\stackrel{?}{=}$ 1

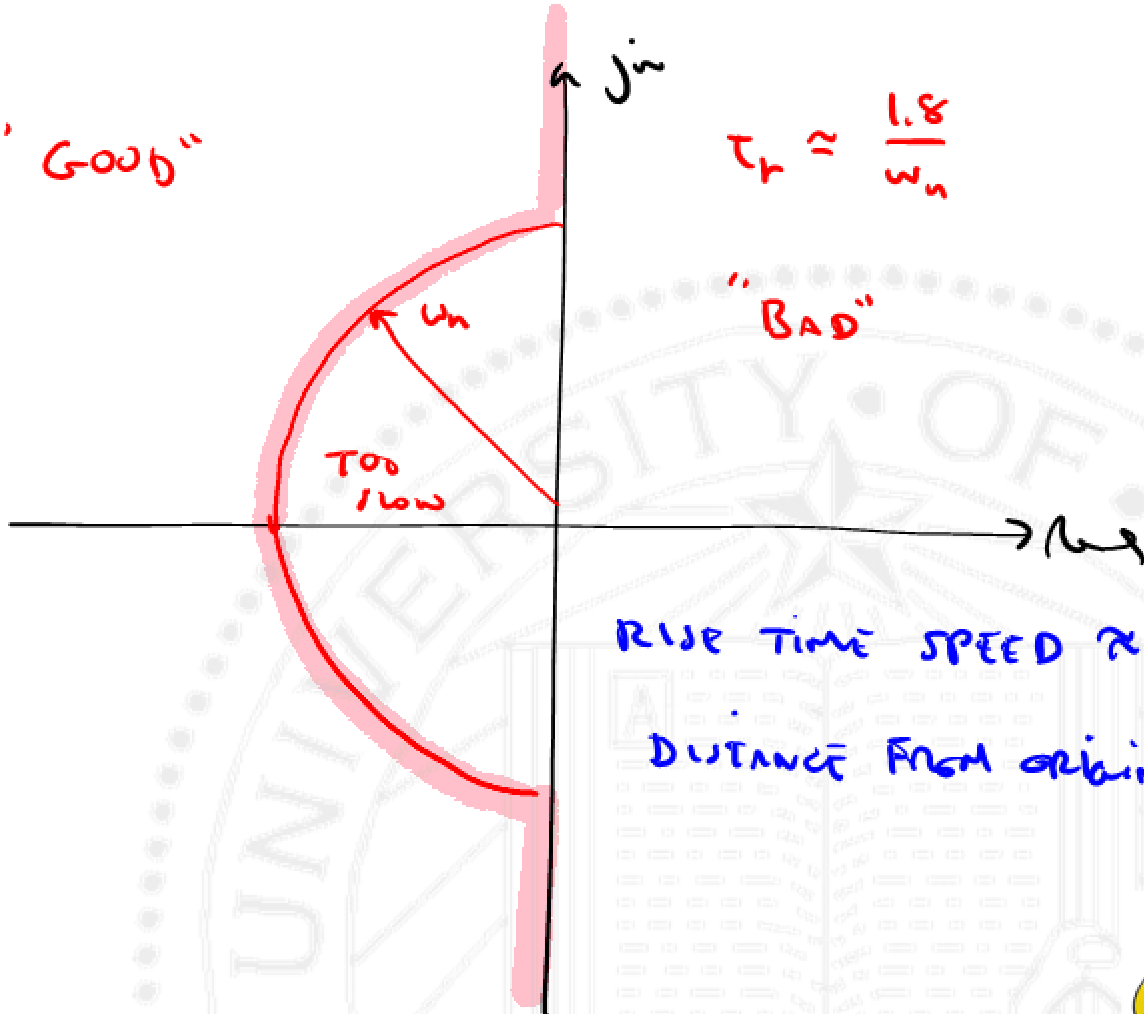
$$y(t) = 1 - e^{-\zeta\omega_n t}\left[\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right]$$



