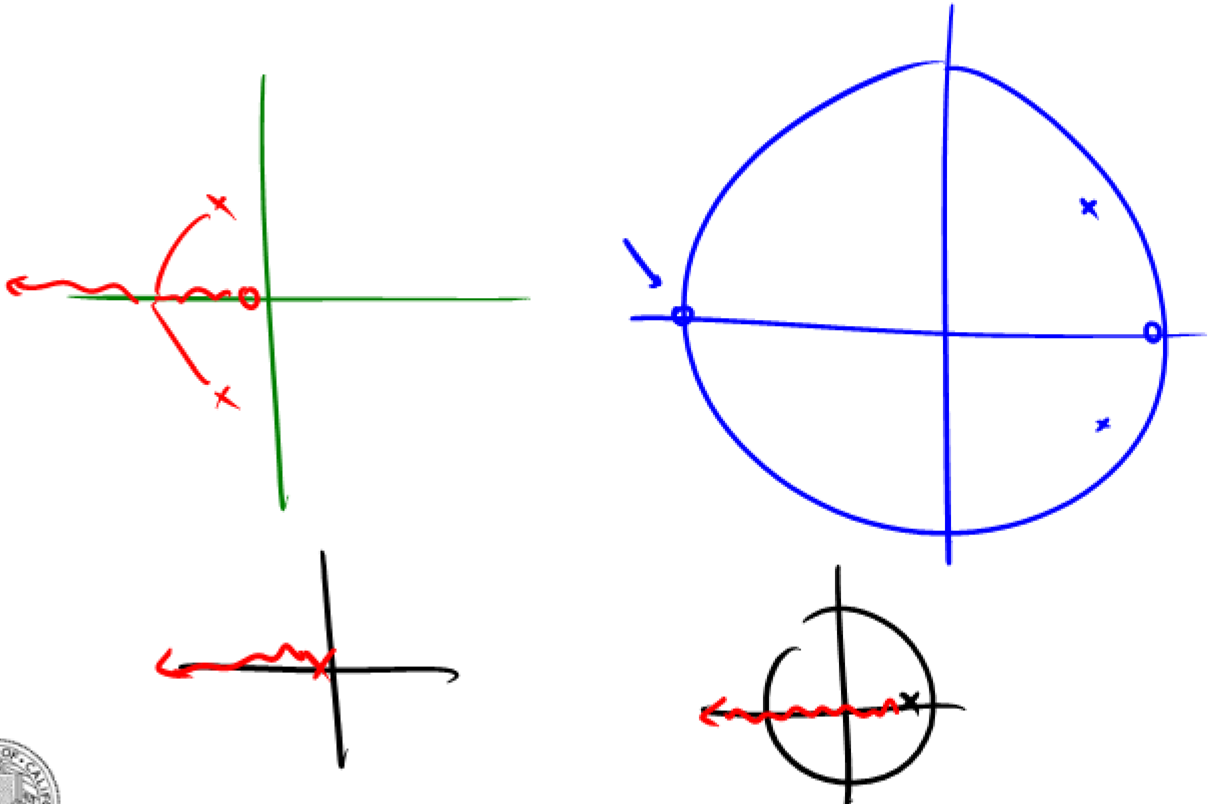
CMPE-242
Applied Feedback Control

Gabriel Hugh Elkaim

Winter 2015

MIDTERM REVIEW

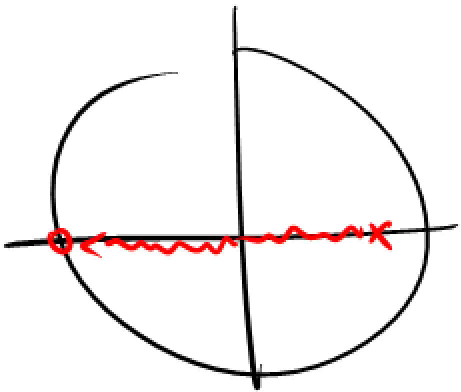


$$G(s) = \frac{a}{s+a}\Bigg|_{s = \frac{2}{T}\frac{z-1}{z+1}}$$

$$G(z) = \frac{a}{\frac{2}{T}\frac{z-1}{z+1} + a}$$

$$= \frac{a(z+1)}{\frac{2}{T}(z-1) + a(z+1)}$$

$$= \frac{a(z+1)}{\left[\left(\frac{2}{T}+a\right)z + \left(a - \frac{2}{T}\right)\right]}$$



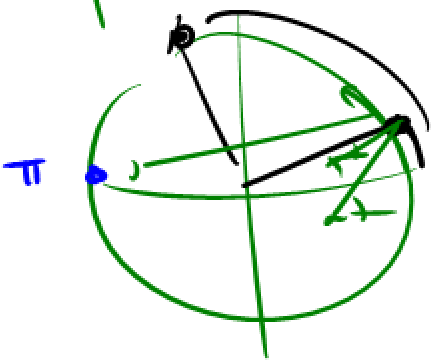
$G(s) \rightarrow G(z) \rightarrow$ bode

