University of California, Santa Cruz
Board of Studies in Computer Engineering

CMPE-242: Applied Feedback Control

Homework #1
Due 17-Jan-2017

1. *Differential Equations*: Consider the system defined by:

   $m\ddot{x} + b\dot{x} = F(t)$ where $x(0) = 0$ and $\dot{x}(0) = 0$.

   a. Find the step response of this system. That is, what is $x(t)$ if $F(t) = \mathbf{1}(t)$. Do this in a way that does NOT use Laplace Transforms. For example, you can solve the equation using the techniques you learning in your first DiffEq class (i.e.: find the forced and natural parts of the response, etc.). So that you might check this, the answer is $x(t) = \frac{1}{b}(t - \frac{m}{b}\left[1 - e^{-\frac{b}{m}t}\right])$.

   b. The impulse response of the system, $h(t)$, is the derivative of the step response. What is $h(t)$?

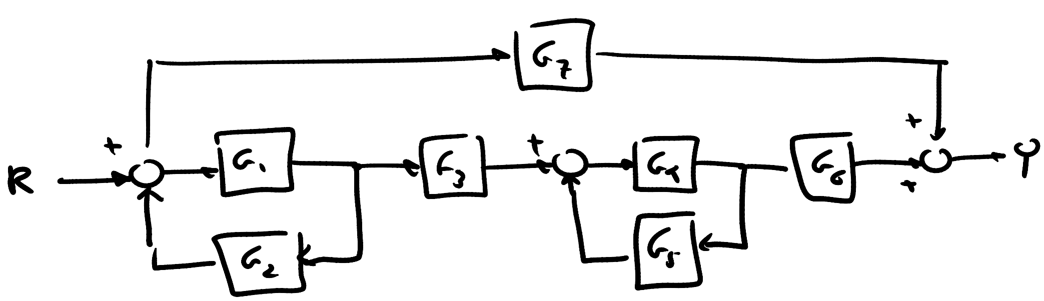
   c. Compute the step response using the convolution integral $x(t) = \int_0^t u(\tau)h(t-\tau)d\tau$, where $u(\tau) = 1$.

   d. What is the transfer function, $\frac{X(s)}{F(s)}$, of this system? What is the Laplace Transform of a step input, $U(s)$?

   e. Find the step response, $x(t)$, by taking the inverse Laplace Transform of $X(s)$. Use the table of Laplace Transforms.

   f. Repeat part (e), but first do a partial fraction expansion of $X(s)$. Note that there are repeated roots (see Appendix A of FPE).

2. *Block diagram reduction*: Write down the transfer function [Y(s)/U(s)] of the block diagram below:



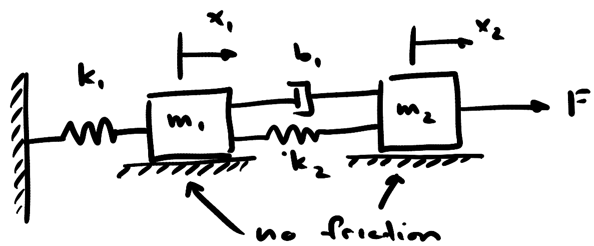
3. *Laplace transform*: find the time function for each of the following using the Inverse Laplace Transforms and partial fraction expansion (look at Appendix A for distinct complex roots):

   a. $F(s) = \frac{2}{s(s+2)}$

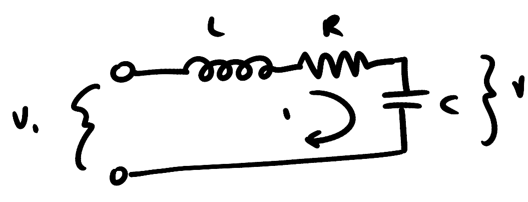
   b. $F(s) = \frac{3s+2}{s^2+4s+20}$

   c. $F(s) = \frac{2(s+2)}{(s+1)(s^2+4)}$

4. *Transfer Functions*: Given the following mass-spring system, derive the transfer function from the position of the leftmost of the masses to the forcing function $X_1(s)/F(s)$:



5. *Transfer Functions*: Given the following simple electrical system, derive the transfer function from the input voltage to the output voltage $[V_2(s)/V_1(s)]$:



6. *Dynamic Response*: Given the following third order system:

$$H(s) = \frac{\alpha\omega_n^2}{(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

The response to a unit step (U(t)=**1**(t)) is:

$$y(t) = 1 + Ae^{-\alpha t} + Be^{-\sigma t}\,\mathrm{SIN}(\omega_d t - \varphi)$$

where:

$$A = \frac{-\omega_n^2}{(\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2)}$$

$$B = \frac{\alpha}{\sqrt{(\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2)(1-\zeta^2)}}$$

$$\varphi = \mathrm{TAN}^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \mathrm{TAN}^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha - \zeta\omega_n}$$

a. Which term dominates as α gets large?
b. Which term dominates as α gets small?
c. Approximate A and B for small values of α.
d. Assume that $\omega_n = 1$ and $\zeta = 0.707$, plot the step response for several values of α. Use MATLAB's *step* command (could you use *impulse*? How?) Comment on where the extra pole becomes unimportant.

e. *Extra credit*: show that this is, indeed, the response to the step input (lots of hairy complex math and trig transformations).