Rise Time

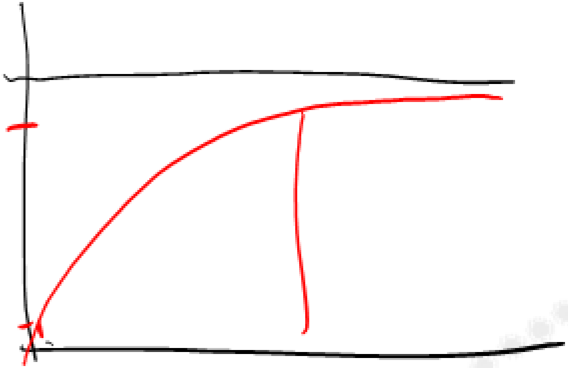


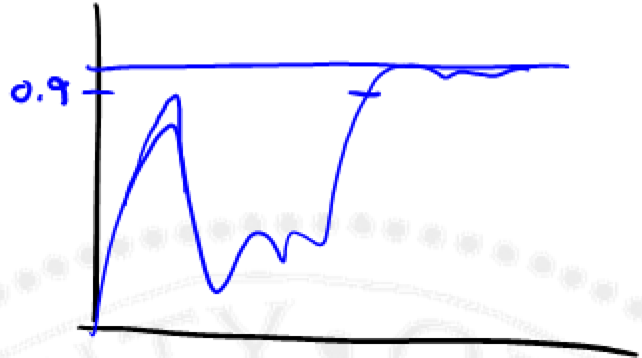
$$\boxed{T_r \simeq \frac{1.8}{\omega_n}}$$ ← rad/sec.