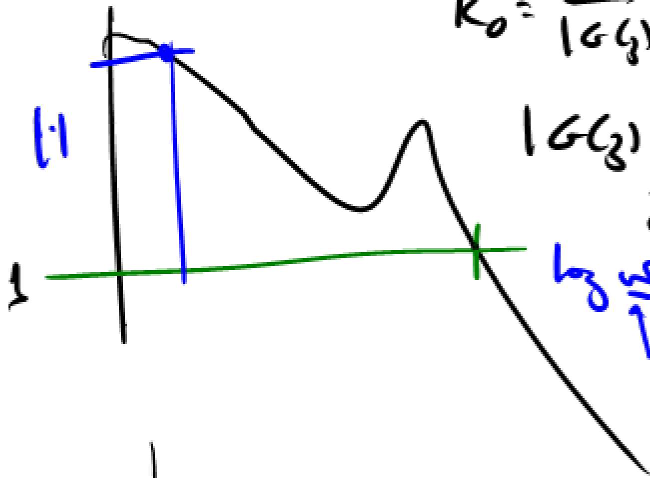
$\omega_{s0} = 5$ rad/sec.

$z = e^{5jT}$ (0.01, 0.2)

$K_0 = \dfrac{1}{|G(z)|}\Big|_{z=e^{j\omega T}}$

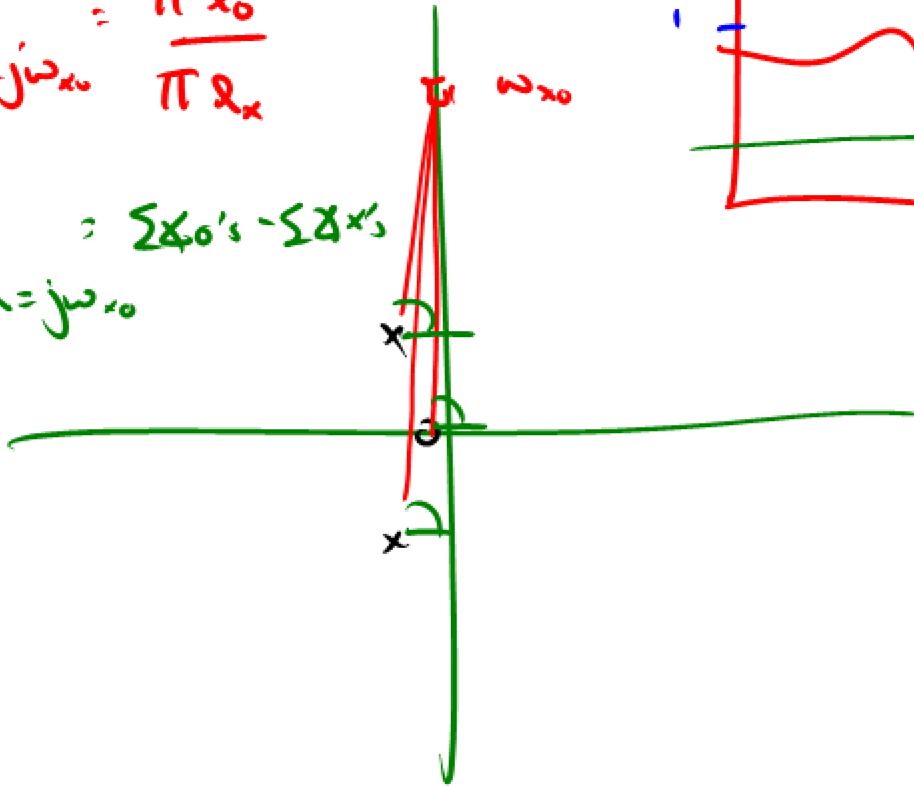
$|G(z)|\Big|_{z=e^{j\omega T}}$

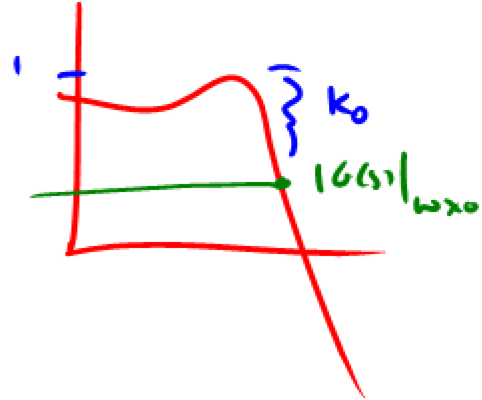




$$|G(s)|_{s=j\omega_{x0}} = \frac{\Pi\, \ell_0}{\Pi\, \ell_x}$$

$$\angle G(s)|_{s=j\omega_{x0}} = \Sigma \angle 0\text{'s} - \Sigma \angle x\text{'s}$$

