CMPE-242
Applied Feedback Control

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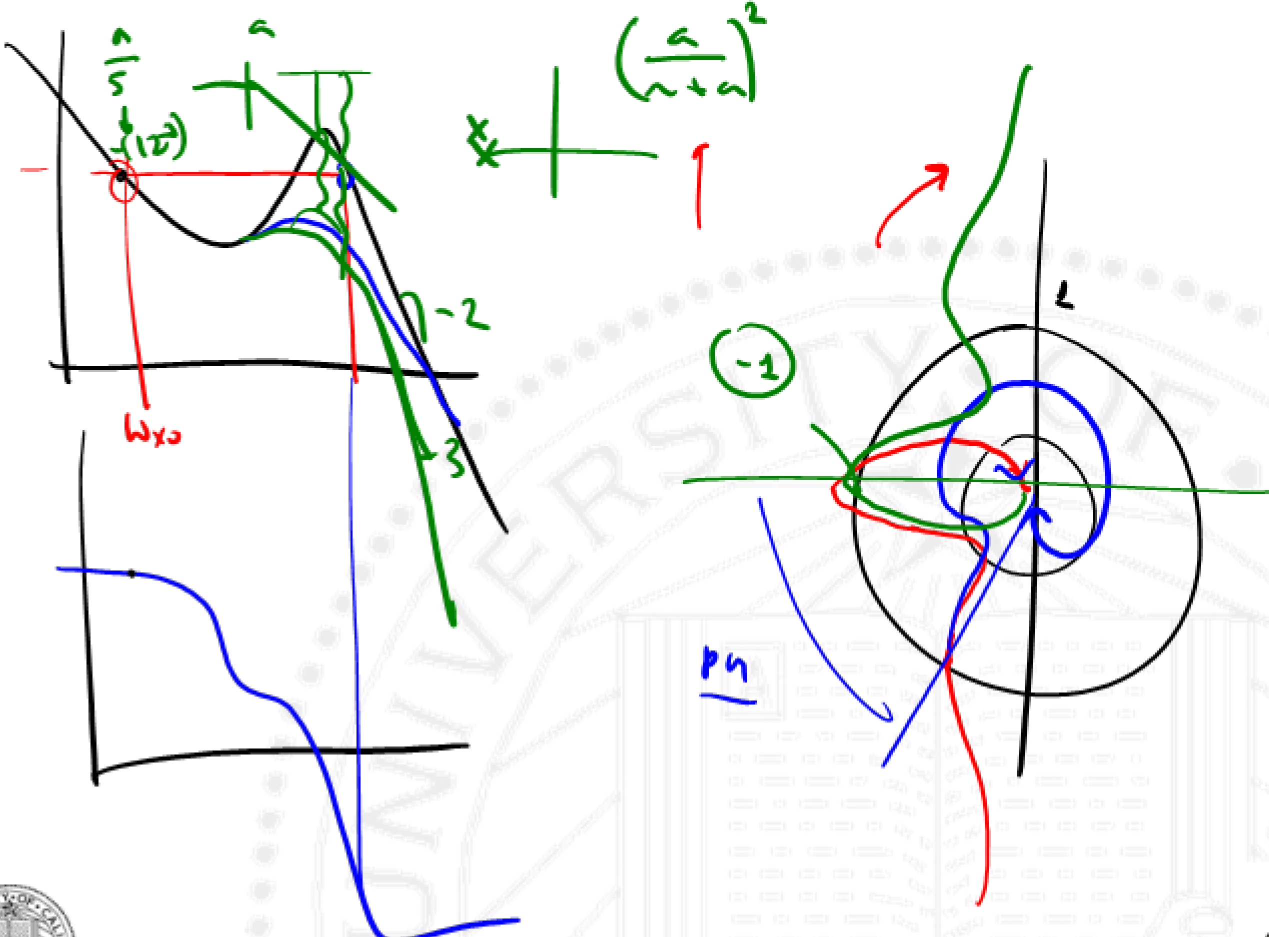
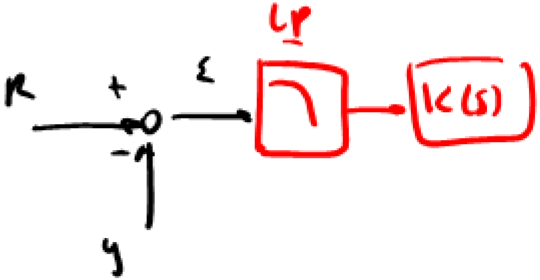
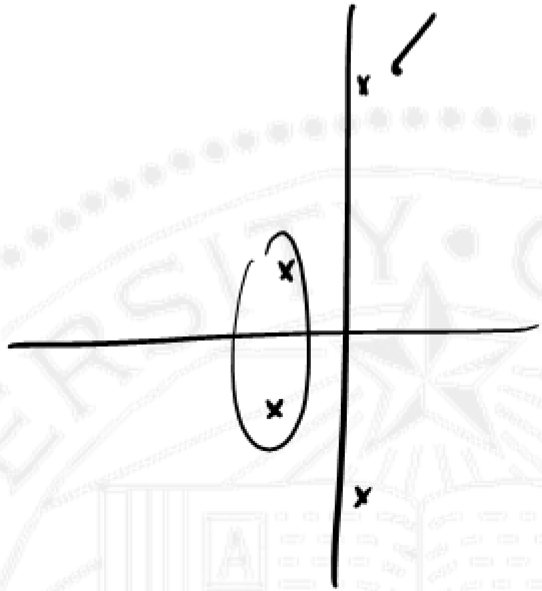
Winter 2016