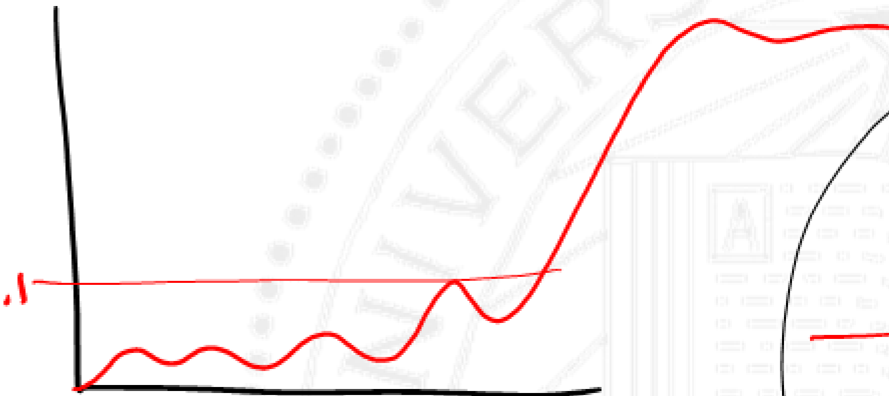


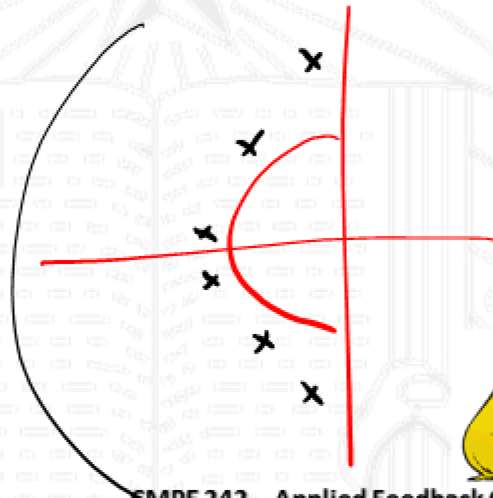
$t_r \approx \frac{1.8}{\omega_n}$

RISE TIME SPEED $\propto$ DISTANCE FROM ORIGIN

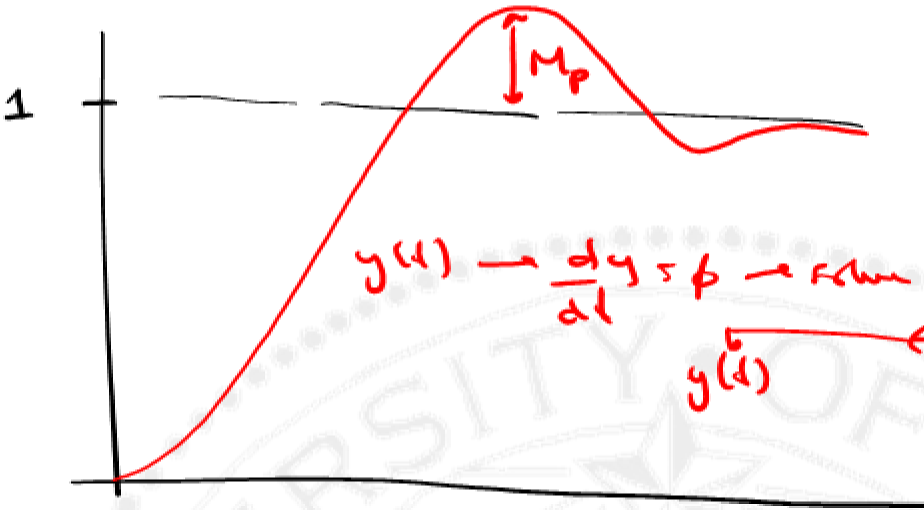



0.9


.1



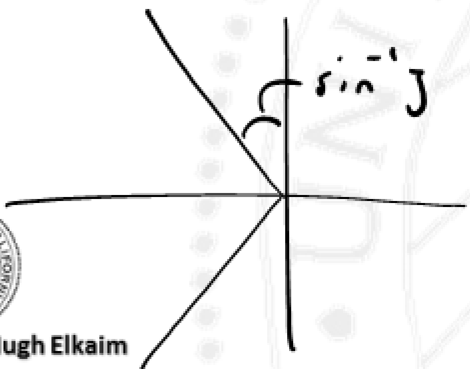
Overshoot



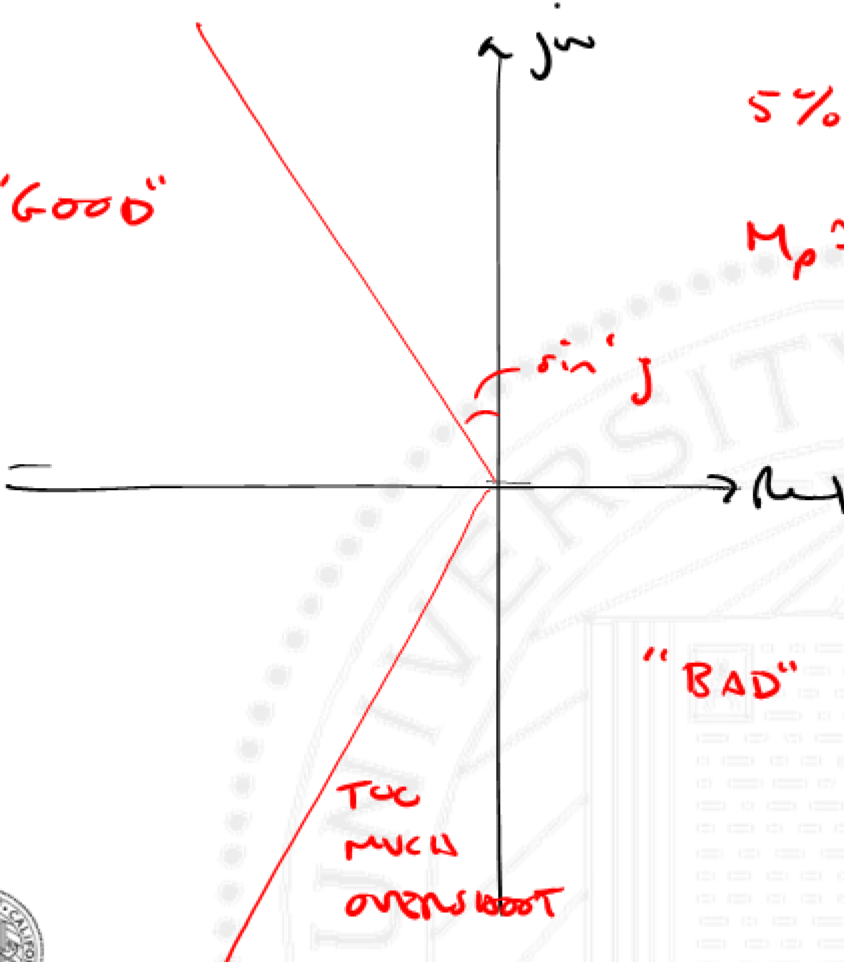
$y(t) \rightarrow \frac{dy}{dt} = 0 \rightarrow$ solve for t