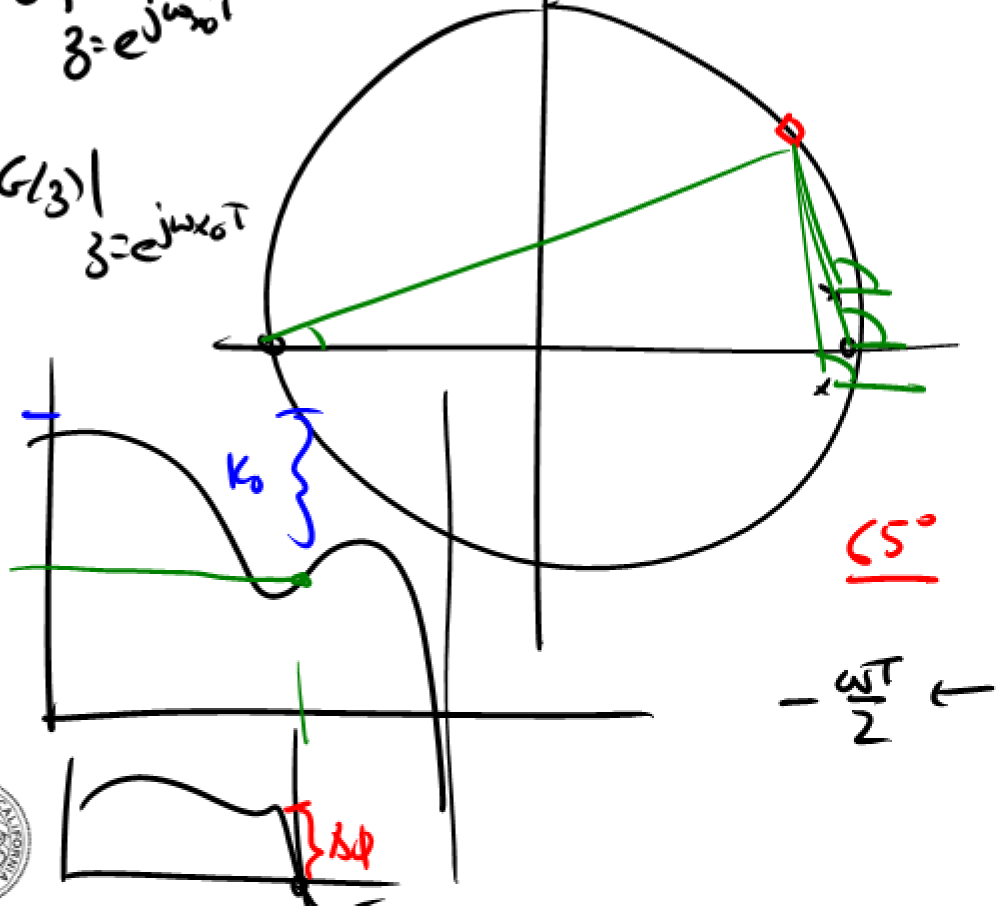


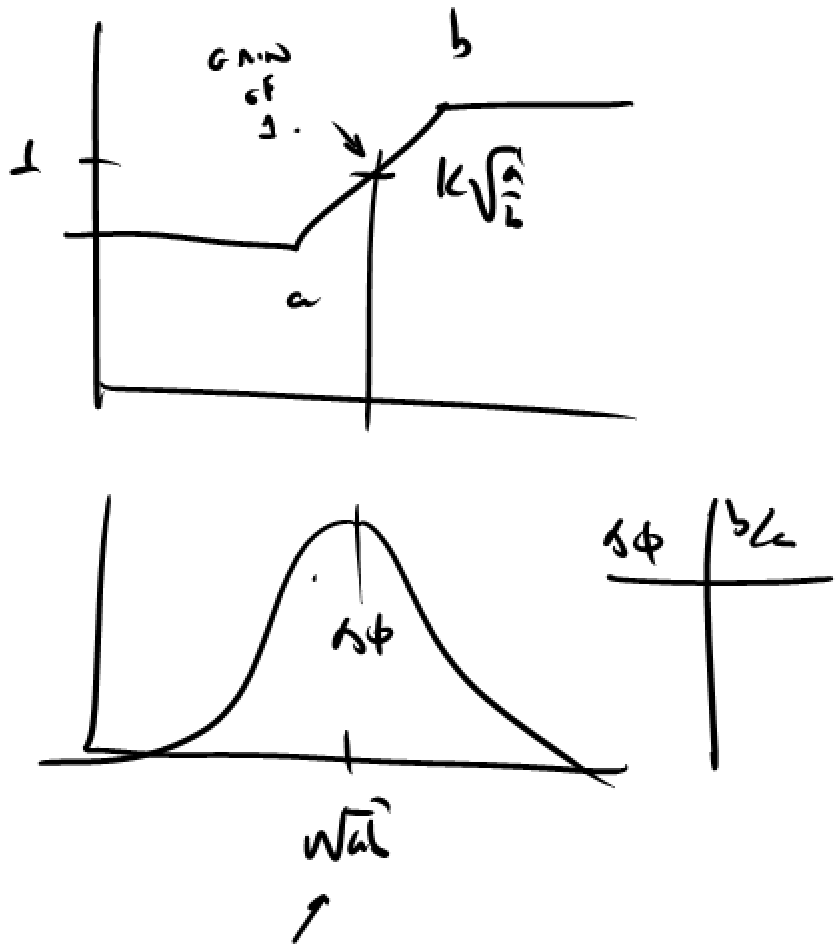


$\left| G(z) \right|_{z=e^{j\omega_0 T}}$

$\measuredangle G(z) \Big|_{z=e^{j\omega_{x0} T}}$

$e^{j\omega T} = \cos \omega T + j \sin \omega T$




GAIN of g.
b
1
a
$K\sqrt{\frac{a}{b}}$
$\Delta\phi$
$\Delta\phi$
b/a
$W\sqrt{ab}$

Lead

$$K\frac{(s+a)}{(s+b)}$$

90

ω_{xo}

TUSTIN w/ PRE-WARP (ω_{xo})

