CMPE-242
Applied Feedback Control

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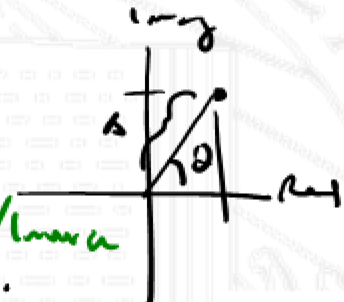
Announcements

Three handouts — (1) General Course Info
(2) Detailed Syllabus
(3) Personal Responsibility

Prereq's —
- Differential Equations (LCCODE)
- Laplace transforms
- Complex Numbers
- Linear Algebra - Det/Eig/Inverse
- Fourier Analysis



$a + bj$

$Ae^{i\theta}$

Announcements (cont'd)

Grading — A/B/F

MATLAB — Control System Toolbox

SISOTool

Shortened Class time

Intro

Classical Control
- Transfer functions
- Root locus
- Bode
- Nyquist

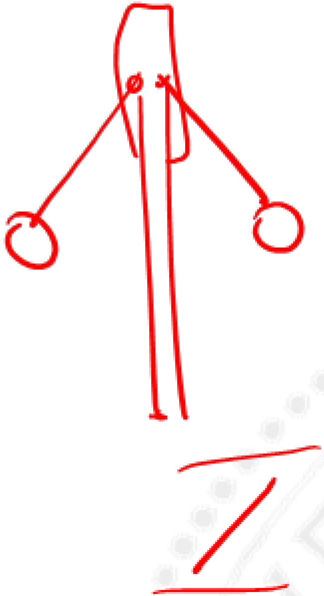
3 weeks

Digital Control
- z-transform
- equivalents

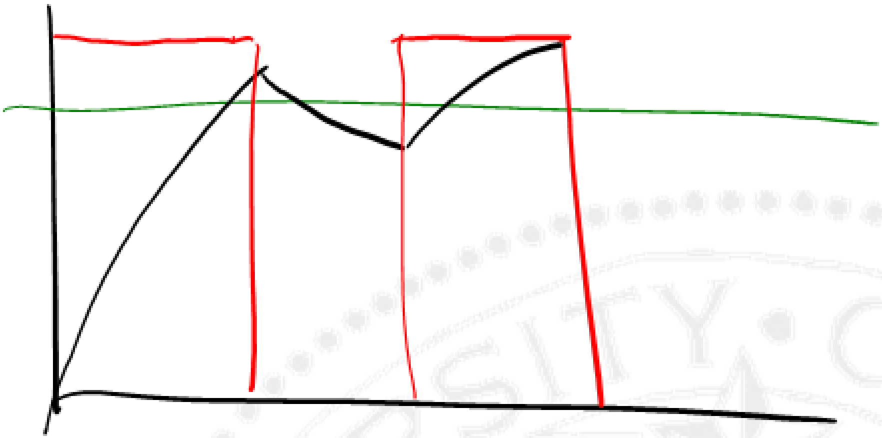