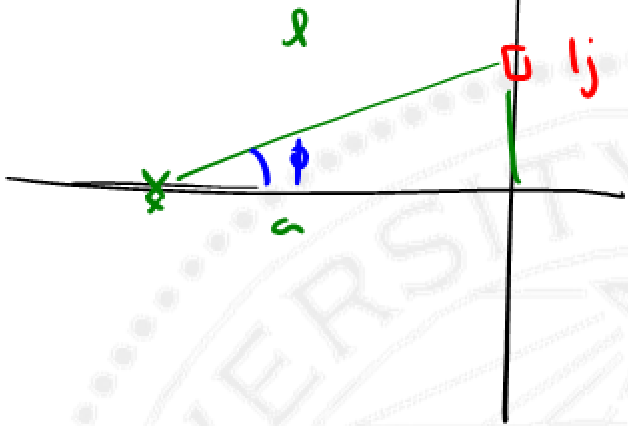

-2
3
ωxo
-1
pn


R
+
-
y
K(s)

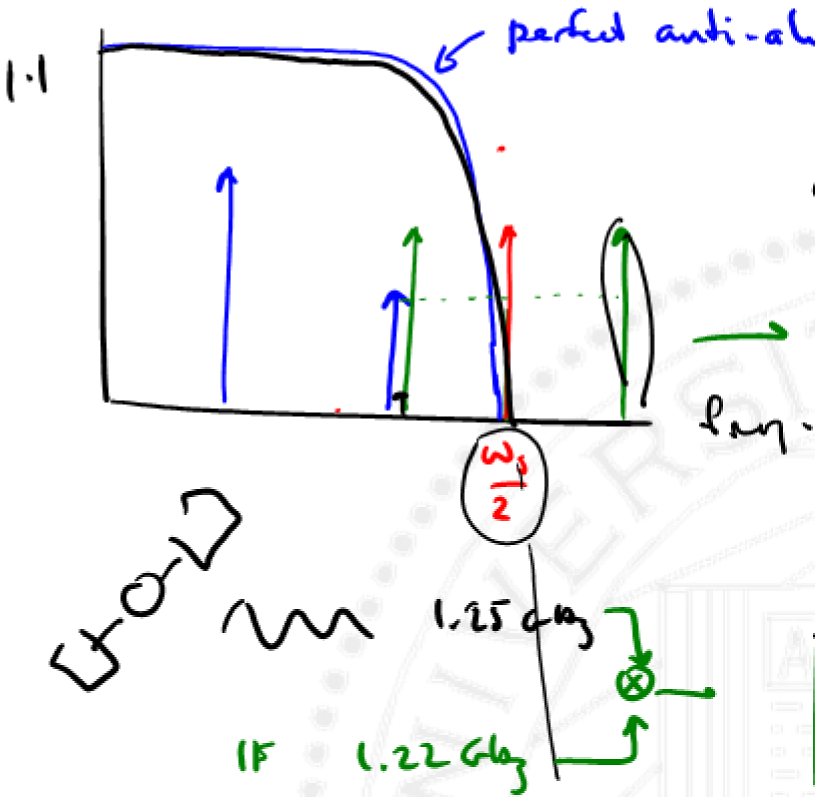


$$\ell = \sqrt{a^2+1}$$

$$\left(\frac{a}{\sqrt{a^2+1}}\right)$$



$$\Delta\phi = -\tan^{-1}\left(\frac{1}{a}\right)$$



$\cos(a)\cos(b) =$

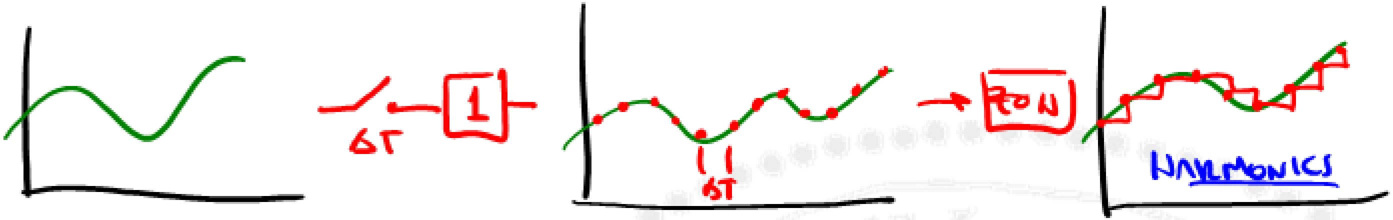
$\left[(\cos(a) + \cos(b)) +\right.$

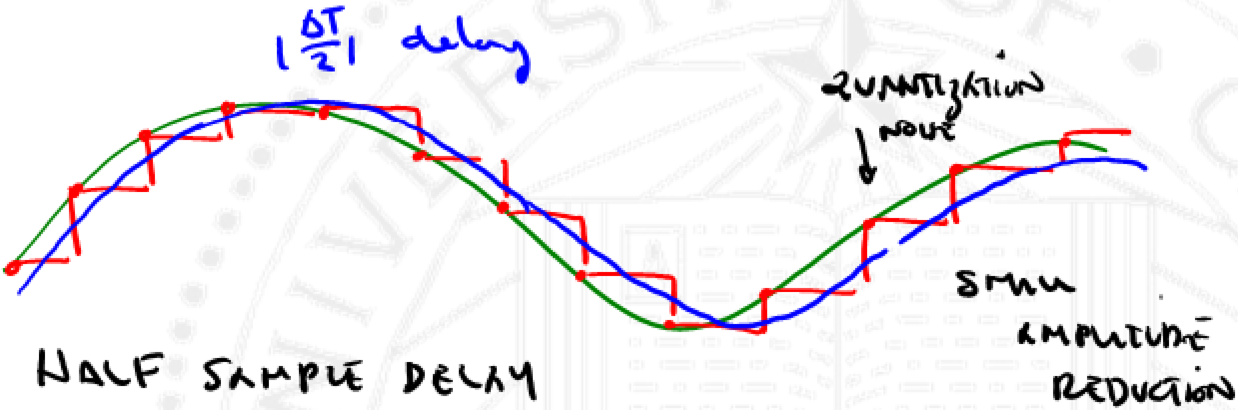
$\left.(\cos(a) - \cos(b))\right]$

Freq. folding

Heterodyning

Problems w/ Digital Control



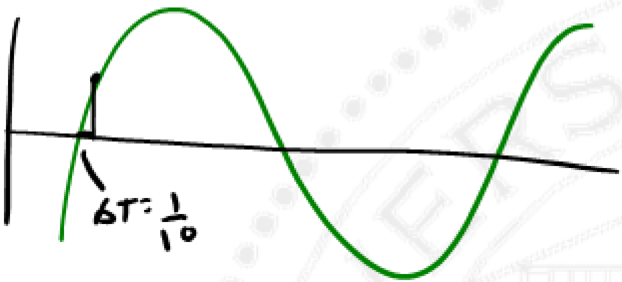