$T = 0.01 \leftarrow -\frac{\omega T}{2} \sim \underline{1.5^\circ}$

$T = 0.2 \searrow -\frac{\omega T}{2} \sim \underline{45^\circ}$

DBODE

LEAD in "S"

↓

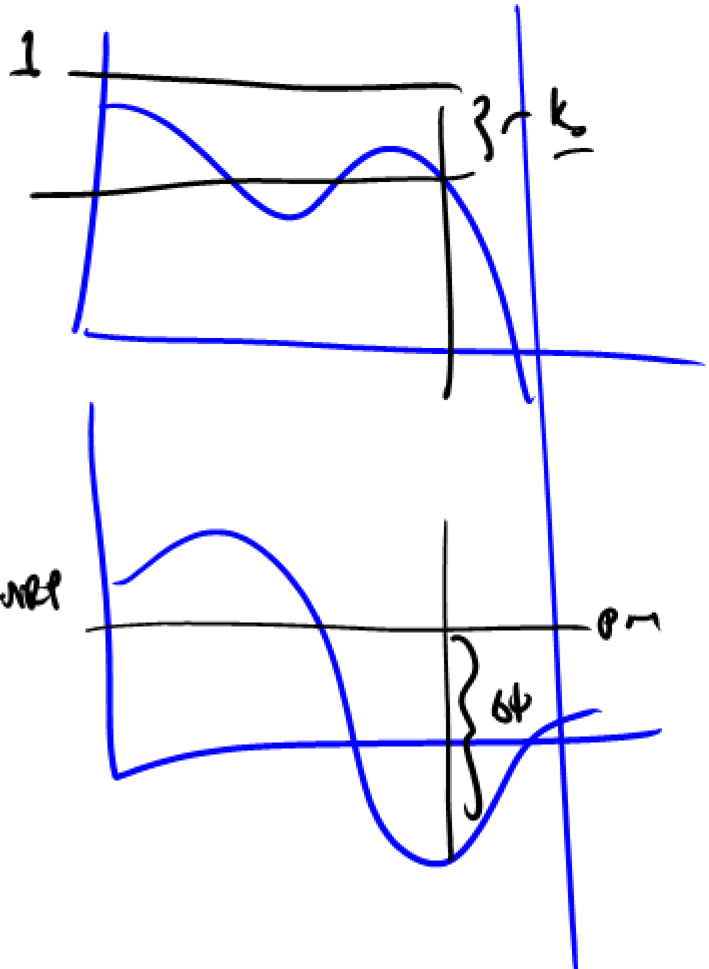
$\Delta\phi$ @ ω_{xo}

↓

TUSTIN w/ PREWARP

↓

$K_{LEAD}(z)$



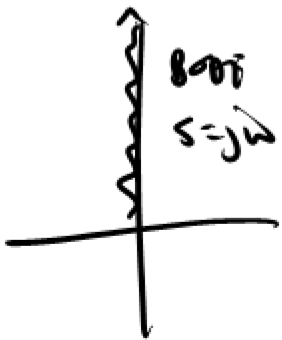
$$K(s) \longleftrightarrow K(z) \quad @ \quad \omega \to K(s) = K(z)$$

if 1 prewarp:

$$K(s)\Big|_{s=j\omega_1} = K(z)\Big|_{z=e^{j\omega_1 T}}$$

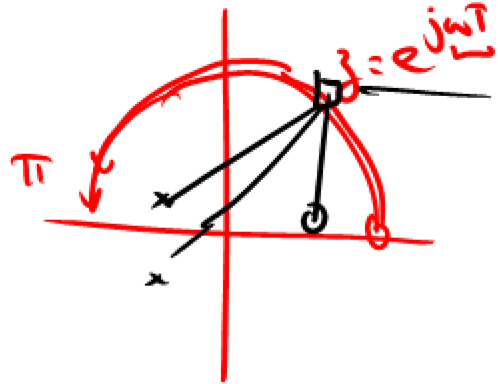
$$\Delta\phi \rightarrow -\frac{\omega T}{2}$$

