CMPE-242
Applied Feedback Control

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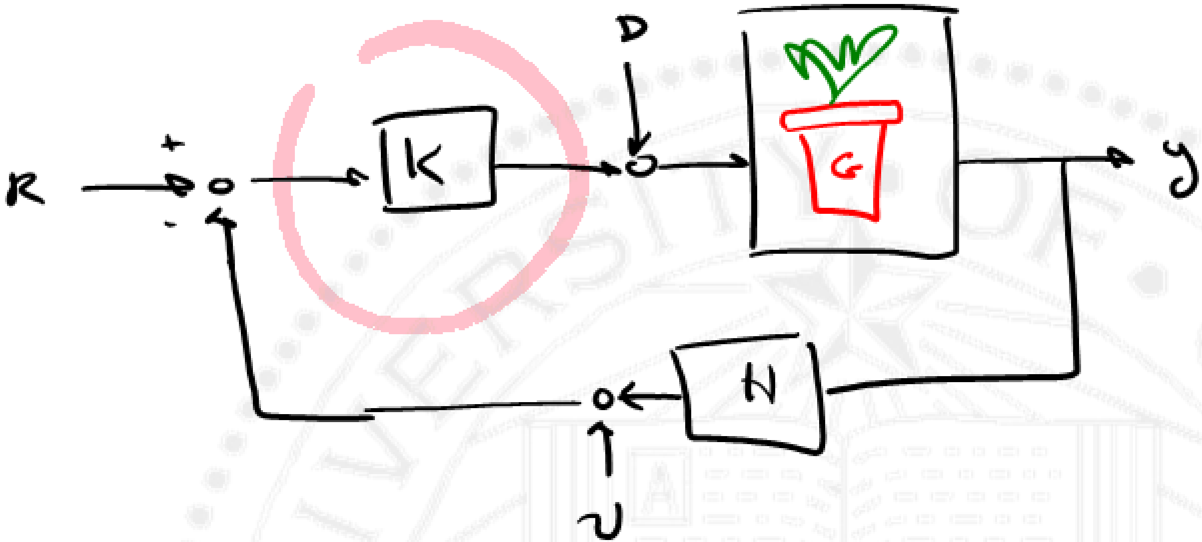
Announcements

Please turn in your Personal Responsibility Form.

Questions?

CONTROLS ≠ APPLIED MATH



10-20% TIME INVESTED IN DESIGNING "K"

$$v_{\lrcorner} = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right) \qquad v_{\uparrow} = \frac{F_0}{m} e^{-\frac{b}{m}t}$$

