CMPE-242
Applied Feedback Control

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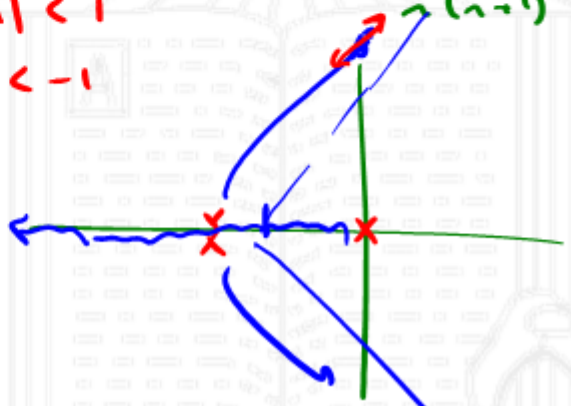
Saturating Actuator

$K_\varepsilon \rightarrow |\varepsilon K_\varepsilon| < 1$ $K_\varepsilon = k$.

$K_\varepsilon = \frac{1}{\varepsilon}$

$$u_{act} = \begin{cases} +1 & u_{cmd} > 1 \\ u_{cmd} & |u_{cmd}| < 1 \\ -1 & u_{cmd} < -1 \end{cases}$$

$G(s) = \frac{1}{(s+1)^2}$



Limit cycle

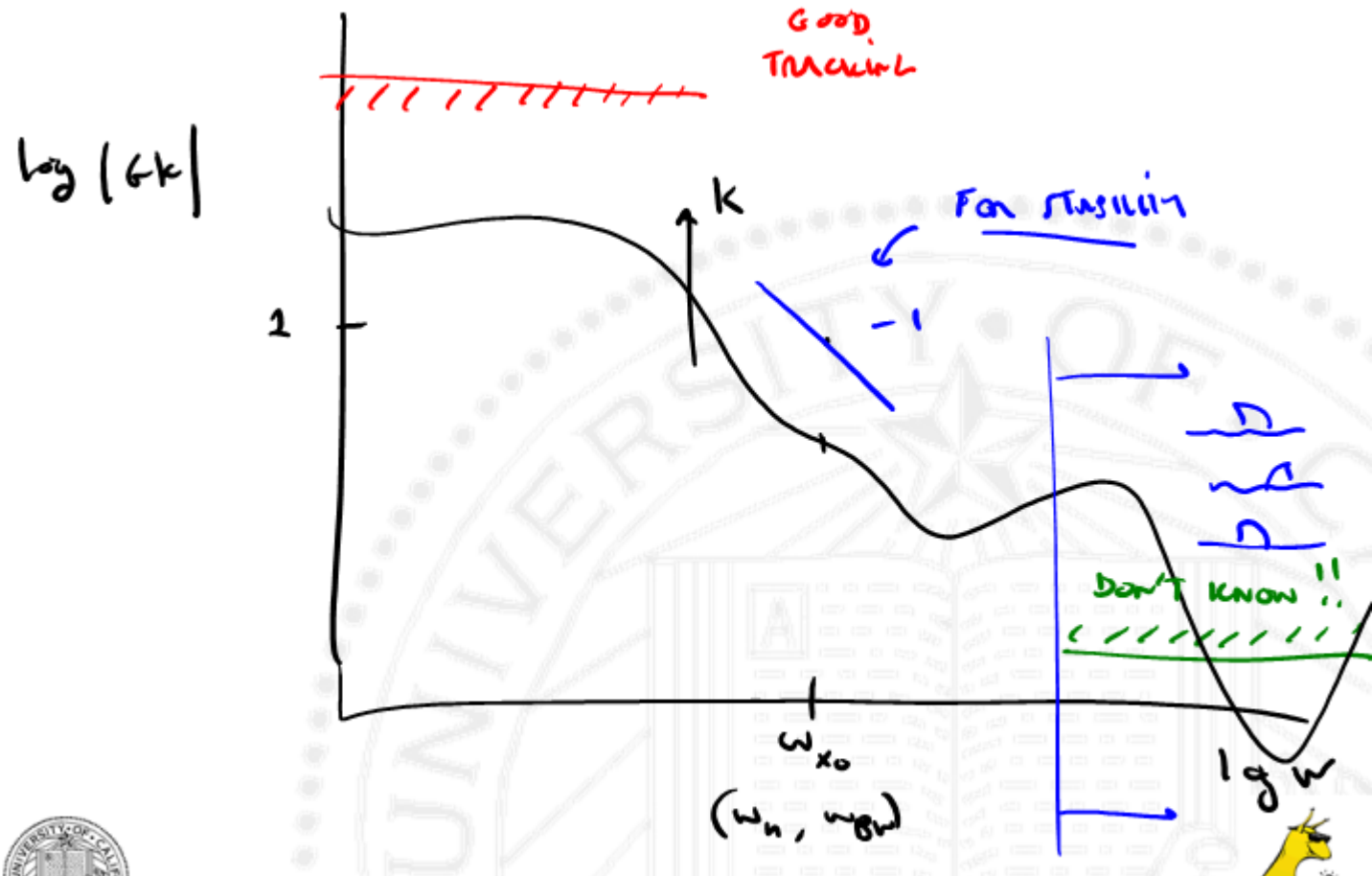
Bode - Tracking Spec / Robustness

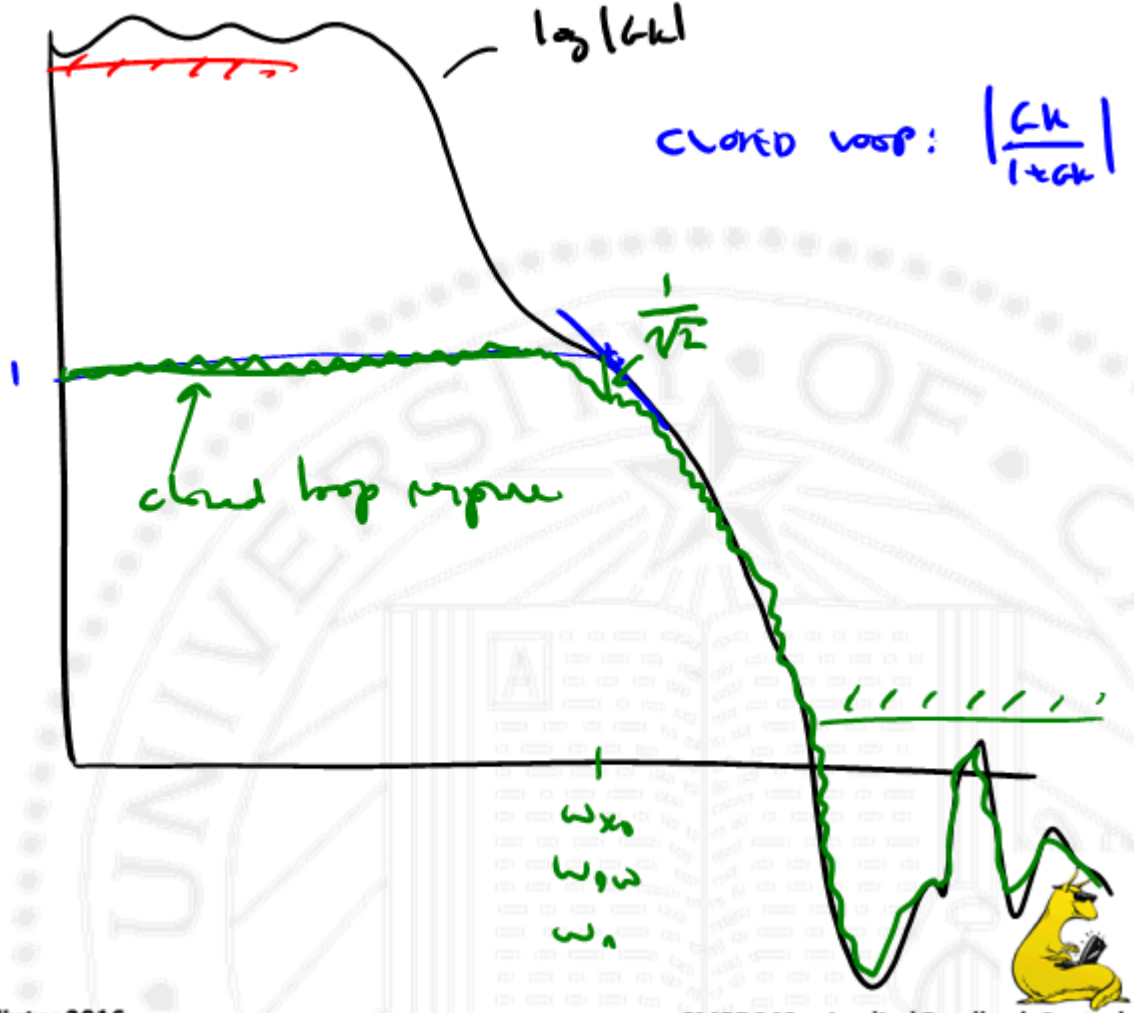
- Model From Design

$\log\left|\frac{B}{A}\right|$ vs $\log\omega$

ϕ vs $\log\omega$

open loop bode


log |Gk|
GOOD TRACKING
k
1
FOR STABILITY
-1
DON'T KNOW !!