$$z = e^{j\omega_{xo}T}$$



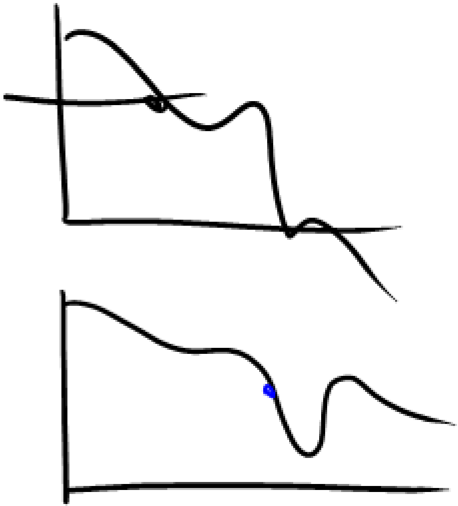
DBode

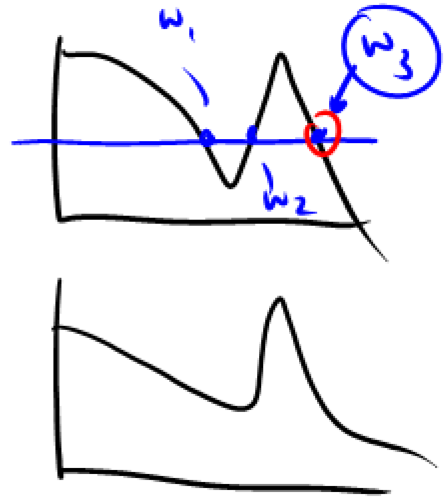
ω
$\downarrow$



PRACTICE MIDTERM

BODE PLOTS of $K_1G(s)$ $K_2G(s)$





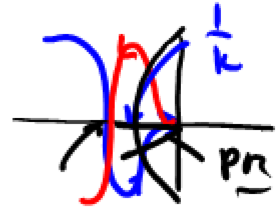
(a) $K_1(s)$

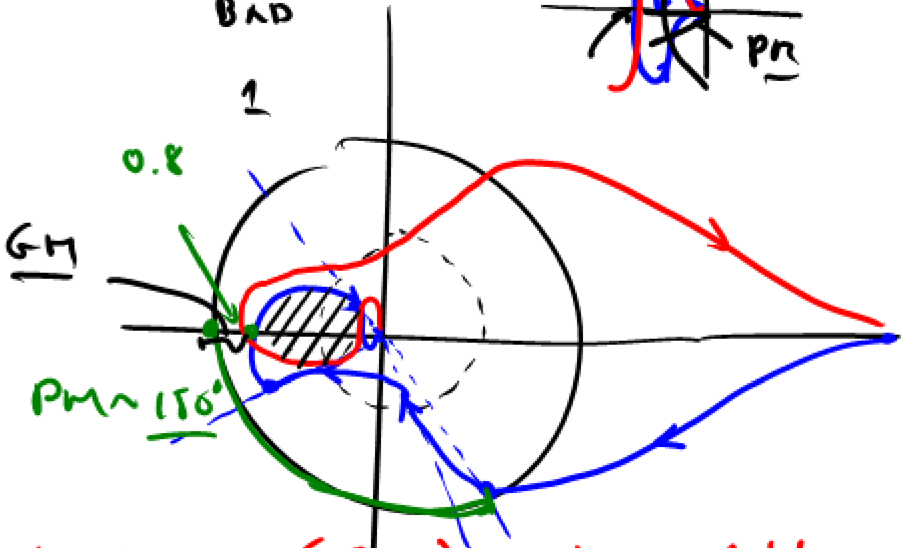
for $K_1 G(s)$

$\lvert\cdot\rvert$	$\measuredangle$
7	ϕ
1	-60
.3	-70
.8	-150
.1	-225
0.05	-150
0	-180

ok 1.2 — 20 ok

BAD





$\forall$ $K<0$ (0° rl) system unstable w/ 1 pole in RHP.

$\forall$ $K>0$ (180° RL) system stable for

$K<1.2$ — Two poles in RHP

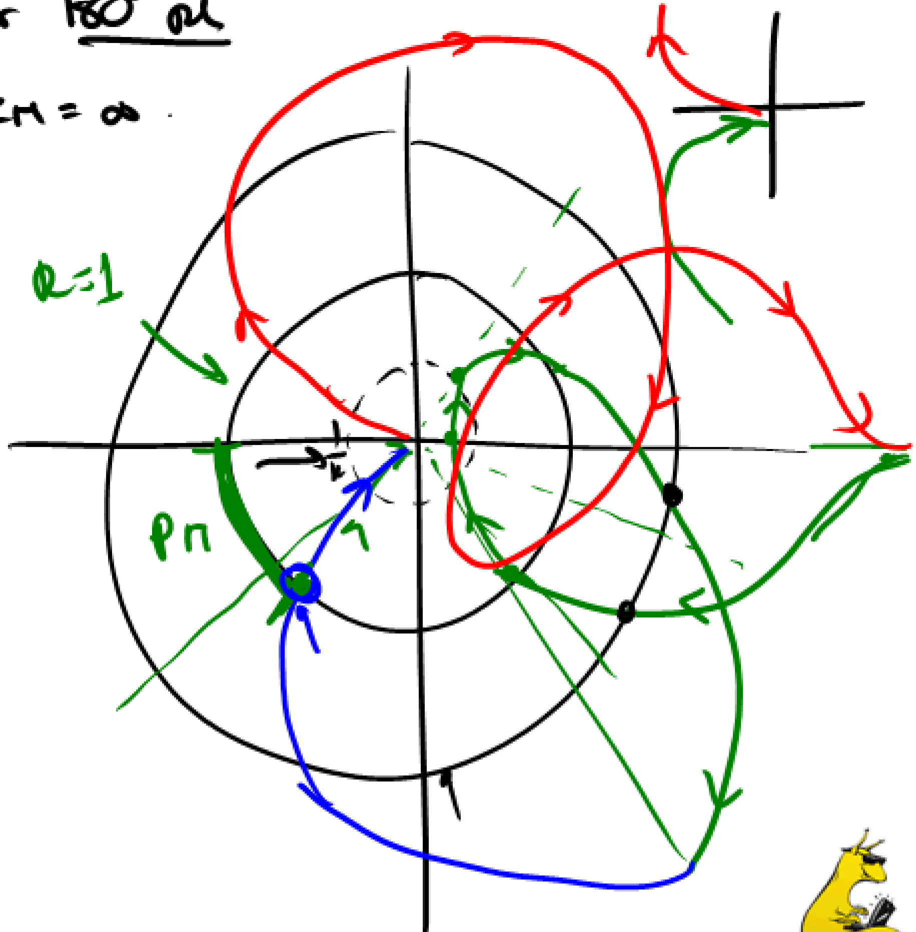
$K>20$ ok

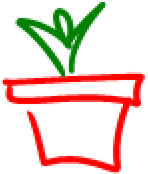
② $K_2(s)$

$\lvert L \rvert$	$\angle$
7	ϕ
5	-30
1	-50
.2	ϕ
0.5	50
1	20
5	-60
.5	-130
0	-180

for 180° ol

GM = ∞.