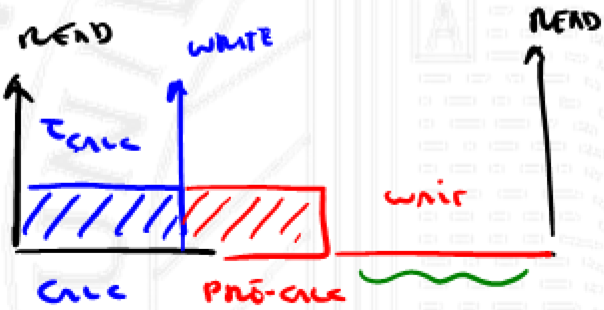
Sample @ 10x ω_{xo}

5x ω_{xo}

$\frac{1}{20}(360^\circ) = \underline{18^\circ \text{ phase loss}}$

$\frac{1}{10}(360^\circ) = \underline{36^\circ \text{ phase loss}}$



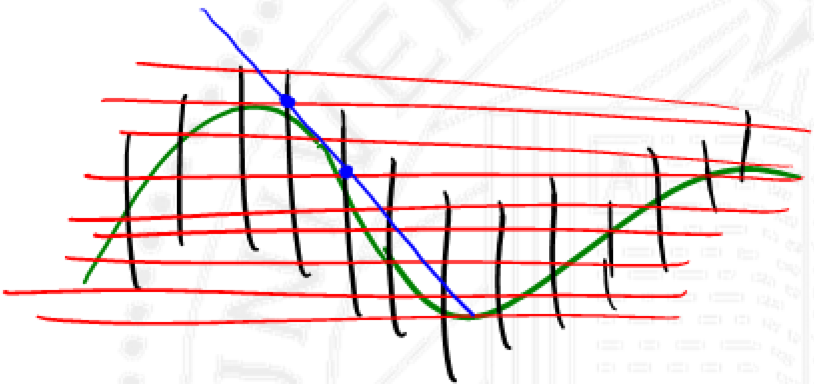


$$\dot{e} \approx \frac{e_k - e_{k-1}}{\Delta T}$$

first order difference

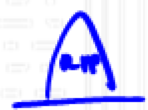
# of bits in my ADC ← quantization error

~ white noise $\frac{LSB}{2}$.



Round off errors

Amplify noise



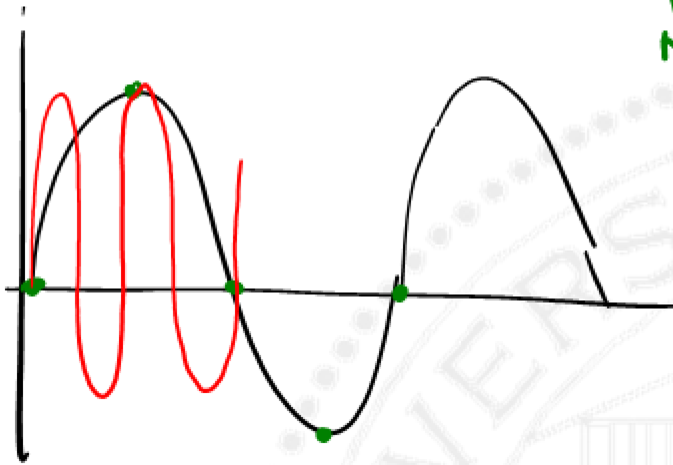
DIGITAL IS <u>NOT</u> FREE.