3 weeks

State Space
- "Modern Stuff" – 1960's
- Linear Algebra
- LQ(-) → R/E/G

Not covered: Non-linear control (Lyapunov)
Bang/Bang +/0/-

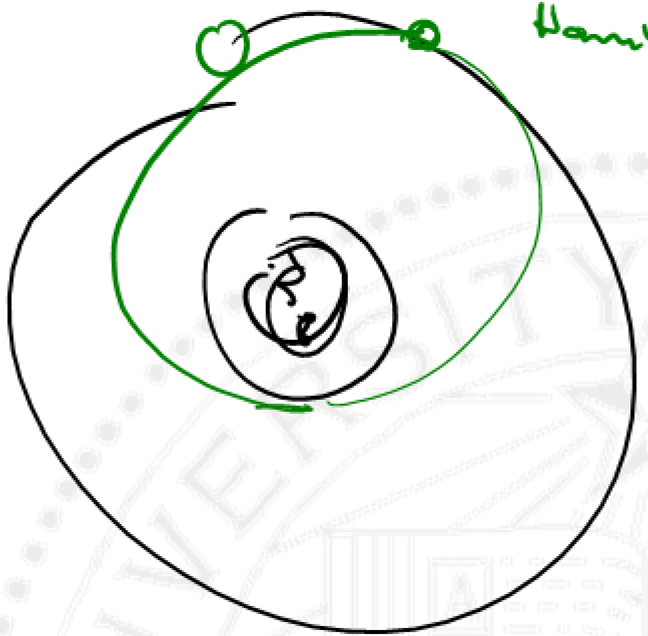


Flyball Governor



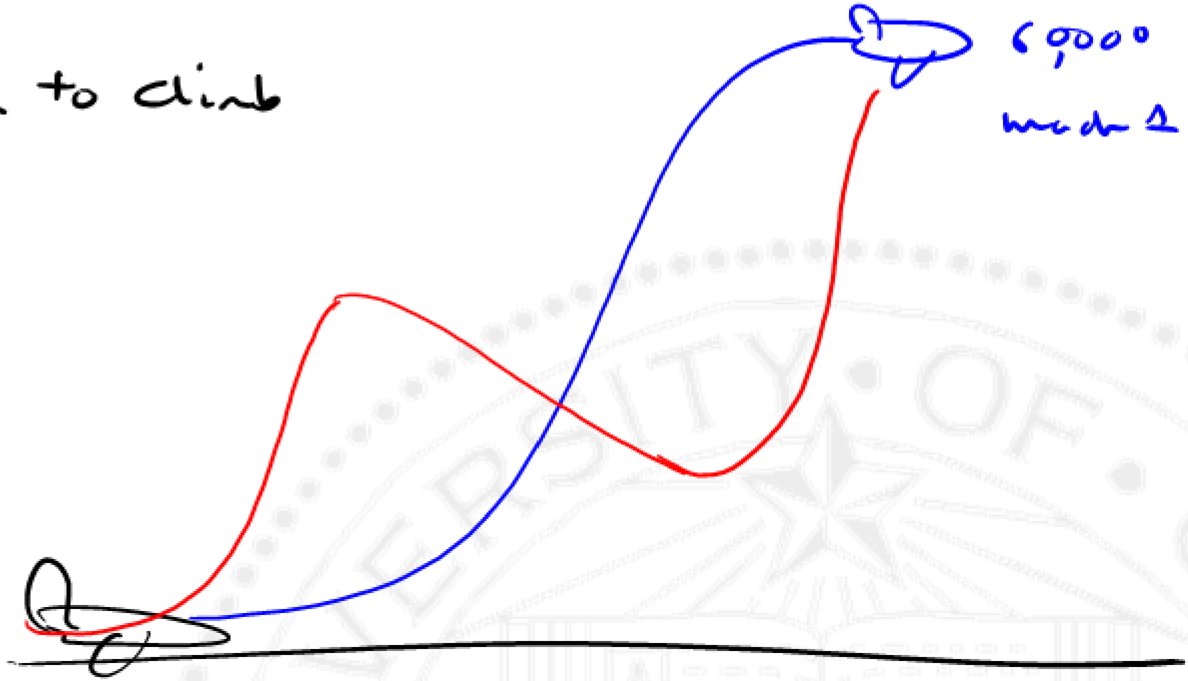
Art Bryson — calculus of variations

$$\int_0^\infty J + \lambda_i \text{ constraints}$$

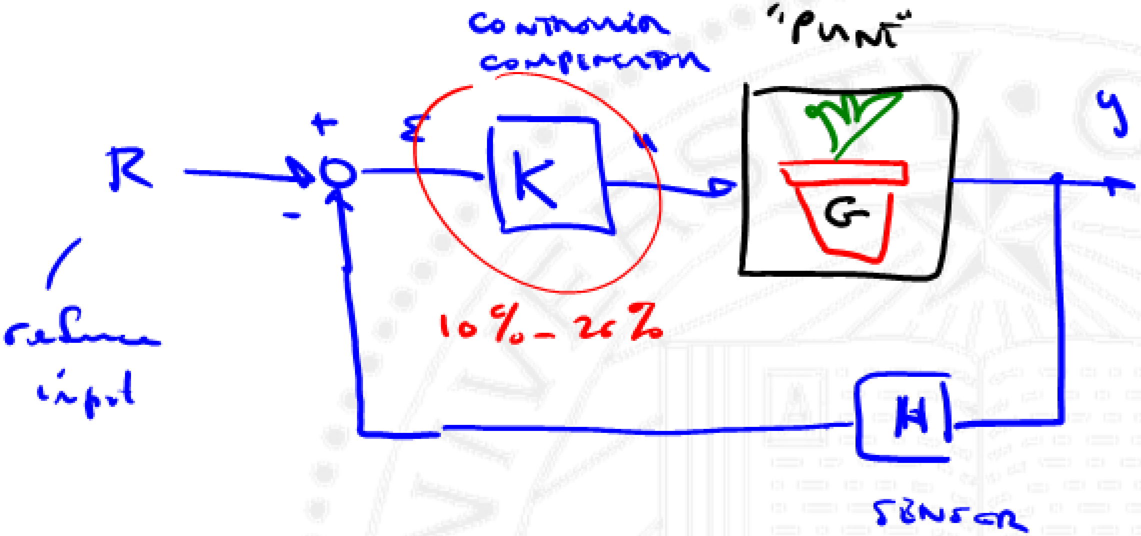


Hohmann Transfer

Time to climb

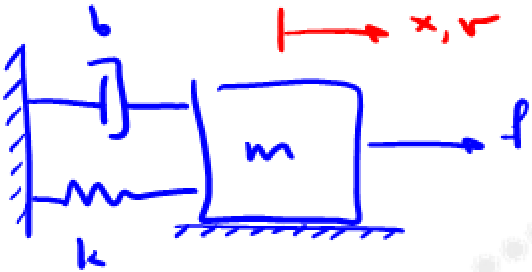


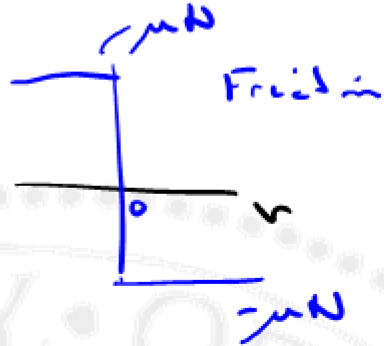
CONTROLS ≠ APPLIED MATH



READ FPE ch. 1-3

Equations of Motion





$$F_f = \mu N \,\mathrm{sign}(v)$$

$$F = \frac{d\vec{p}}{dt} = ma$$

$$m\ddot{x}$$

$$m\dot{v}$$

$\neq$

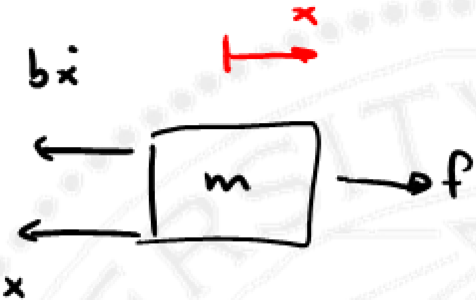