$y(t)$

$$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}} \xrightarrow{\sim} 1 - \zeta/0.6 \qquad 0 \le \zeta \le 0.6$$



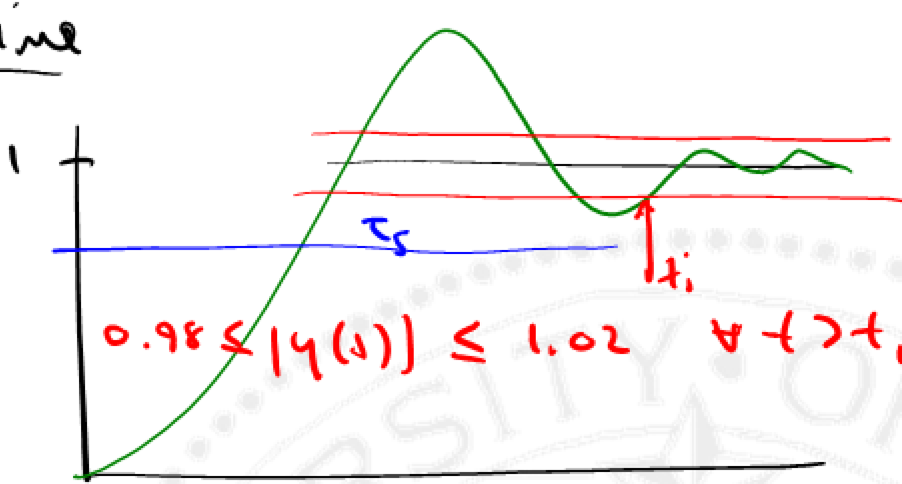
$\zeta = \frac{1}{\sqrt{2}} \rightarrow 5\%$ overshoot