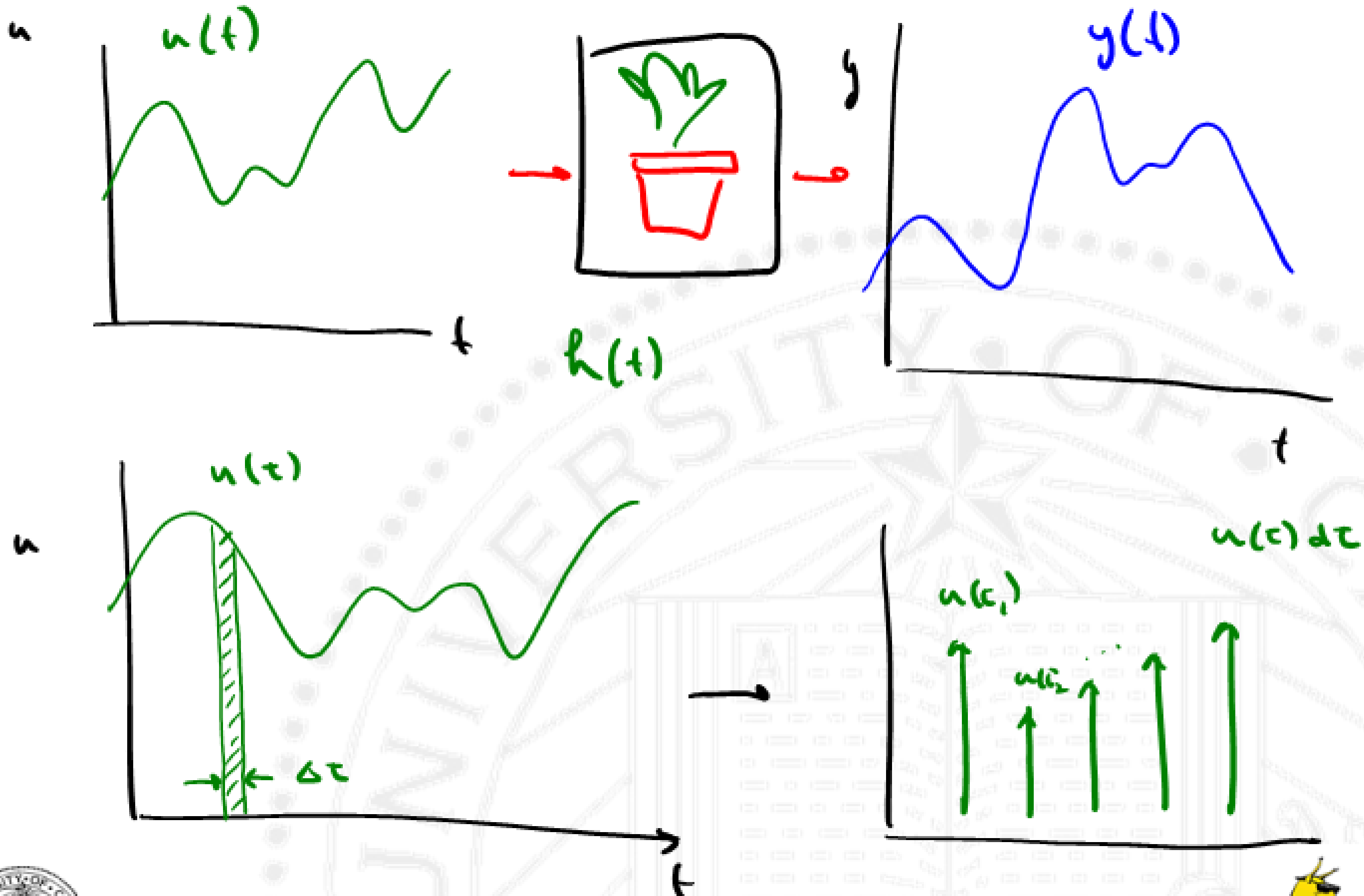
$$\frac{d}{dt}\left(v_{\lrcorner}\right) = v_{\uparrow} \rightarrow \frac{d}{dt}\left(\frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)\right) = \frac{d}{dt}\left[\cancelto{0}{\frac{F_0}{b}} - \frac{F_0}{b} e^{-\frac{b}{m}t}\right]$$

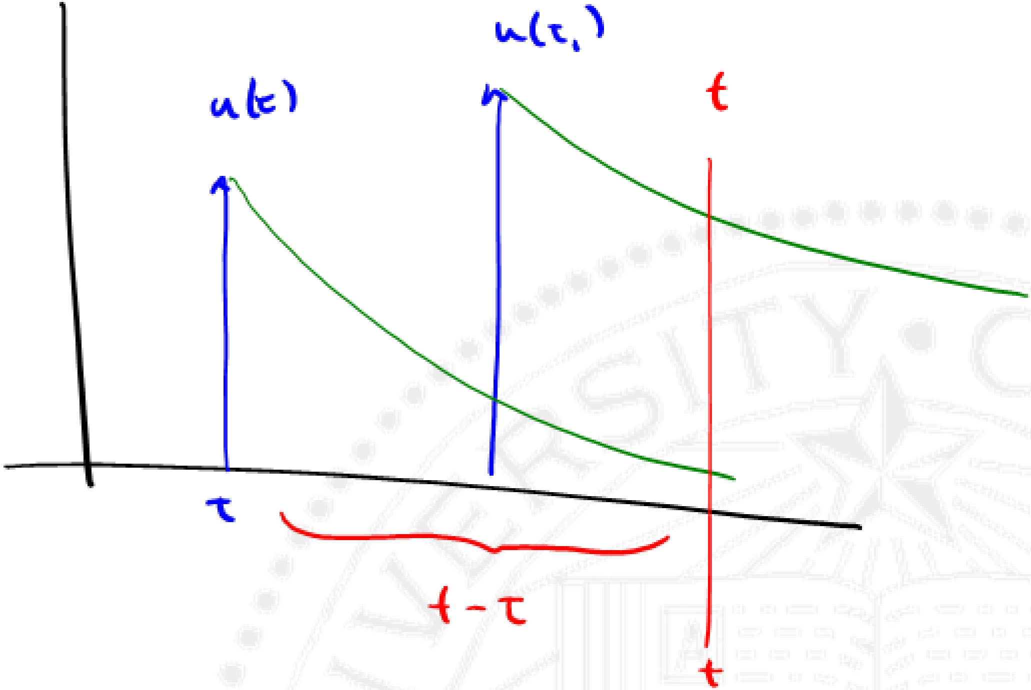
$$= -\frac{F_0}{b}\left[-\frac{b}{m}\right] e^{-\frac{b}{m}t} = \boxed{\frac{F_0}{m} e^{-\frac{b}{m}t}}$$