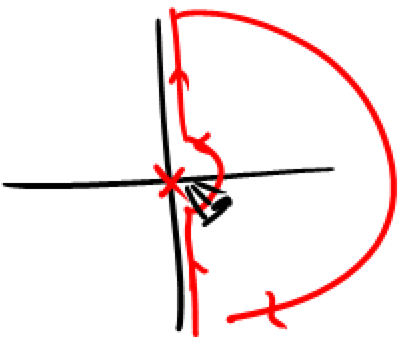


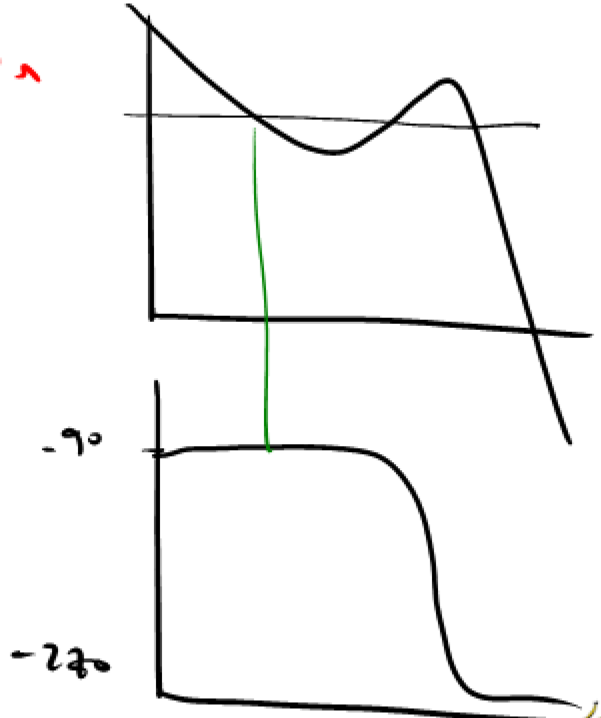
$$G(s) = \frac{1\,(0.3)}{s\,(s^2+0.6s+1)} \leftarrow \underline{\text{UE}}$$

$0.3s$

$1.1\ [0.3 \quad 0.8 \quad 1.1]$

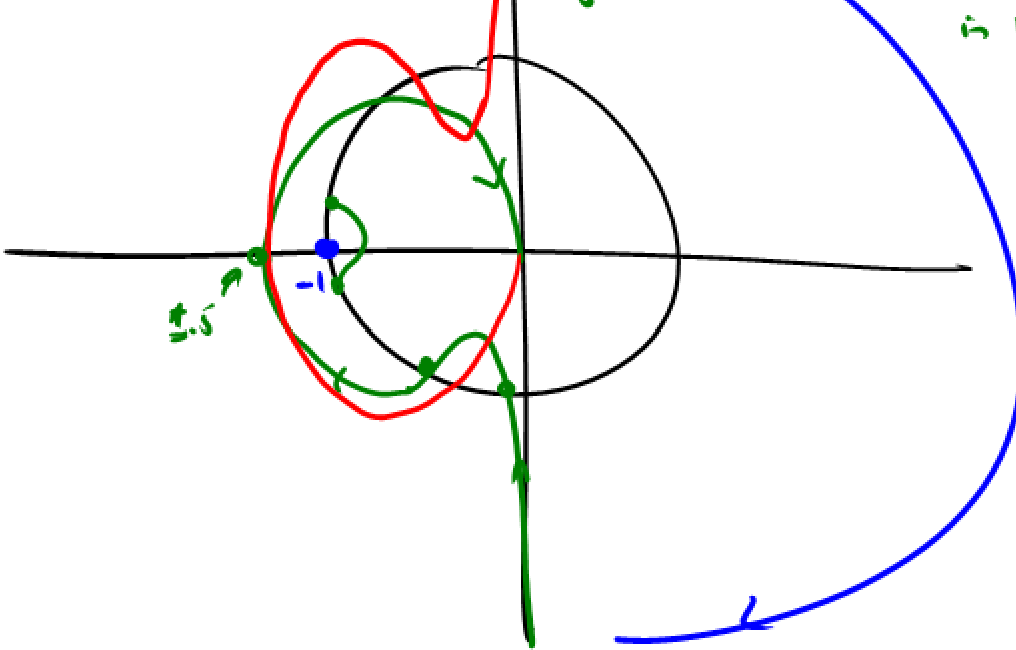
$\angle\ [-98^\circ, -115^\circ, -225^\circ]$