$$F = b\dot{x}$$

$$bv$$

$$F = kx$$

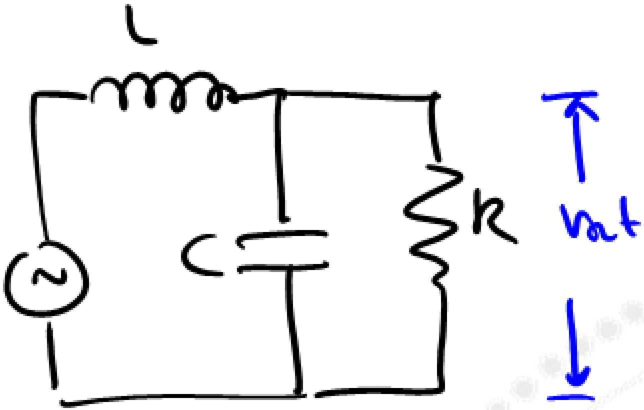
$$= k\int v\,dt$$

$$\Sigma F = \boxed{m\ddot{x} = f - b\dot{x} - kx}$$



F. B. D

$$m\ddot{y} + b\dot{x} + kx = f$$



Kirchov's Laws

$$\sum i_{\odot} = \phi$$

$$\sum v_{\circlearrowright} = \phi$$

 $i = C\dot{v}$

 $i = v/R$

 $i = \frac{1}{L}\int v\,dt$

Equations of Motion

$$F = m\ddot{x} + b\dot{x} + kx$$

Linear, Constant Coefficient Ordinary Diff. Eq'n.

LCCODE

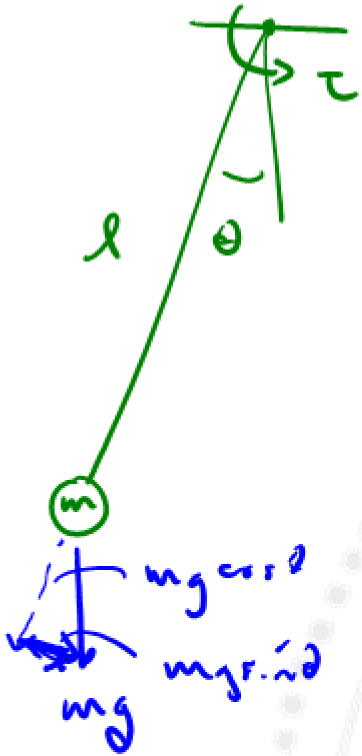
"LINEAR TIME INVARIANT" – LTI

$$F_1 \longrightarrow x_1$$

$$F_2 \longrightarrow x_2$$

$$F_1 + F_2 \longrightarrow x_1 + x_2$$

$$\alpha F_1 \longrightarrow \alpha x_1$$