$$F_0 \triangleq 1$$

$$v_{\uparrow} = \underline{\frac{1}{m} e^{-\frac{b}{m}t}}$$

$h(t) \triangleq$ impulse response


u
u(t)
t
h(t)
y
y(t)
t
u(τ)
u
Δτ
t
u(τ1)
u(τ2)
u(τ)dτ



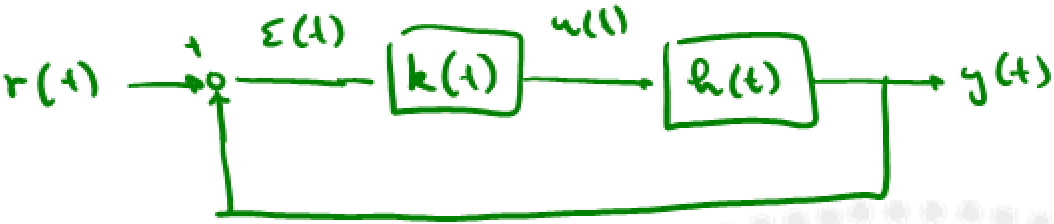
$$y(t) = u(\tau)d\tau\, h(t-\tau) + u(\tau_1)d\tau_1\, h(t-\tau_1) + \sum_{\text{impulses}}^{\text{all}}$$

$$\boxed{y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau}$$ ← CONVOLUTION INTEGRAL

CONVOLUTION INTEGRAL

$$y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = u(t) * h(t)$$



$$y(t) = u(t) * h(t)$$

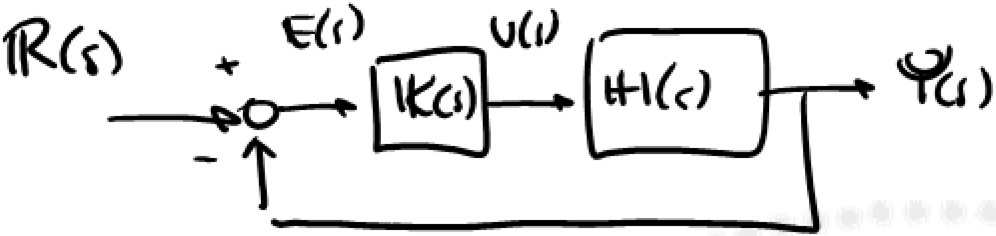
$$u(t) = \varepsilon(t) * k(t) = (r(t) - y(t)) * k(t)$$

$$\varepsilon(t) = r(t) - y(t)$$

$$y(t) = r(t) * k(t) * h(t) - y(t) * k(t) h(t)$$

$$y(t)\,(1 + h(t) * k(t)) = r(t) * k(t) h(t)$$

$$\mathcal{L}\left[\, y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau \,\right] \Rightarrow Y(s) = U(s) \times H(s)$$