5% → 45° line

$$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

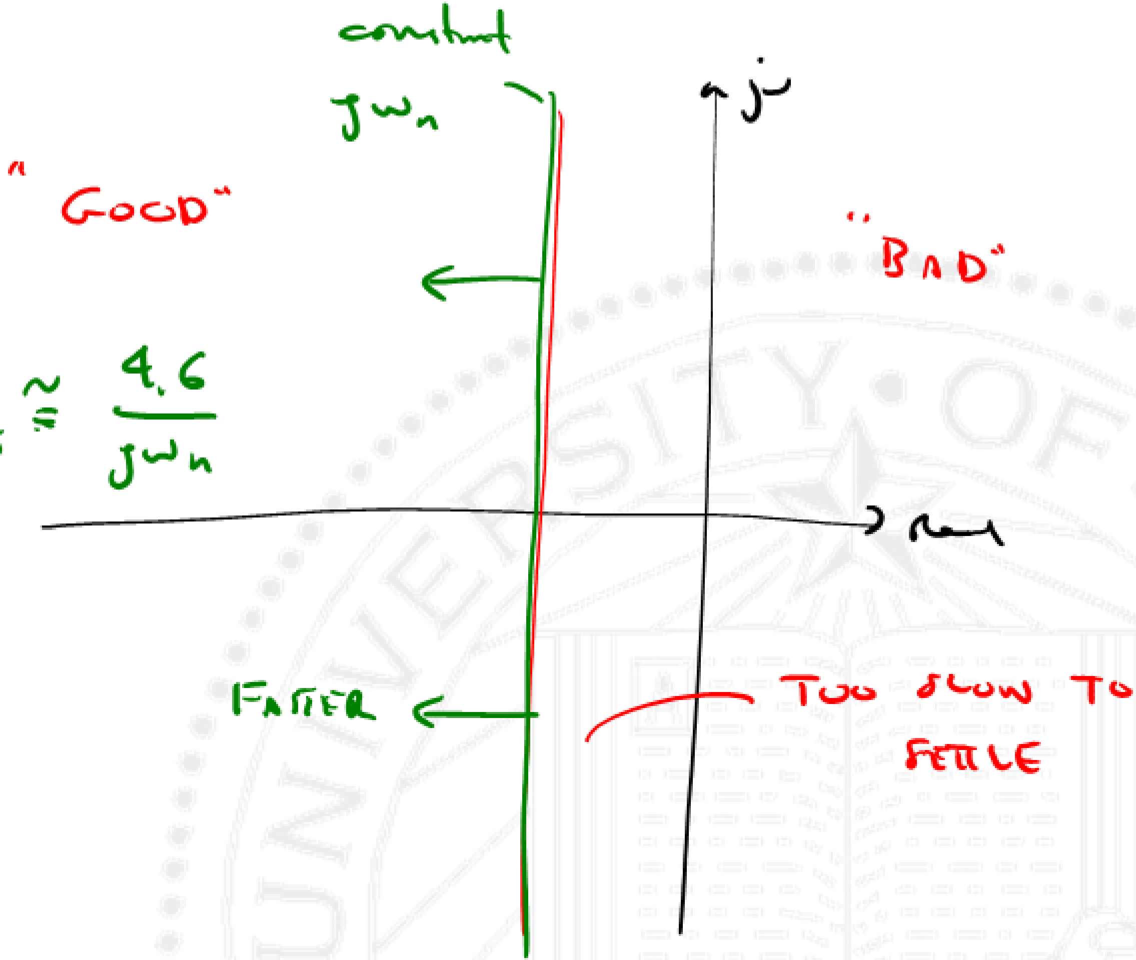
Settling Time



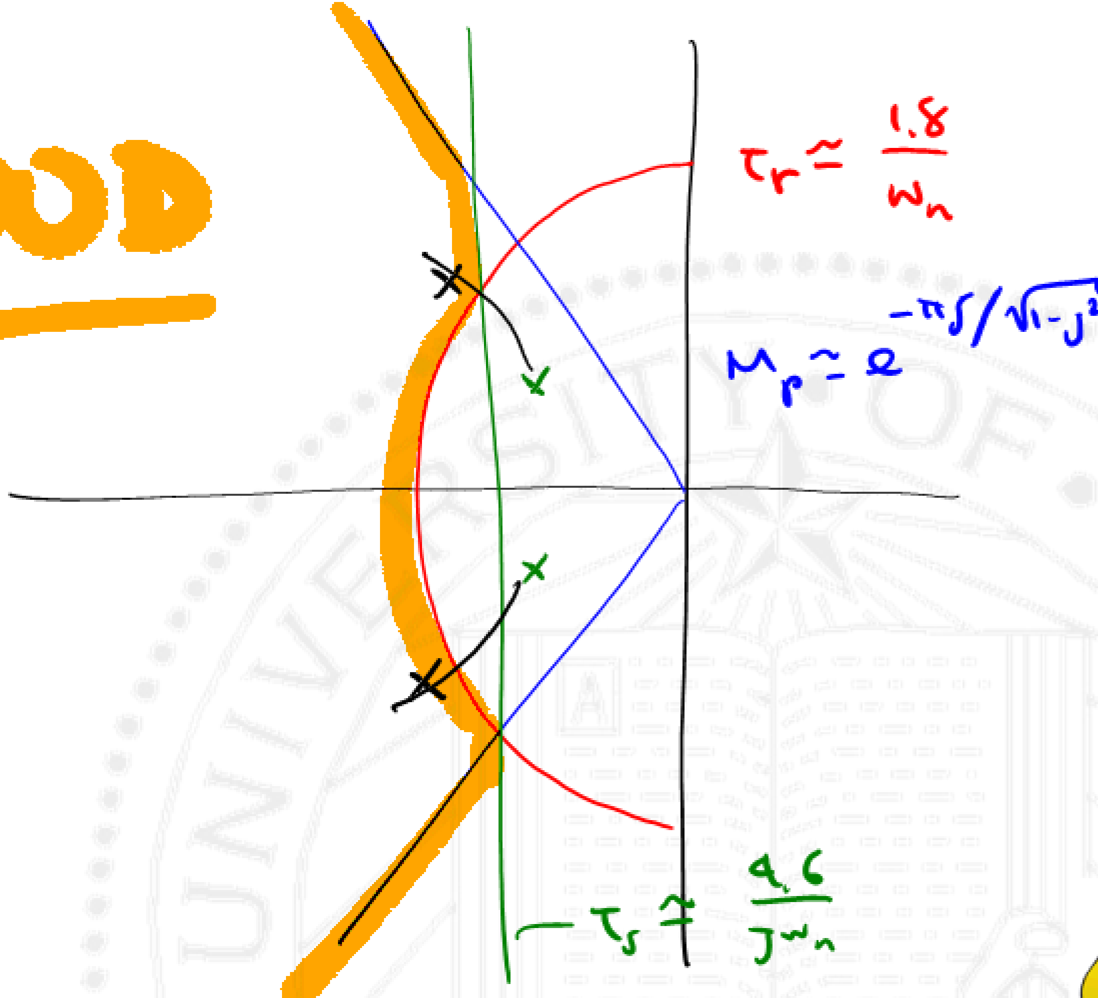