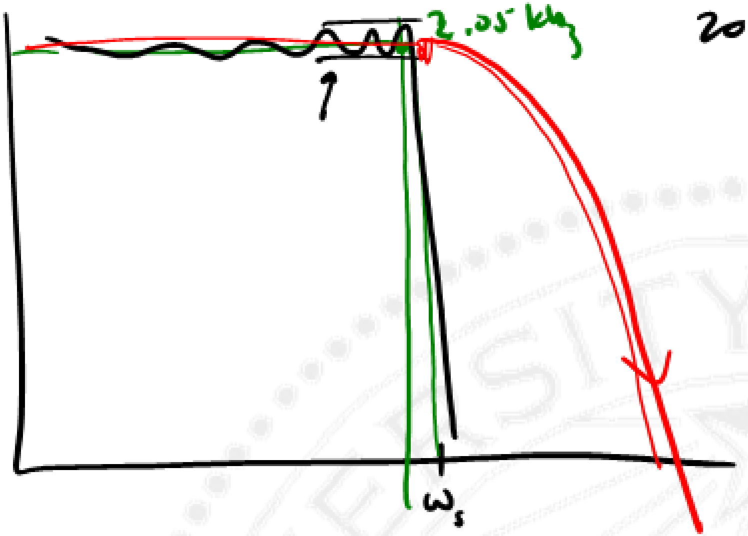
Alias

$$0 \le \omega \le \frac{\omega_s}{2}$$

Nyquist freq.

Shannon's Sampling Theorem





20 Hz – 20 kHz

44.1 kHz

$\frac{\omega_s}{2} = 22.05$

96 kHz

48 kHz

MP3's

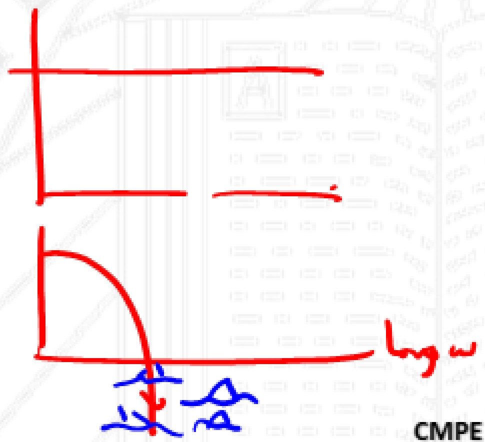
Time delay $\sim \frac{\Delta T}{2}$

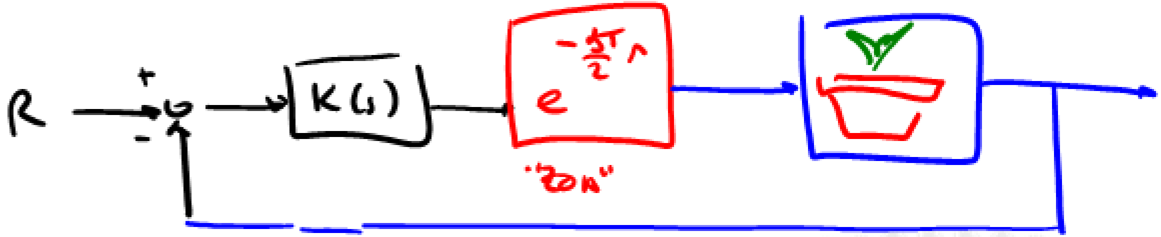
$$\mathcal{L}\{\text{time delay}\} = e^{-\frac{\Delta T}{2}s}$$

$$e^{-\frac{\Delta T}{2}s}\Big|_{s=j\omega} = 1\, e^{-\left(\frac{\Delta T}{2}\omega\right)j}$$