$$Y(s) = U(s) \times H(s)$$

$$U(s) = E(s) \times K(s)$$

$$E(s) = R(s) - Y(s)$$

$$Y(s) = H(s)K(s)\left[R(s) - Y(s)\right]$$

$$Y(s) = \frac{K(s)H(s)}{1 + K(s)H(s)} R(s)$$

$$\boxed{\frac{Y}{R}(s) = \frac{KH}{1+KH}}$$

LAPLACE TRANSFORM

$$\mathcal{L}\{y(t)\} \triangleq Y(s) = \int_0^\infty y(t)\, e^{-st}\, dt$$

initial condition

$$\mathcal{L}\{\dot{y}\} = sY(s) - y(0)$$

$$\mathcal{L}\{\ddot{y}\} = s^2 Y(s) - s\,y(0) - \dot{y}(0)$$

$$\mathcal{L}\{f = m\ddot{x} + b\dot{x} + kx\} =$$

$$F = m[s^2X - sx_0 - \dot{x}_0] + b[sX - x_0] + kX$$

$$F = [ms^2 + bs + k]X - [ms + b]\cancel{x_0} - m\cancel{\dot{x}_0}$$

$$\frac{X}{F} = \underbrace{\frac{1}{ms^2 + bs + k}}_{H(s)}$$

Assuming $x_0 = 0$, $\dot{x}_0 = 0$

$$Y(s) = H(s) \times F(s)$$

$$\mathcal{L}\{\delta(t)\} = \int_0^\infty \delta(t)\, e^{-st}\, dt = 1$$

$$\mathcal{L}\{1\} = \int_0^\infty 1\, e^{-st}\, dt = \frac{e^{-st}}{s}\Big|_0^\infty = 0 - \left(\frac{-1}{s}\right) = \frac{1}{s}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^2\} = \frac{1}{2s^3}$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\boxed{\mathcal{L}^{-1}\{X(s)\} \triangleq x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} e^{st} X(s)\, ds}$$

$$Y(s) = TF(s) \times R(s) \leftarrow R(s) = \int_0^\infty r(t)e^{-st}\,dt \quad \text{"easy"}$$

impulse response "easy"

$$Y(s) = \frac{A}{(s+a)^2} \rightarrow y(t)$$

Method ϕ:

$$y(t) = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} e^{st}\frac{A}{(s+a)^2}\,ds \rightarrow y(t)$$

Method 1: look it up in a table #14

$$y(t) = (1+at)e^{-at}$$

Method 2: Break it up into parts that are in the table. P.F.E

$$Y(s) = \frac{s^2 + as + a^2}{s(s+a)^2} = \frac{a}{s(s+a)} + \frac{s}{(s+a)^2}$$

PARTIAL FRACTION EXPANSION

PARTIAL FRACTION EXPANSION

$$Y(s) = \frac{(s+d)(s+e)}{(s+a)(s+b)(s+c)} \longrightarrow \frac{A}{(s+a)} + \frac{B}{(s+b)} + \frac{C}{(s+c)}$$

residues

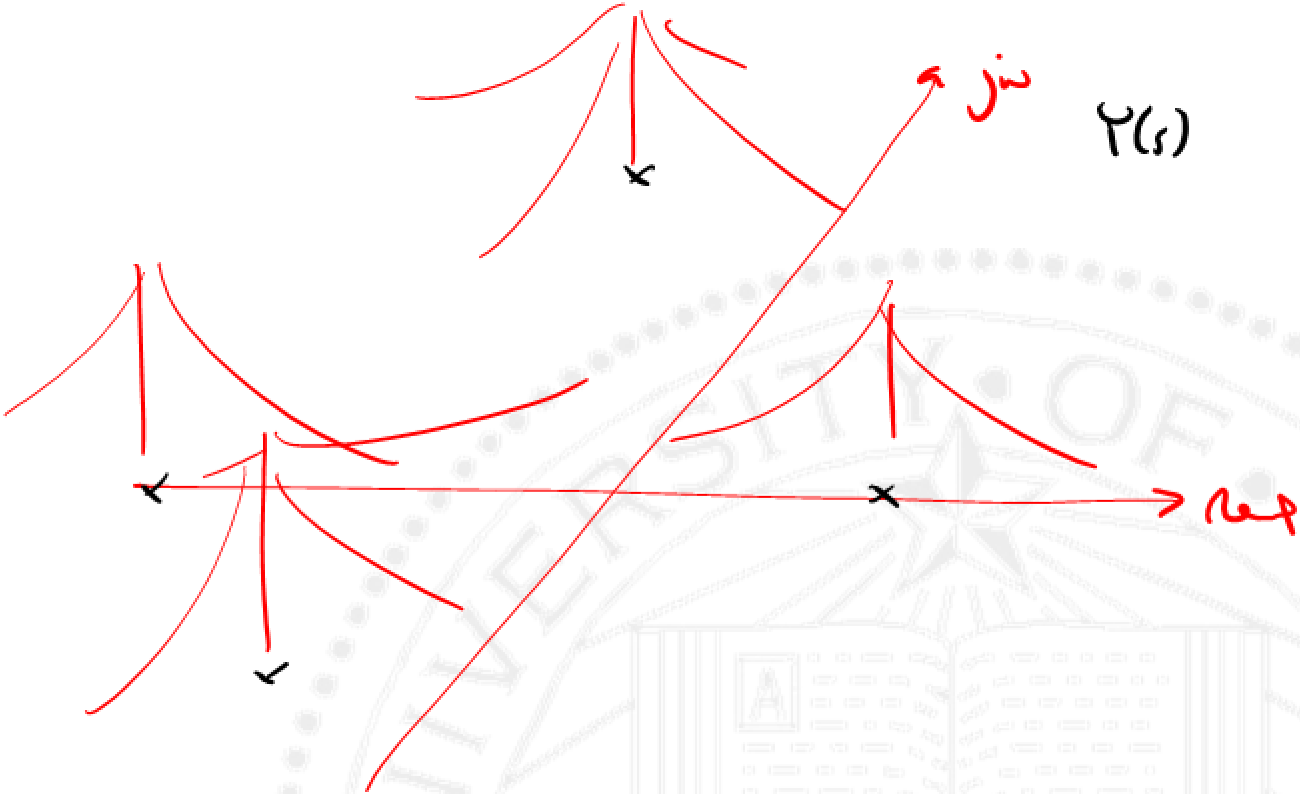
$$y(t) = \underbrace{Ae^{-at}} + \underbrace{Be^{-bt}} + \underbrace{Ce^{-ct}}$$