$$ml^2\ddot{\theta} + mgl\sin\theta = \tau$$

$$\theta \triangleq \underset{\text{constant}}{\theta_0} + \underset{\text{small}}{\delta\theta} \qquad \tau \triangleq \tau_0 + \delta\tau$$

$$\sin(\theta_0+\delta\theta) = \sin\theta_0 \overset{\Delta}{\cancel{\cos\delta\theta}} + \cos\theta_0 \overset{\delta\theta}{\cancel{\sin\delta\theta}}$$

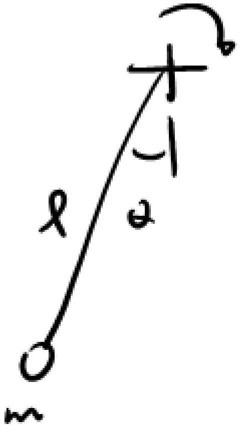
$$mgl\sin\theta_0 = \tau_0 \qquad \theta_0 = \sin^{-1}\left(\frac{\tau_0}{mgl}\right)$$

$$m\ell^2 \ddot{\delta\theta} + mg\ell\left(\sin\theta_0 + \cos\theta_0\,\delta\theta\right) = \tau_0 + \delta\tau$$

$$m\ell^2 \ddot{\delta\theta} + \cancel{mg\ell\sin\theta_0} + mg\ell\cos\theta_0\,\delta\theta = \cancel{\tau_0} + \delta\tau$$

