CMPE-242
Applied Feedback Control

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$$\left[\int_0^t e^{A\tau}\,d\tau\right]B \qquad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = A$$

$$e^{A\tau} = I + A\tau + \frac{A^2\tau^2}{2!} + \frac{A^3\tau^3}{3!}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \tau & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \tau & 1 \end{bmatrix} =$$

$$\Phi = e^{A\tau} = \begin{bmatrix} 1 & 0 \\ \tau & 1 \end{bmatrix}$$

$$e^{A\tau} = \begin{bmatrix} 1 & 0 \\ \tau & 1 \end{bmatrix}$$

$$\int_0^T e^{A\tau}\,d\tau = \int_0^T \begin{bmatrix} 1 & 0 \\ \tau & 1 \end{bmatrix} d\tau = \begin{bmatrix} T & 0 \\ \frac{T^2}{2} & T \end{bmatrix}$$

CLOSED FORM SOLUTION: if $\underline{A^{-1}}$ exists

$$\int_0^T e^{A\tau}\,d\tau = A^{-1}\left[e^{AT} - I\right]$$

Part e: Sym. root locus rlocus$\cancel{(-G*G)}$.

sigh.input/output

$G = ss(A, B, C, D)$

$[z, p, k] = zpkdata(G)$

$Gmin = zpk(-z, -p, k)$.

rlocus (G*Gmin)

$\dot{x} = Ax + Bu$

$y = Cx$

$u \in \mathbb{R}^{1 \times 1}$

$$Lqr - \int_0^\infty x^TQx + u^TRu$$

ρ

y^TQy

C^TQ_yC

Q into lqr

y^TQy

$C^T\left[\frac{1}{y_{max}^2}\right]C$

$y = Cx \quad y^T = x^TC^T$

$q \times q$

$$\dot{x} = Ax + Bu \qquad u = -\underline{K}(\hat{x} - \overset{0}{\cancel{x^c}})$$

$$y = Cx + v$$

$$\dot{\hat{x}} = A\hat{x} + Bu + \underline{L(y - \hat{y})}$$

$$\hat{y} = C\hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A & -BK \\ \hline LC & A-BK-LC \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} BK \\ BK \end{bmatrix} x^c$$

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & -K \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} x^c$$

$$\dot{x} = Ax - BK\hat{x} + BKx^c$$

$$\dot{\hat{x}} = A\hat{x} - BK\hat{x} + BKx^c + LCx - LC\hat{x}$$

$$\dot{\hat{x}} = (A - BK - LC)\hat{x} + LCx + BKx^c$$

$$\begin{bmatrix} y \\ u \\ \tilde{x} \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & -K \\ I & -I \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix} r^c$$