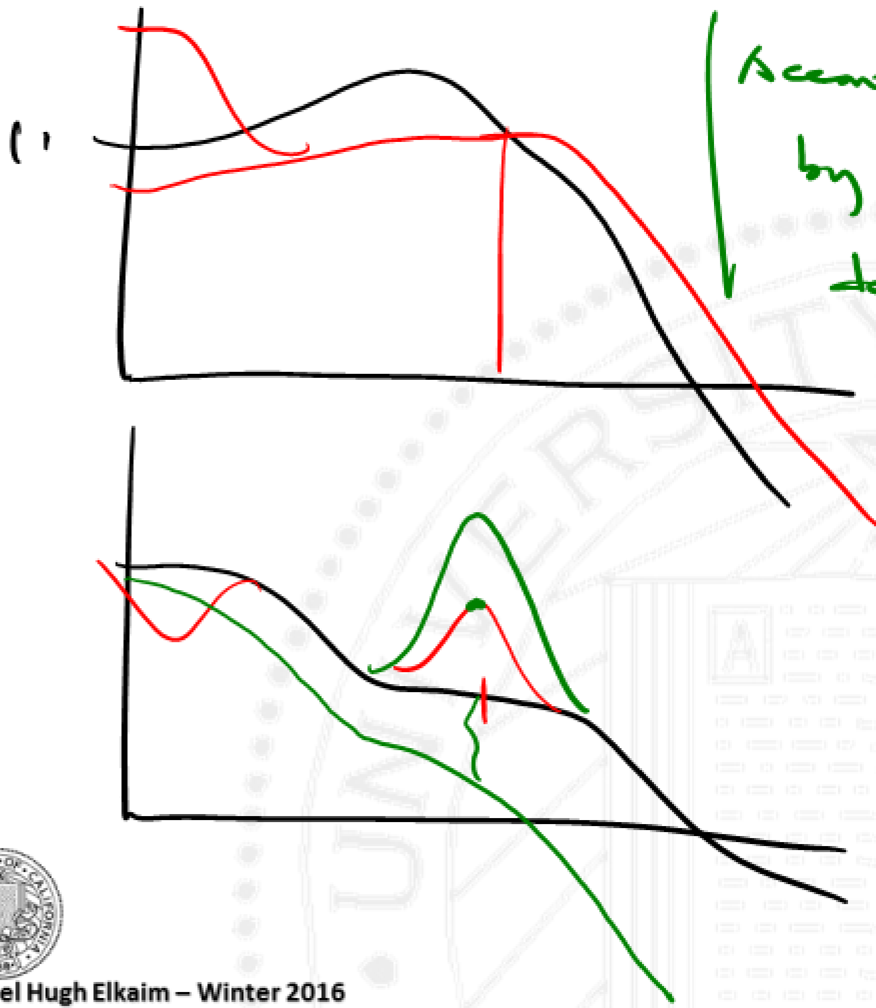
$|\cdot| = 1 \;\forall\, \omega$

$\measuredangle = -\frac{\Delta T}{2}\omega$





DESIGN IN BODE AS NORMAL, BUT ACCOUNT FOR $\frac{\Delta T}{2}\omega$ added phase delay.



Account for digital by adding phase loss to bode plot.

$$\Delta\phi_{hc} = -\frac{\Delta T}{2}\omega_{co}$$

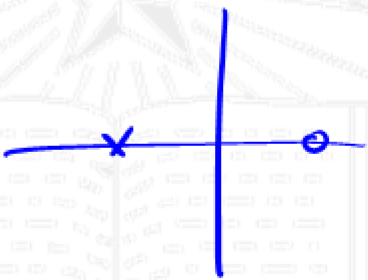
ROOT LOCUS

$$e^{-\frac{\Delta T}{2}s}$$

$$e^{-\frac{\Delta T}{2}s} = 1 - \frac{\Delta T}{2}s + \left(\frac{\Delta T}{2}\right)^2 \frac{s^2}{2!} - \left(\frac{\Delta T}{2}\right)^3 \frac{s^3}{3!} + \dots$$

PADÉ APPROXIMATION

$$e^{-\frac{\Delta T}{2}s} \simeq - \frac{(s-a)}{(s+a)}$$



$$-\frac{(s-a)}{(s+a)} = \frac{1 - s/a}{1 + s/a}$$

$$\boxed{1 - \frac{2s}{a} + \frac{2}{a^2}s^2 - }$$

$$1 + s/a \,\overline{\big)\, 1 - s/a}$$

$$\frac{1 + s/a}{}$$

$$\frac{2s}{a}$$

$$-\frac{2s}{a} - \frac{2s^2}{a^2}$$

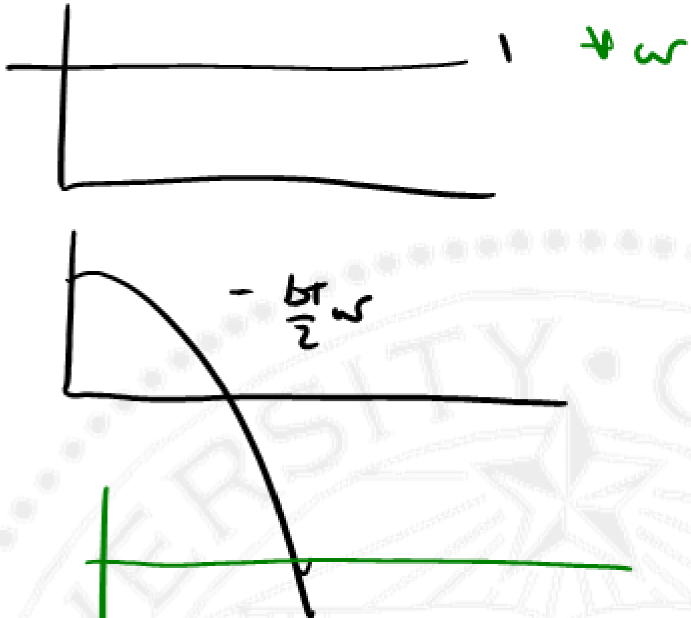
$$a = \frac{4}{\Delta T}$$

$$e^{-\frac{\delta T}{2}s} = \boxed{1} - \frac{\delta T}{2}s + \frac{\delta T^2}{8} - \frac{\delta T^3}{48} + \cdots$$

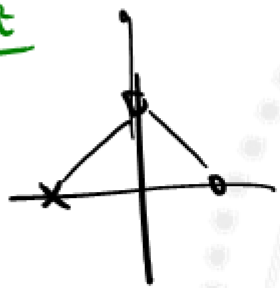
$$\frac{-\left(s - \frac{4}{\delta T}\right)}{s + \frac{4}{\delta T}} = \boxed{1} - \frac{\delta T}{2}s + \cancel{\frac{\delta T^2}{16}}\, s^2 - \cdots$$

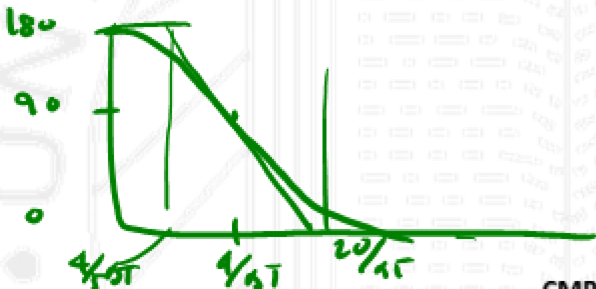
Root locus $\longrightarrow$ $\underline{180^\circ}$ to $\underline{0^\circ}$