$$m\ell^2 \ddot{\delta\theta} + mg\ell\cos\theta_0\,\delta\theta = \delta\tau$$

Perturbation about trim



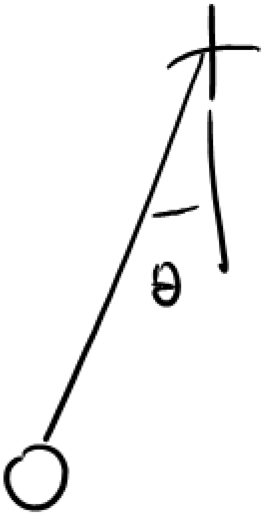
$$\tau = mgl\sin\theta + \delta u$$

$$ml^2\ddot{\theta} + \cancel{mgl\sin\theta} = \cancel{mgl\sin\theta} + \delta u$$

$$ml^2\ddot{\theta} = \delta u$$

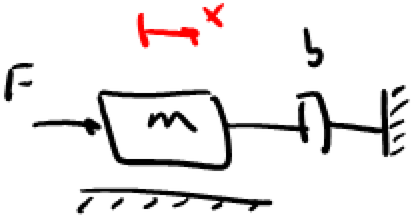
Feedback Linearization

PLANT INVERSION



$\sin\theta \simeq \theta$ if θ is small

Linearize + Ignore



$$F - bv = m\dot{v}$$

$$\boxed{m\dot{v} + bv = F + 0} \quad \text{E.O.M}$$

Natural/Homogeneous

Forced

complex

$$v = Ae^{st}$$

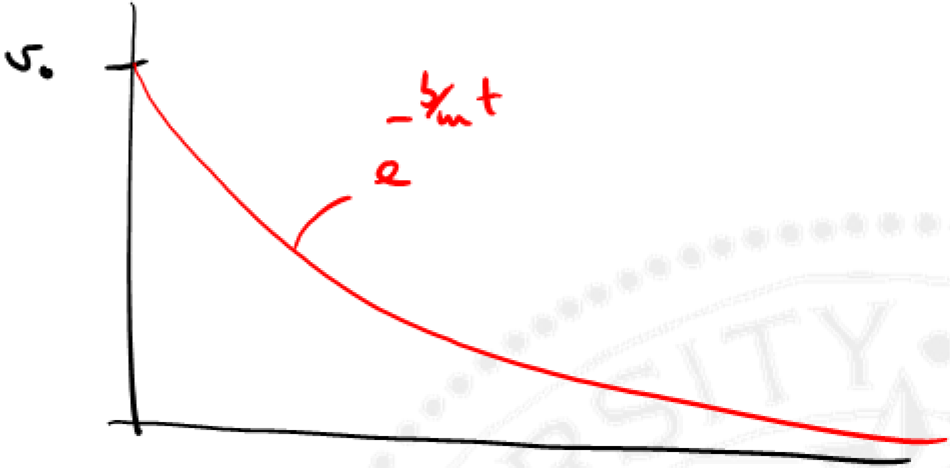
$$\dot{v} = Ase^{st}$$

$$m\cancel{A}s\cancel{e^{st}} + b\cancel{A}\cancel{e^{st}} = 0$$

$$ms + b = 0 \longrightarrow s = -\frac{b}{m}$$

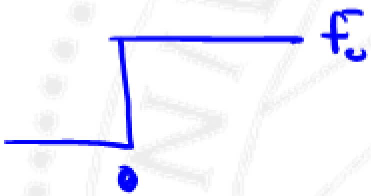
$$v = Ae^{-\frac{b}{m}t} \longrightarrow v(0) = Ae^{-\frac{b}{m}(0)} = A$$