$(x - \hat{x})$

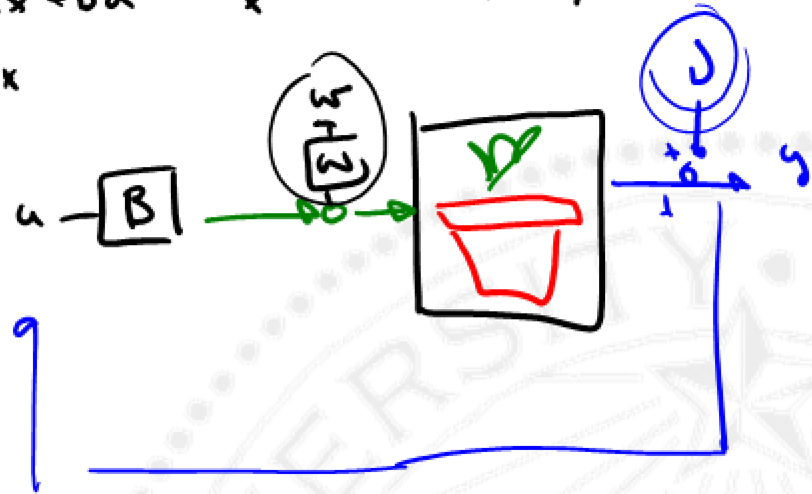
<u>initial</u>

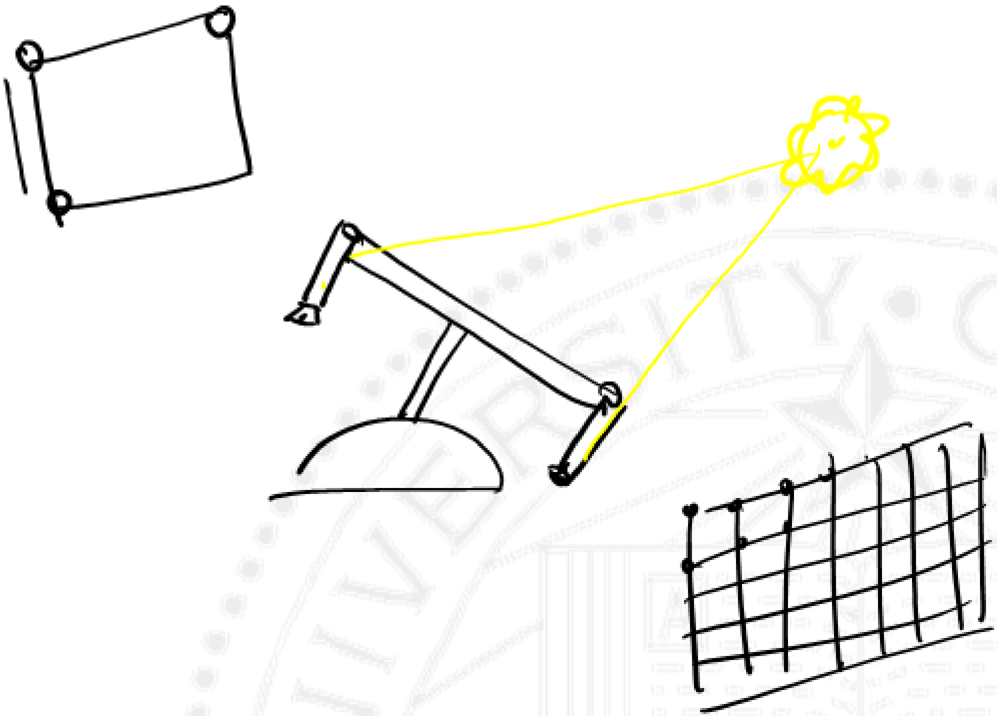
<u>step</u>

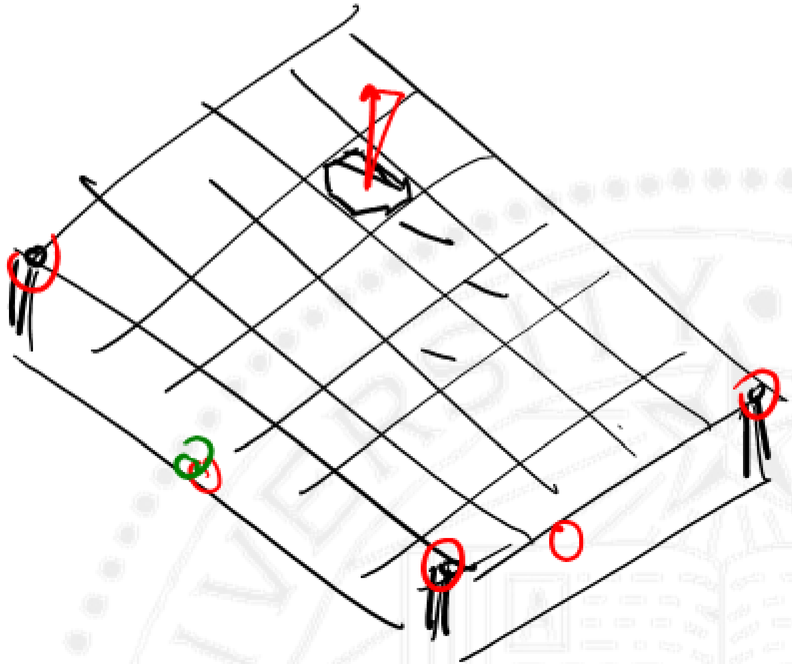
<u>lsim</u>

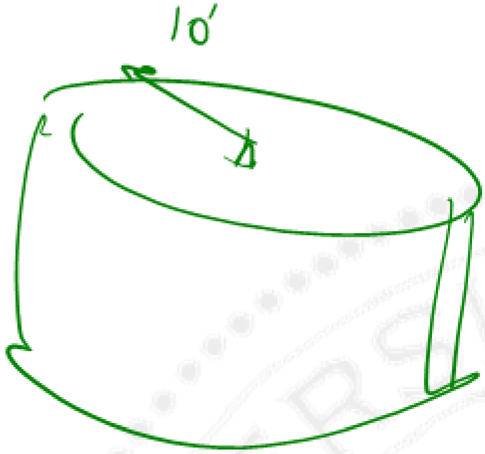
$\dot{x} = Ax + Bu$ $\quad \dot{\hat{x}} \cdots \quad x^c = \phi$.

$y = Cx$

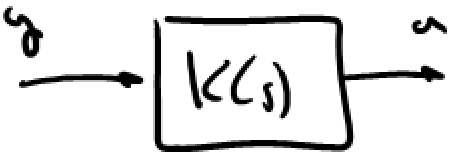







10'

$$\boxed{K(s) = -K[sI-(A-BK-LC)]^{-1}L}$$



$\dot{x} = Ax + Bu \qquad u = -Kx$

$y = Cx$

$\dot{\hat{x}} = A\hat{x} + Bu + L(y-\hat{y})$

$\hat{y} = Cx$

output

$u = -K\hat{x}$

input

$\dot{\hat{x}} = (A-BK-LC)\hat{x} + Ly$

$\dot{\hat{x}} = (A-BK-LC)\hat{x} + Ly$

$u = -K\hat{x}$