"modes"

Non-repeated roots

"cover up method"

$$A = (s+a)Y(s)\Big|_{s=-a} = \frac{\cancel{(s+a)}(s+d)(s+e)}{\cancel{(s+a)}(s+b)(s+c)} = \frac{(d-a)(e-a)}{(b-a)(c-a)}$$


jω
Re

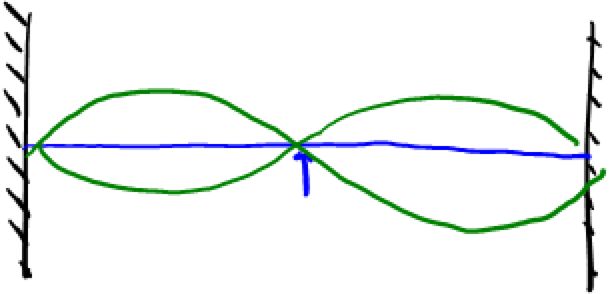
$$Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3} = \frac{8}{3}\frac{1}{s} - \frac{3}{2(s+1)} - \frac{\frac{1}{6}}{(s+3)}$$

$$y(t) = \frac{8}{3} - \frac{3}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

$$A = sY(s)\Big|_{s=0} = \frac{(s+2)(s+4)}{(s+1)(s+3)}\Bigg|_{s=0} = \frac{(2)(4)}{(1)(3)} = \frac{8}{3} = A$$

$$B = (s+1)Y(s)\Big|_{s=-1} = \frac{(-1+2)(-1+4)}{(-1)(-1+3)} = -\frac{3}{2}$$

$$C = (s+3)Y(s)\Big|_{s=-3} = \frac{(-1)(+1)}{(-3)(-2)} = \frac{-1}{6}$$



$\mathcal{O}$ observability

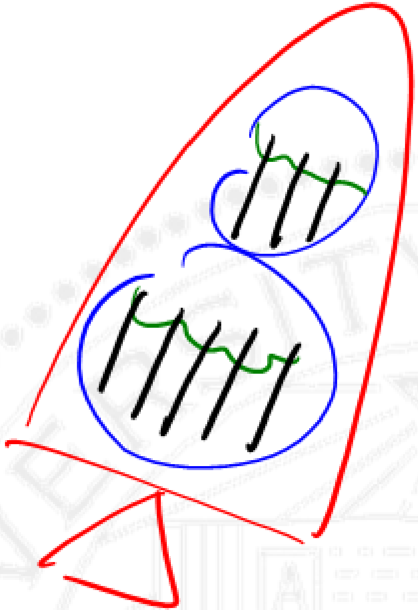
$\mathcal{C}$ controllability

Repeated Roots

$$Y(s) = \frac{s+3}{(s+1)(s+2)^2} = \frac{C_1}{s+1} + \frac{C_2}{(s+2)} + \frac{C_3}{(s+2)^2}$$