$$0.98 \le |y(t)| \le 1.02 \quad \forall\, t > t_1$$

$$\boxed{t_s \cong \frac{4.6}{\zeta \omega_n}}$$ ← rad/sec

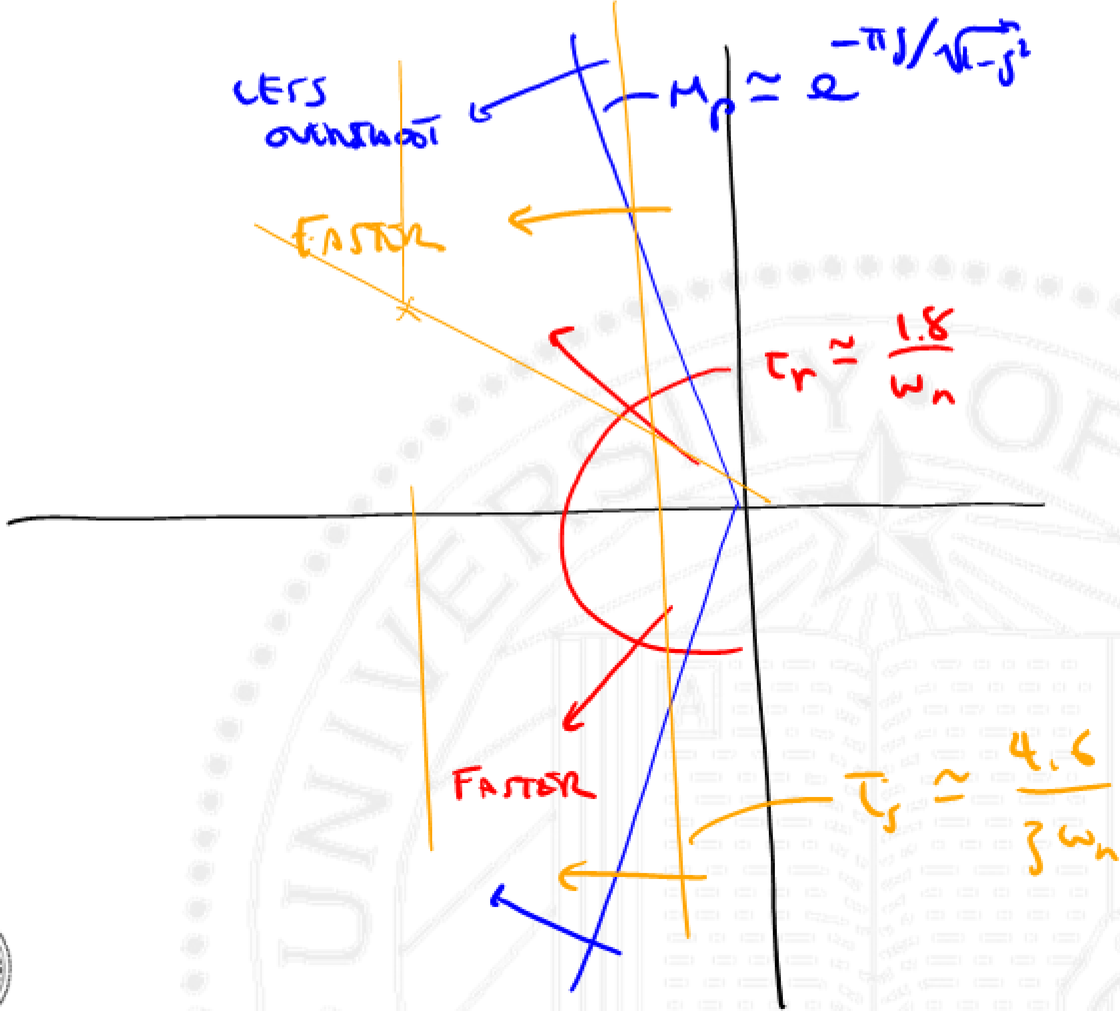
$$t_s \approx \frac{4.6}{\zeta\omega_n}$$