$$\boxed{v(t) = v_0 e^{-b/m\, t}}$$ ← natural response



$v_0 = \phi$

Forced Response



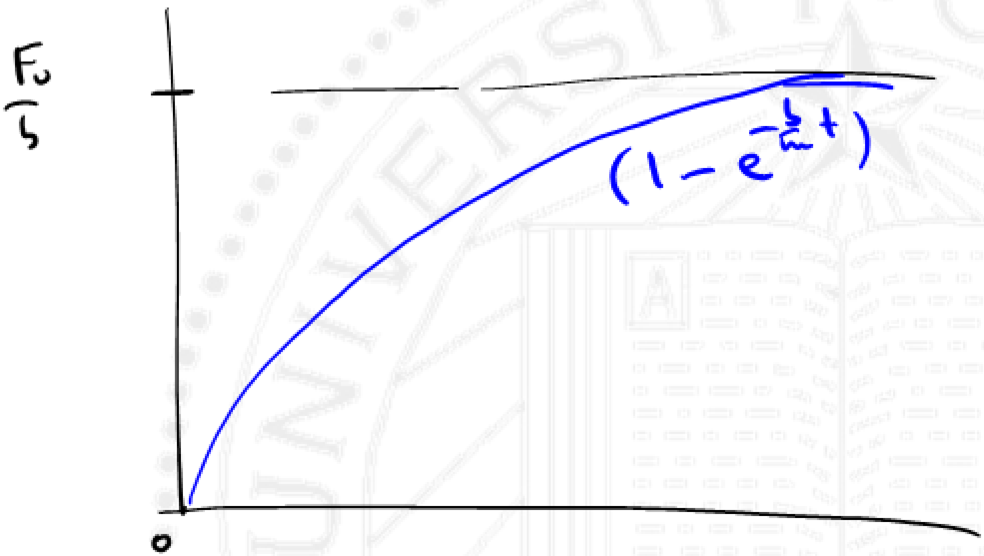
$$\overset{0}{\cancel{m\dot{v}}} + bv = F_0$$