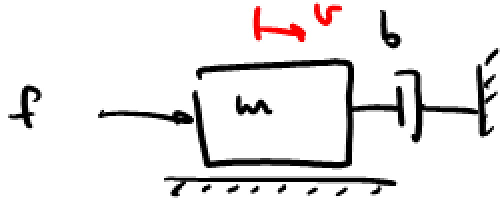
$$C_2 = \frac{d}{ds}\left[(s+2)^2 F(s)\right]\Big|_{s=-2}$$

$$C_3 = (s+2)^2 F(s)\Big|_{s=-2}$$



Complex Roots

$$\frac{}{(s^2 + as + b)} \longrightarrow \frac{c_1}{s + p_1} + \frac{c_1^*}{s + p_1^*} \longrightarrow \frac{[As + B]}{s^2 + as + b}$$



$$m\dot{v} = F - bv$$

$$\mathcal{L}\{m\dot{v} + bv = F\}$$

$$\mathcal{L}\{v\} = V$$

$$\mathcal{L}\{\dot{v}\} = sV - \overset{\text{i.c.}}{v_0}$$

$$F(s) = \text{step} = \frac{F_0}{s}$$

$$sV - v_0 + \frac{b}{m}V = \frac{F}{m} \rightarrow V(s) = \frac{\frac{1}{m}}{(s+\frac{b}{m})}F + \frac{v_0}{(s+\frac{b}{m})}$$

$$V(s) = \underbrace{\overset{\#11}{\frac{\frac{1}{m}}{s(s+\frac{b}{m})}}F_0}_{\text{FORCED}} + \underbrace{\overset{\#7}{\frac{v_0}{(s+\frac{b}{m})}}}_{\text{NATURAL}}$$

$$\boxed{v(t) = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right) + v_0 e^{-\frac{b}{m}t}}$$

Final Value Theorem (FVT)

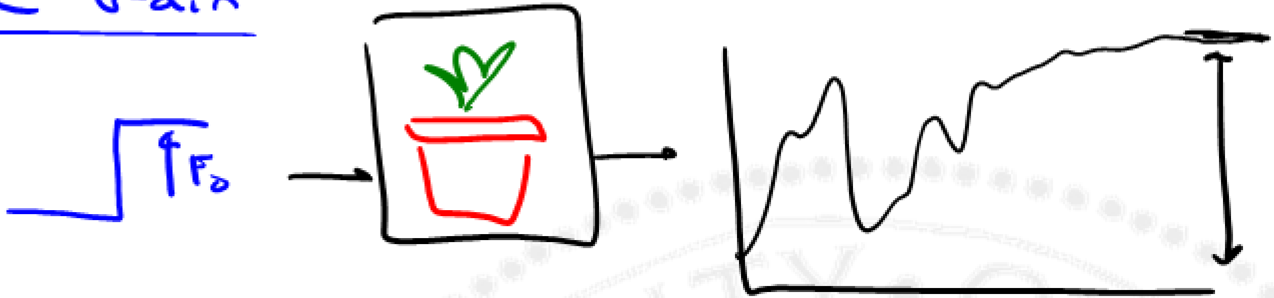
$$y(\infty) = \lim_{s \to 0} sY(s)$$

I.F.F. system is STABLE

Initial Value Theorem (IVT)

$$y(0^+) = \lim_{s \to \infty} sY(s)$$

DC Gain



$$\text{DC gain} \triangleq \frac{y(\infty)}{F_0}$$

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \cancel{s} H(s) \frac{F_0}{\cancel{s}} = H(s)F_0 \Big|_{s=0}$$

$$\boxed{\text{DC Gain} = H(0)}$$

only for stable systems.

$$Y(s) = \frac{N(s)}{D(s)}$$

← residuals "how much"