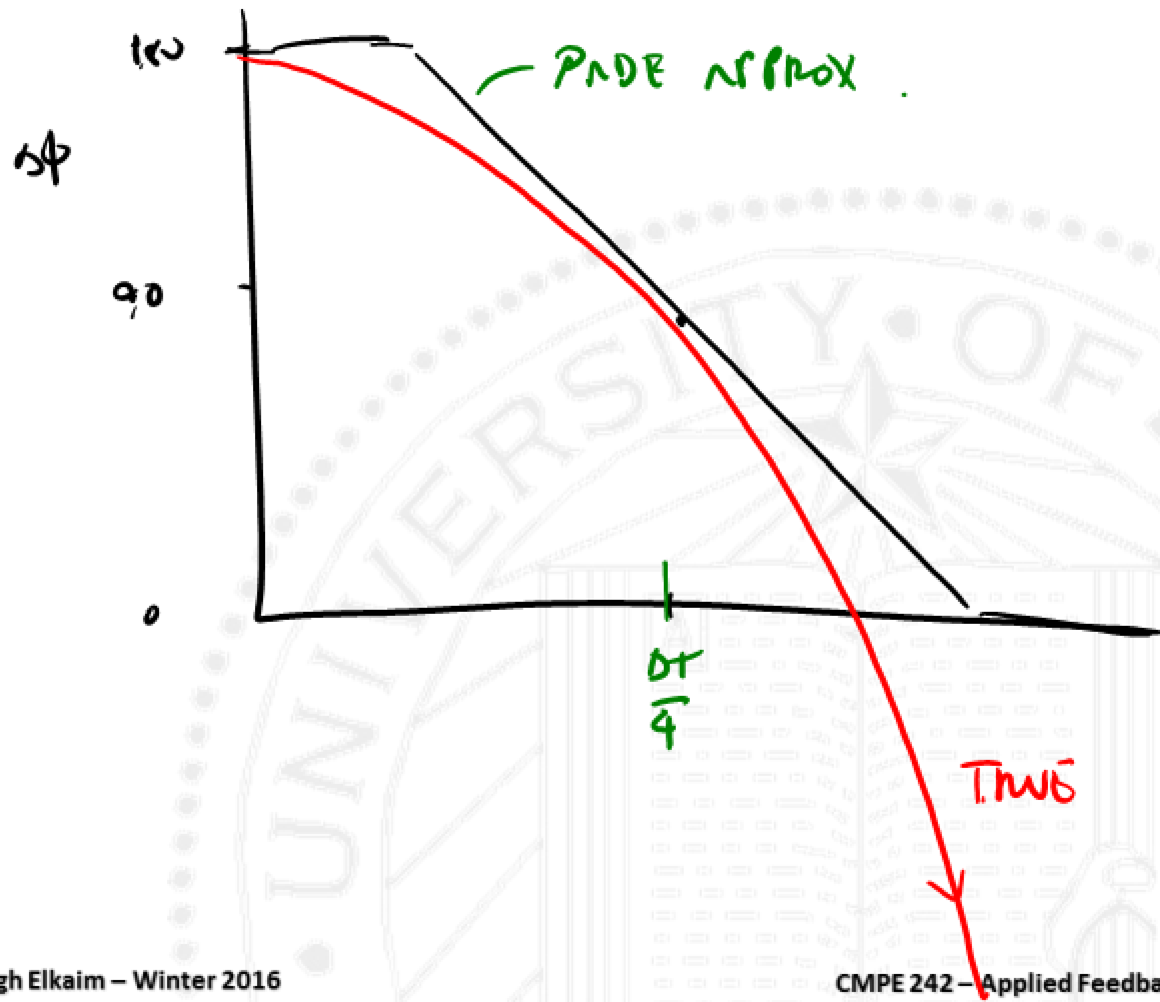
$e^{-\frac{\Delta T}{2}s}$:



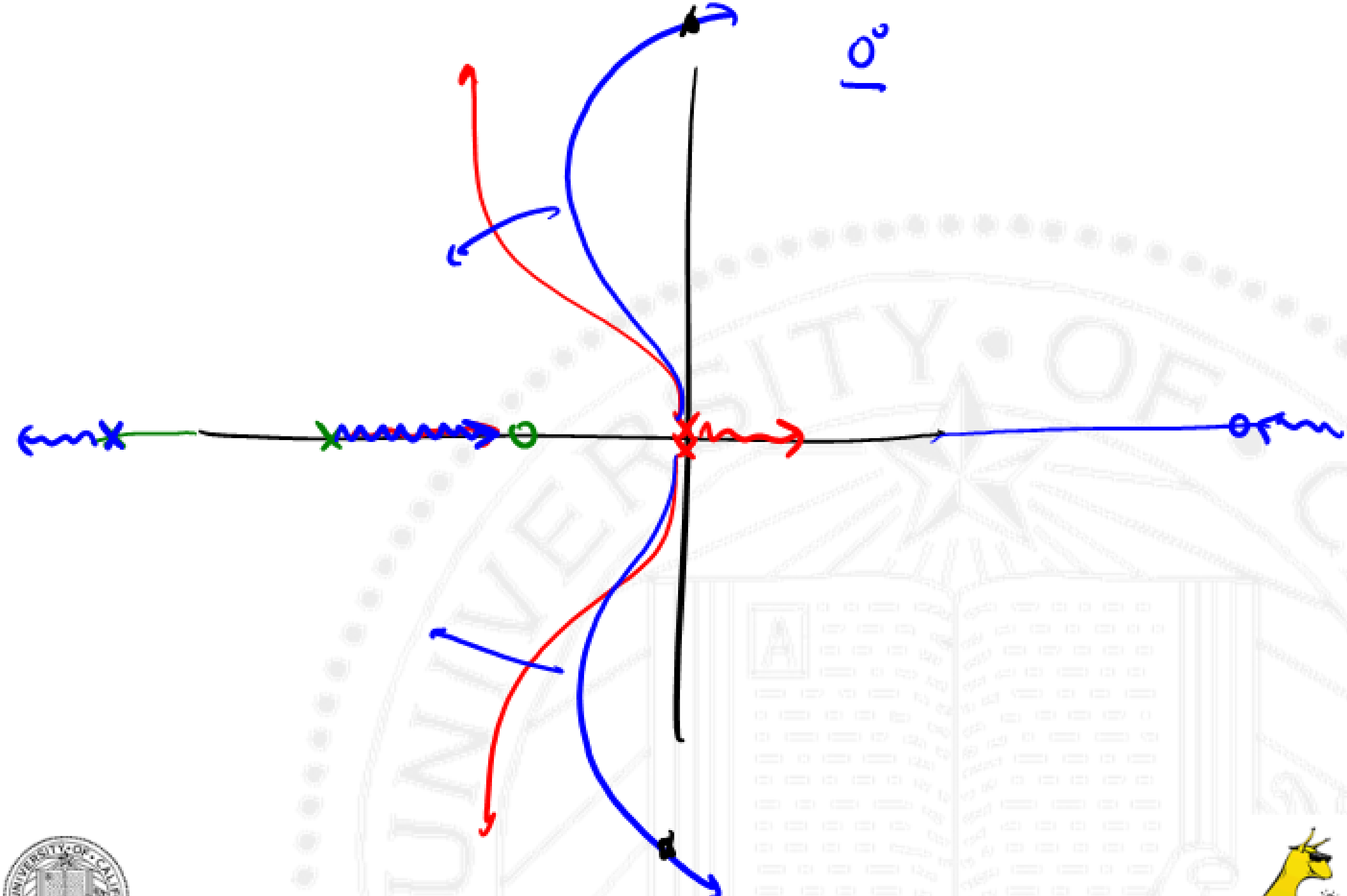
Padé







180
90
0
PADE APPROX
TRUE


10°

PID loop

$$u = K_p \varepsilon + K_d \dot{\varepsilon} + K_I \int_0^t \varepsilon \, dt$$

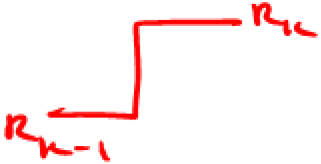
$$U = K_p E + s K_d E + \frac{K_I}{s} E$$

$$\frac{U}{E} = \frac{\left[s^2 K_D + s K_P + K_I \right]}{s} = K(s)$$

$K(s) \rightarrow$ ODE $\Longrightarrow K(z) \rightarrow$ OΔE

$$U_P = K_P \, \varepsilon_k \leftarrow \text{proportional part}$$

$$\dot{\varepsilon} \simeq \frac{\varepsilon_k - \varepsilon_{k-1}}{\Delta T} \longrightarrow U_D = \frac{K_D}{\Delta T}\left[\varepsilon_k - \varepsilon_{k-1}\right] \leftarrow \text{derivative part}$$



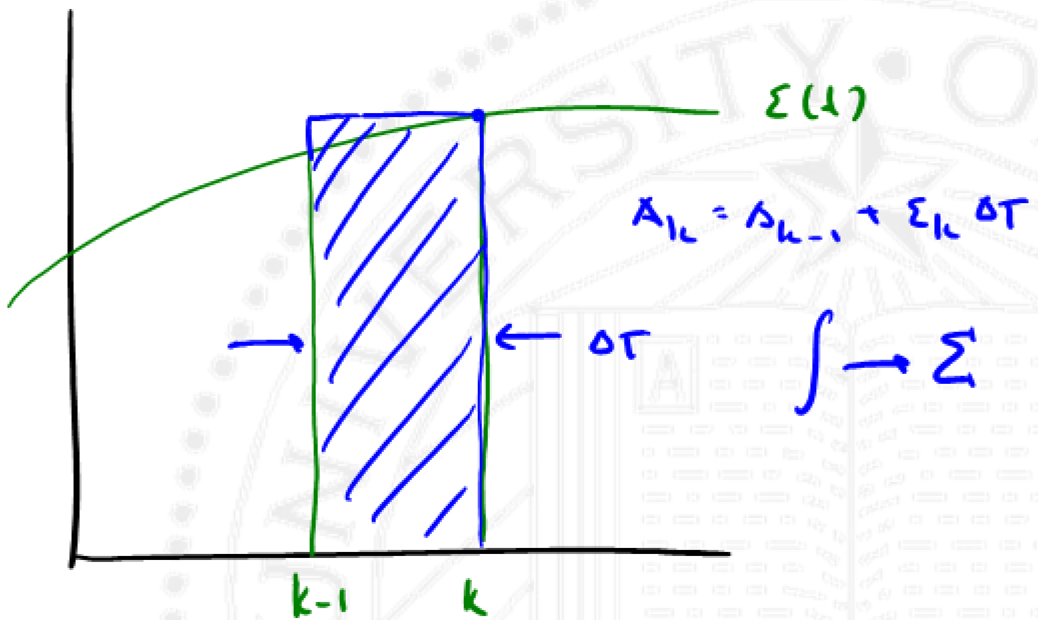
$$\varepsilon_k = R_k - Y_k$$

$$U_D = \frac{K_D}{\Delta T}\left[Y_{k-1} - Y_k\right] \leftarrow \text{derivative part}$$