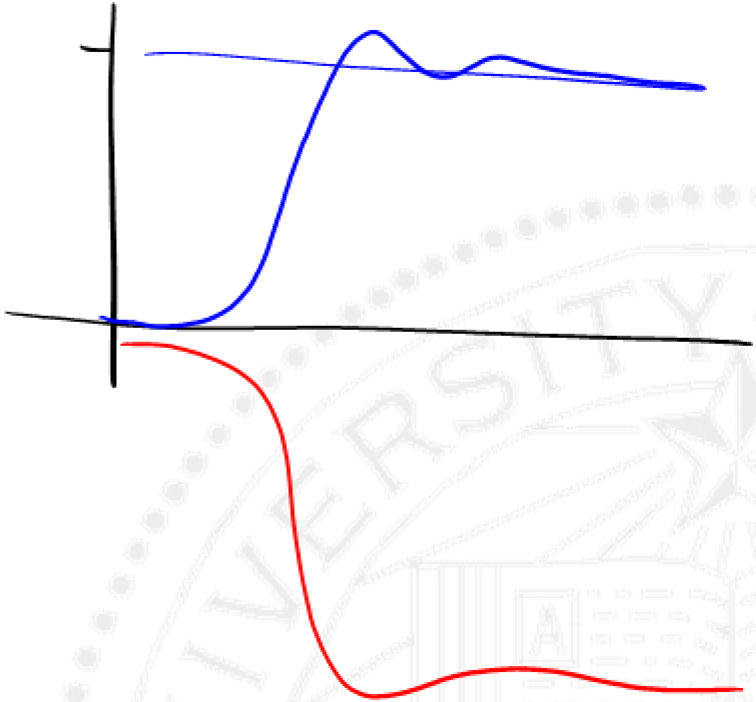


GOOD



$t_r \approx \frac{1.8}{\omega_n}$
$M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
$t_s \approx \frac{4.6}{\zeta\omega_n}$


LESS overshoot
$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
FASTER
$t_r \simeq \frac{1.8}{\omega_n}$
FASTER
$t_s \simeq \frac{4.6}{\zeta\omega_n}$



NMP