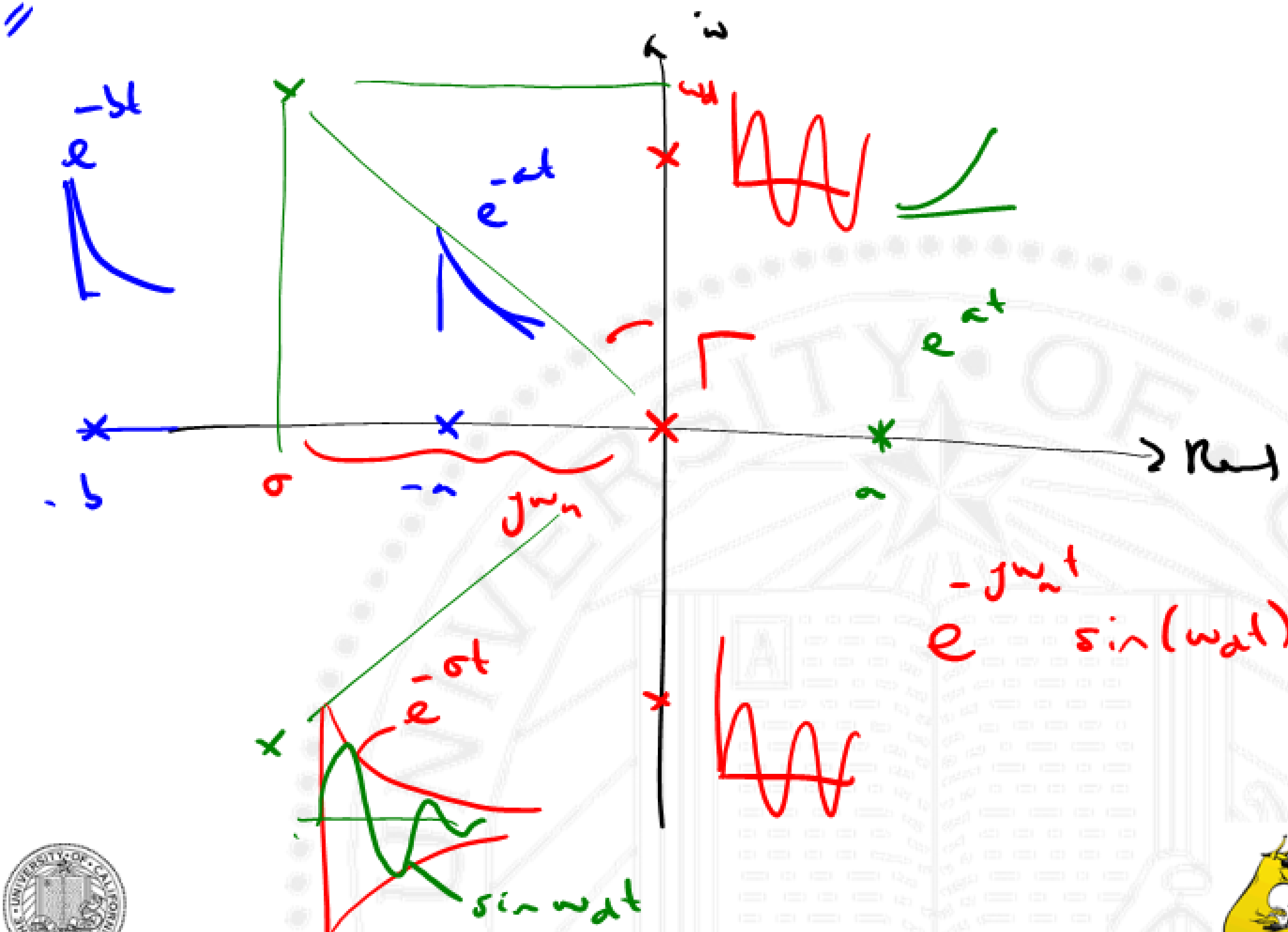
Poles: "type of responses", "modes"

$$Y(s) = H(s)\,U(s) \qquad U(s) = \delta(t) = 1$$

$$H(s) = \frac{\sim\sim}{(s+a)\left[(s+b)^2 + c^2\right]} = \frac{\sim}{(s+a)} + \frac{\sim}{(s+b)^2 + c^2}$$

modes

$\Delta(s)$ characteristic polynomial



jω
ω_d
e^{-bt}
e^{-at}
e^{at}
Real
σ
-b
-a
a
jω_n
e^{-σt}
sin ω_d t
e^{-jω_n t} sin(ω_d t)