$$v = \frac{F_0}{b}$$

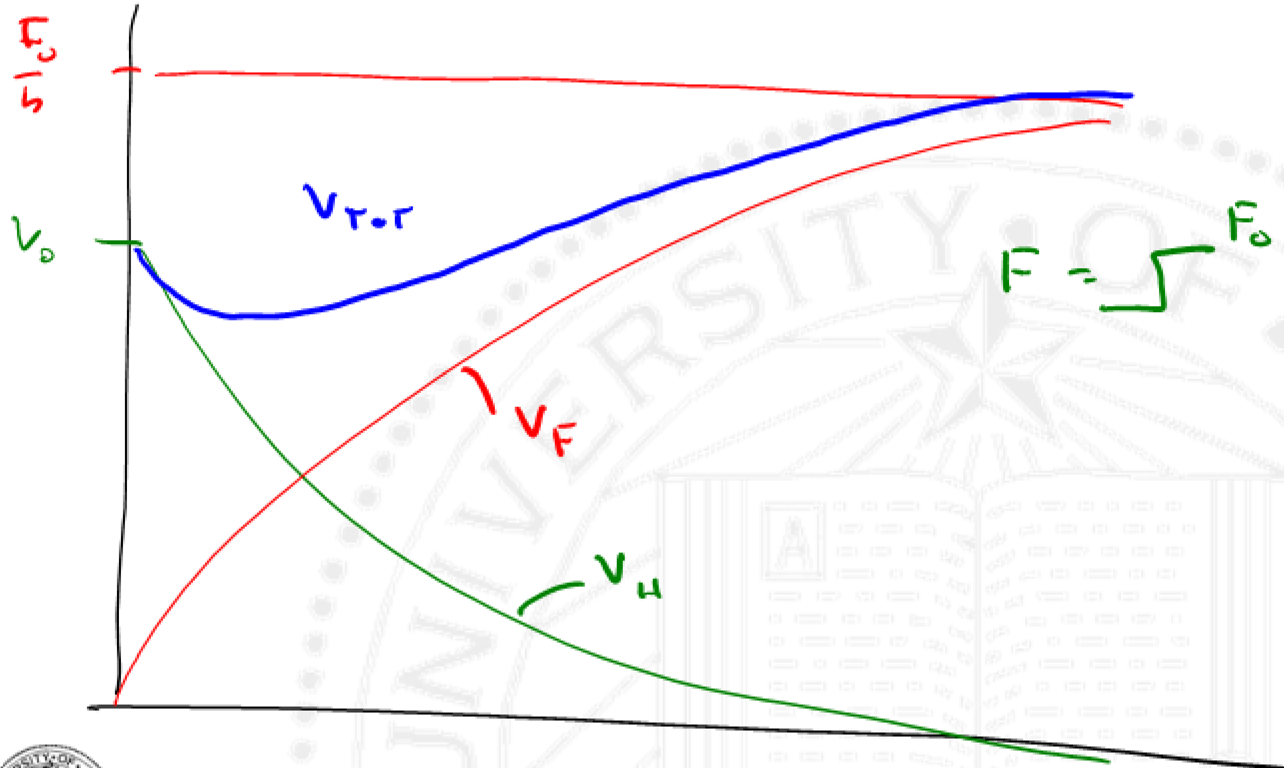
$$v = Ae^{-\frac{b}{m}t}$$

$$v = \frac{F_0}{b} + A e^{-\frac{b}{m}t} \qquad v(0) = 0. \qquad 0 = \frac{F_0}{b} + A \;\therefore\; A = \frac{-F_0}{b}$$

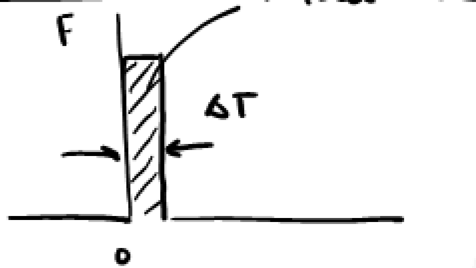
$$v_F(t) = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$



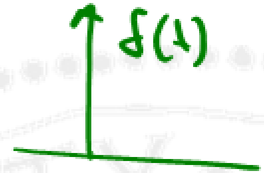
$$v_{TOT} = v_{NAT} + v_F = v_0 e^{-\frac{b}{m}t} + \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