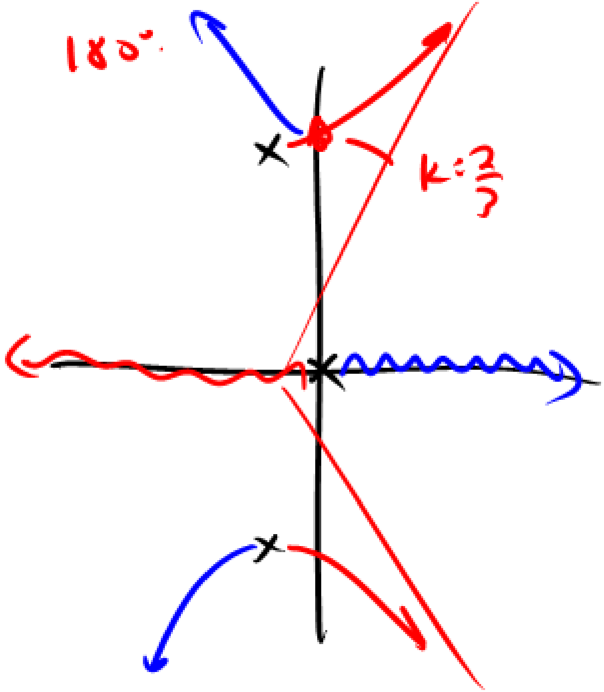




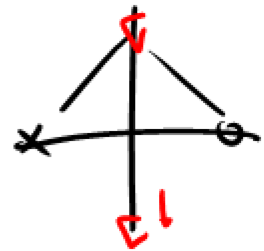
For $-K$ ($0°$ loc) unstable w/ 1 pole in RHP $\forall K$.

For K ($180°$ loc) stable for $K < \frac{2}{3}$, otherwise unstable w/ 2 poles in RHP.