OUTPUT NOT ERROR

$$U_I = K_I\left(A_{k-1} + \varepsilon_k \Delta T\right)$$ ← Integral part

Read in subroutine



$A_{k-1} = \phi$

$u_{k-1} = \phi$

READ Y_k

READ R_k

$\varepsilon_k = R_k - Y_k$

$U_P = K_P \varepsilon_k$

$U_D = \frac{K_D}{\delta T}(Y_{k-1} - Y_k)$

$\boxed{A_k = A_{k-1} + \varepsilon_k \delta T}$

$U_I = K_I \cdot A_k$

$u = U_D + U_P + U_I$; ← check if saturated.

write u

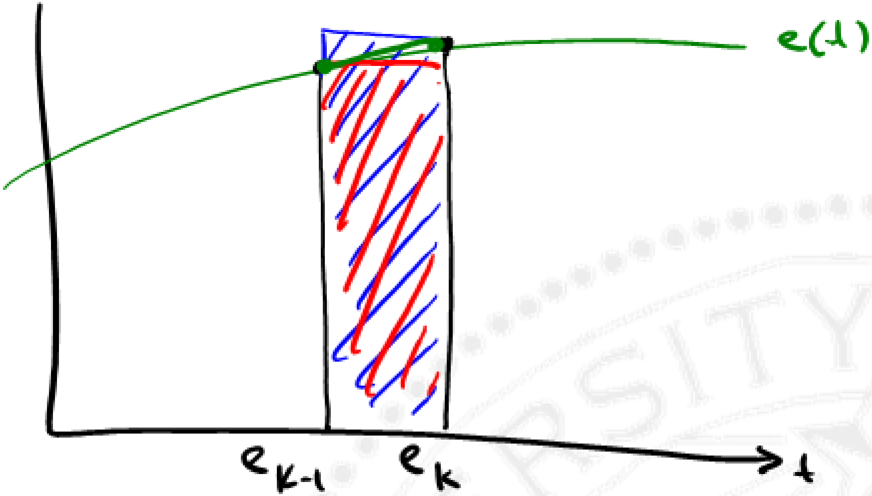
$u_{k-1} = u_k$

$A_{k-1} = A_k$.

anti windup

$\boxed{A_k = A_k - \varepsilon_k \delta T}$

$|u| \geq |u_{max}|$.



$$A_k = A_{k-1} + \Delta T\, e_k$$ (Backward Integration)

$$A_k = A_{k-1} + \Delta T\, e_{k-1}$$ (Forward Integration)

$$A_k = A_{k-1} + \Delta T \left(\frac{e_k + e_{k-1}}{2} \right)$$ (Trapezoidal/Tustin)

Lag

$$\frac{u}{E} = K\frac{s+a}{s+b} \quad \therefore \quad \dot{u} + bu = K(\dot{e} + ae)$$

$$\frac{u_k - u_{k-1}}{\Delta T} + b u_k = K\left[\frac{E_k - E_{k-1}}{\Delta T} + aE_k\right]$$

$$u_k = \frac{\left[u_{k-1} + K(1+\Delta T a)E_k - KE_{k-1}\right]}{(1 - \Delta T b)}$$

$\dot{u} \to su$ (Laplace)

$u_{k-1} = z^{-1} u_k$ (Z-transform) $\quad z^{-1} \triangleq$ unit delay.

$q = z^{-1}$

$$se = \dot{e} \Rightarrow \frac{e_k - e_{k-1}}{\Delta T} = \frac{e_k - z^{-1} e_k}{\Delta T} = \left(\frac{z-1}{z \Delta T}\right) E_k$$

$$s = \left(\frac{z-1}{z \Delta T}\right)$$ ← mapping

"Backwards Integration"

$$\frac{U}{\varepsilon} = \frac{K(s+a)}{(s+b)}\Bigg|_{s=\left(\frac{z-1}{zT}\right)} = \frac{K\left(\frac{z-1}{zT} + a\right)}{\left(\frac{z-1}{zT} + b\right)} = \frac{K(z-1 + Tza)}{z-1 + Tzb}$$

$$\frac{U}{\varepsilon}(z) = \frac{K\left((1+aT)z - 1\right)}{(1-bT)z - 1}$$

$$\frac{u_k}{\varepsilon_k} = \frac{K\left[(1+aT) - z^{-1}\right]}{(1-bT) - z^{-1}} \rightarrow (1-bT)u_k - u_{k-1} = k\left((1+aT)\varepsilon_k - k\varepsilon_{k-1}\right)$$

$$u_k = \frac{1}{(1-bT)}\left[u_{k-1} + k(1+aT)\varepsilon_k - k\varepsilon_{k-1}\right]$$