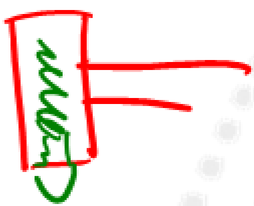


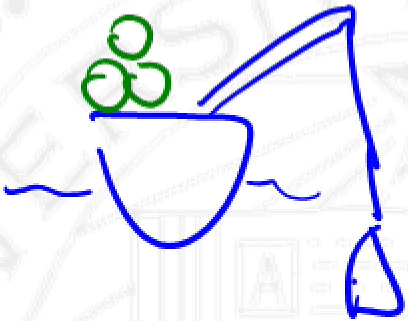
Impulse

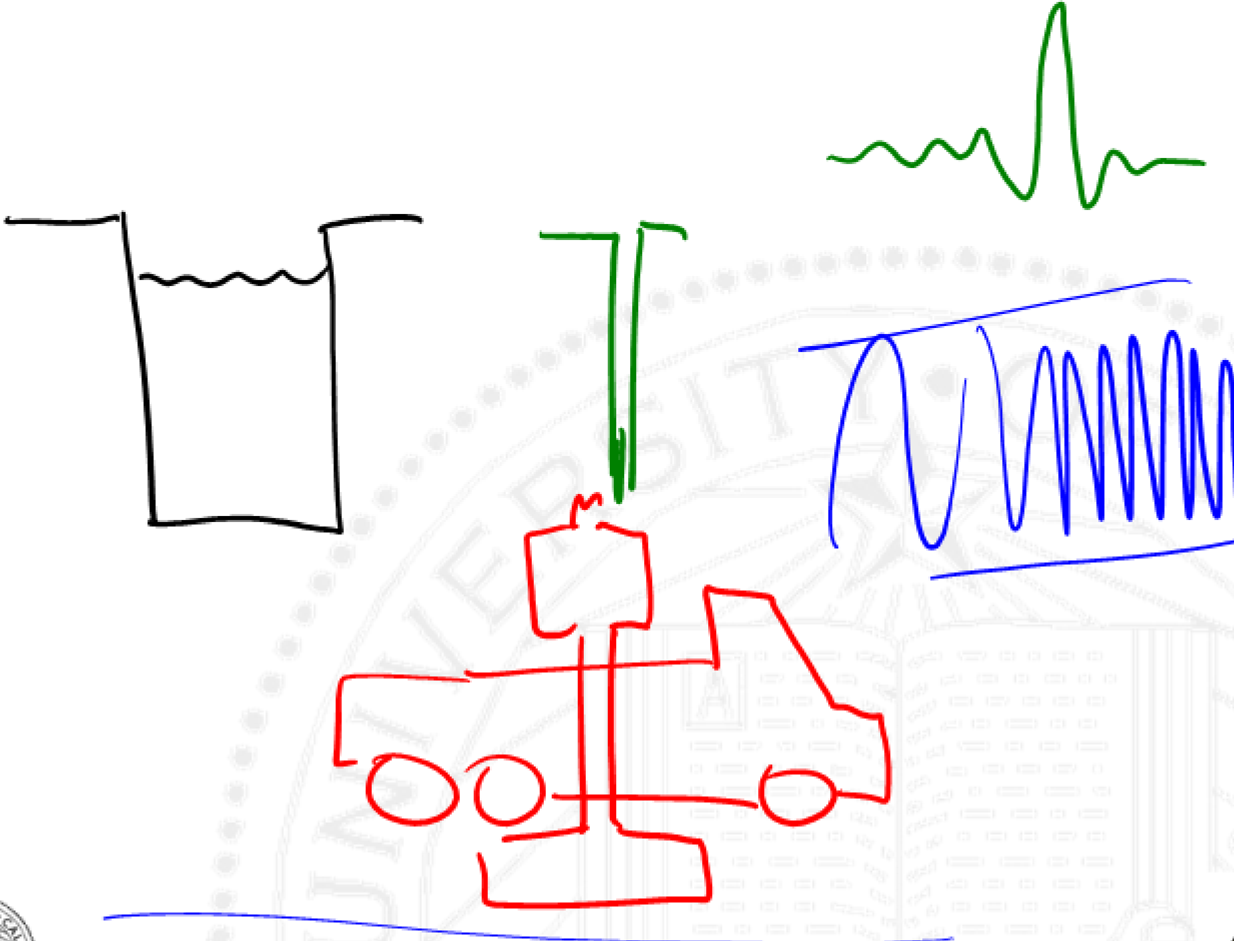


$$\int \delta(t)\,dt = 1$$

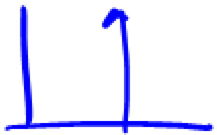






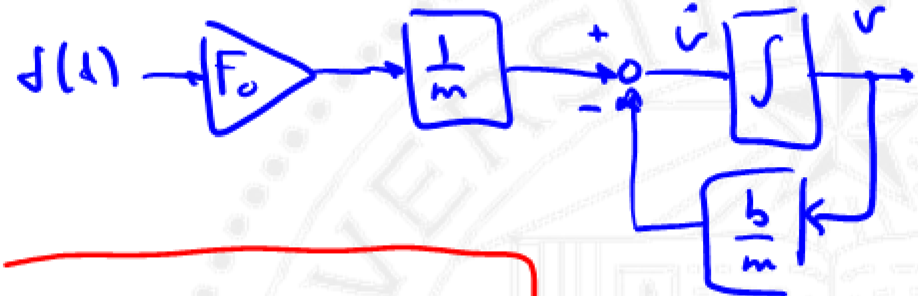


Impulse



$$\int \delta(t)\,dt = 1$$

$$m\dot{v} + bv = F_0\,\delta(t)$$



$0^+ : v = \frac{F_0}{m}$

$v_0 = \frac{F_0}{m}$

$$\boxed{v(t) = \frac{F_0}{m} e^{-\frac{b}{m}t}}$$