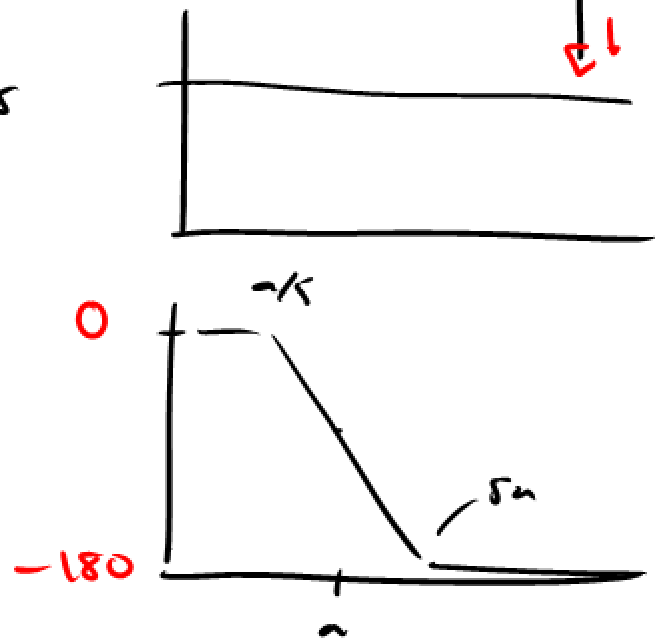


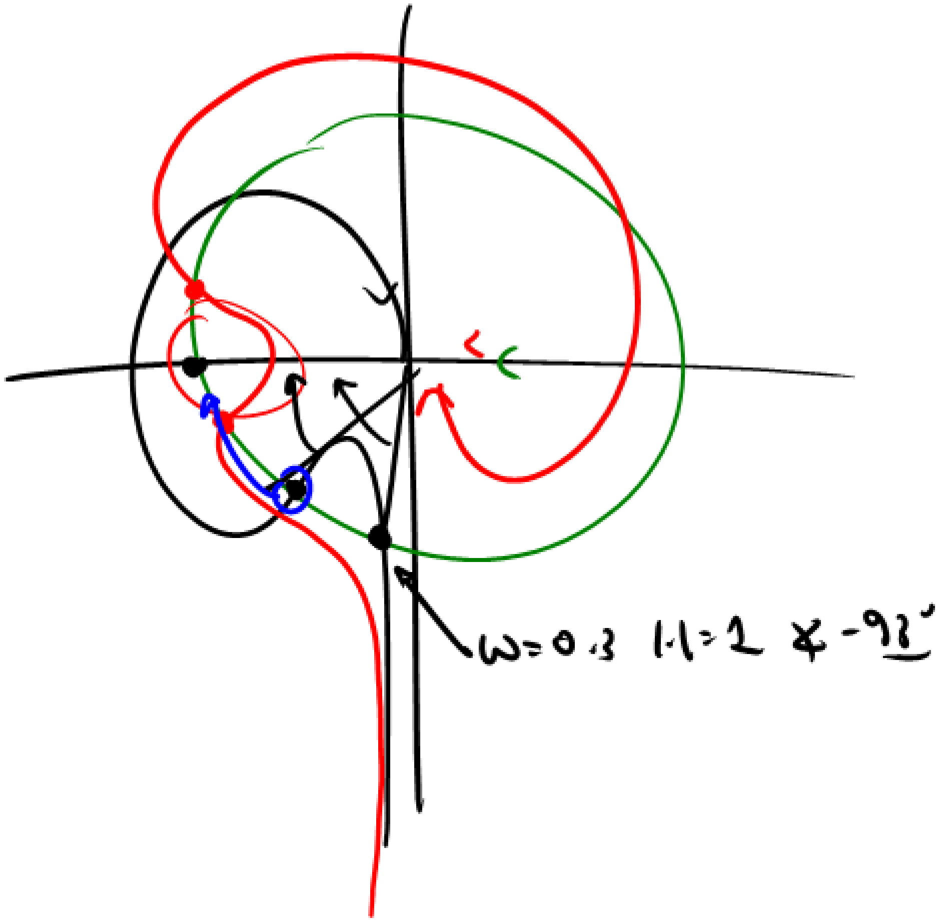

180°
k = 3/7

$$K(s) = \frac{-(s-a)}{(s+a)} \qquad (\text{PADÉ})$$



$$|\cdot| = 1 \ \forall \omega$$

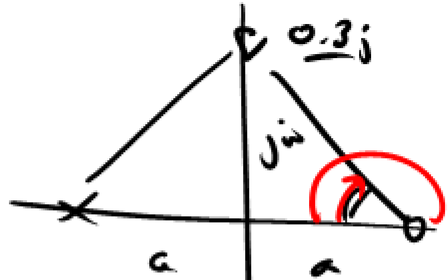




ω=0.3 |L|=1 ∡-98°

$@\ \omega = 0.3 \quad |\cdot| = 1 \quad \measuredangle\ -\underline{93}^\circ$

add $\underline{-87^\circ}$ of phase.

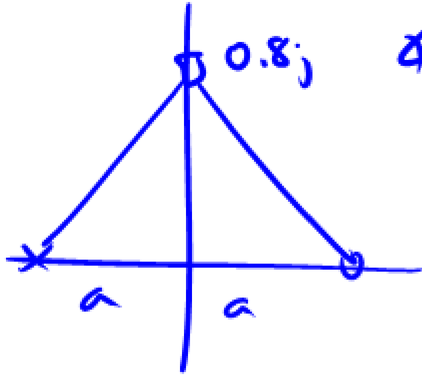


$(-)$

$$\measuredangle K(0.3j) = -\cancel{180} + \underbrace{\cancel{180} - \tan^{-1}\left(\frac{0.3}{a}\right)}_{0} \underbrace{- \tan^{-1}\left(\frac{0.3}{a}\right)}_{x} = -87^\circ$$

$$-2\tan^{-1}\left(\frac{0.3}{a}\right) = -87^\circ \rightarrow a = \underline{0.3161}$$

@ $\omega = 0.8 \quad |\cdot| = 1 \quad \measuredangle\ -\underline{115^\circ}$.



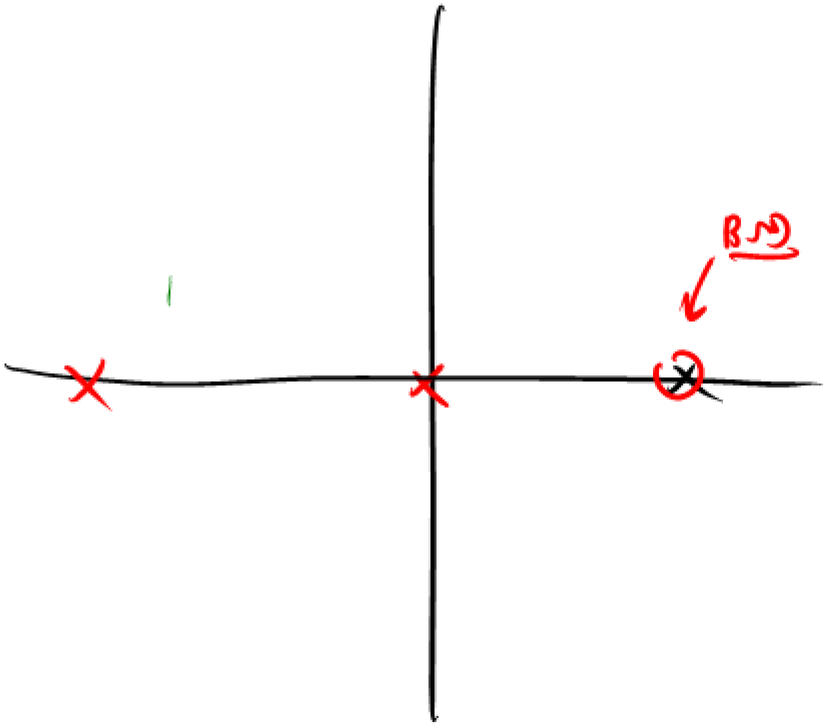
$$\measuredangle K(s)\Big|_{s=0.8j} = -\underline{65^\circ}$$

$$\measuredangle(0.8j) = -180 + 180 - \tan^{-1}\left(\frac{0.8}{a}\right) - \tan^{-1}\left(\frac{0.8}{a}\right)$$

$$-2\tan^{-1}\left(\frac{0.8}{a}\right) = -65$$

$$\therefore\ \underline{a = 1.256}$$

$$K(s) = \frac{-(s-a)}{(s+a)} \qquad .311 < a < 1.2557.$$



Open Book / Open Notes

NO COMPUTERS — NO MATLAB

NO INTERNET

END OF CLASS