ω_{x_0}
(ω_n, ω_{BW})
log ω


log|GH|
CLOSED LOOP: $\left|\frac{GH}{1+GH}\right|$
$\frac{1}{\sqrt{2}}$
1
closed loop response
ω_{x_0}
ω_{gw}
ω_n

Cookbook Approach

K for ω_{x0}

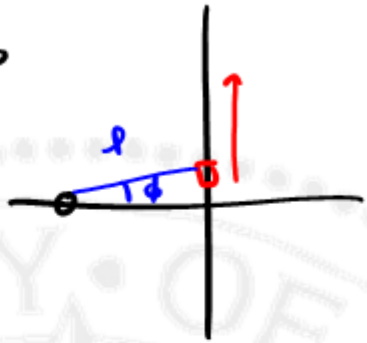
LEAD for PM

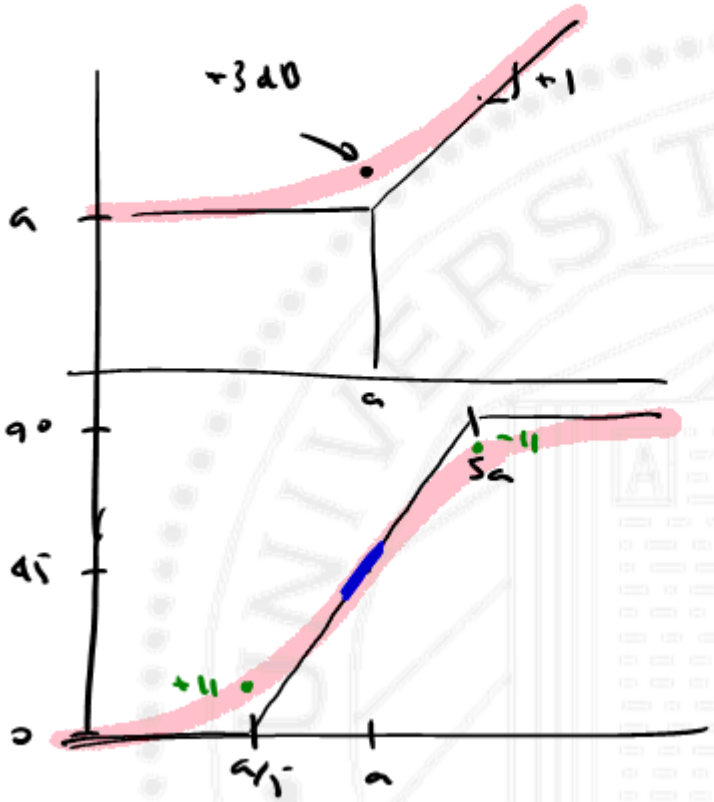
LAG for Low freq. gain

NOTCH/LAG/LOW PASS for high freq. attenuation.

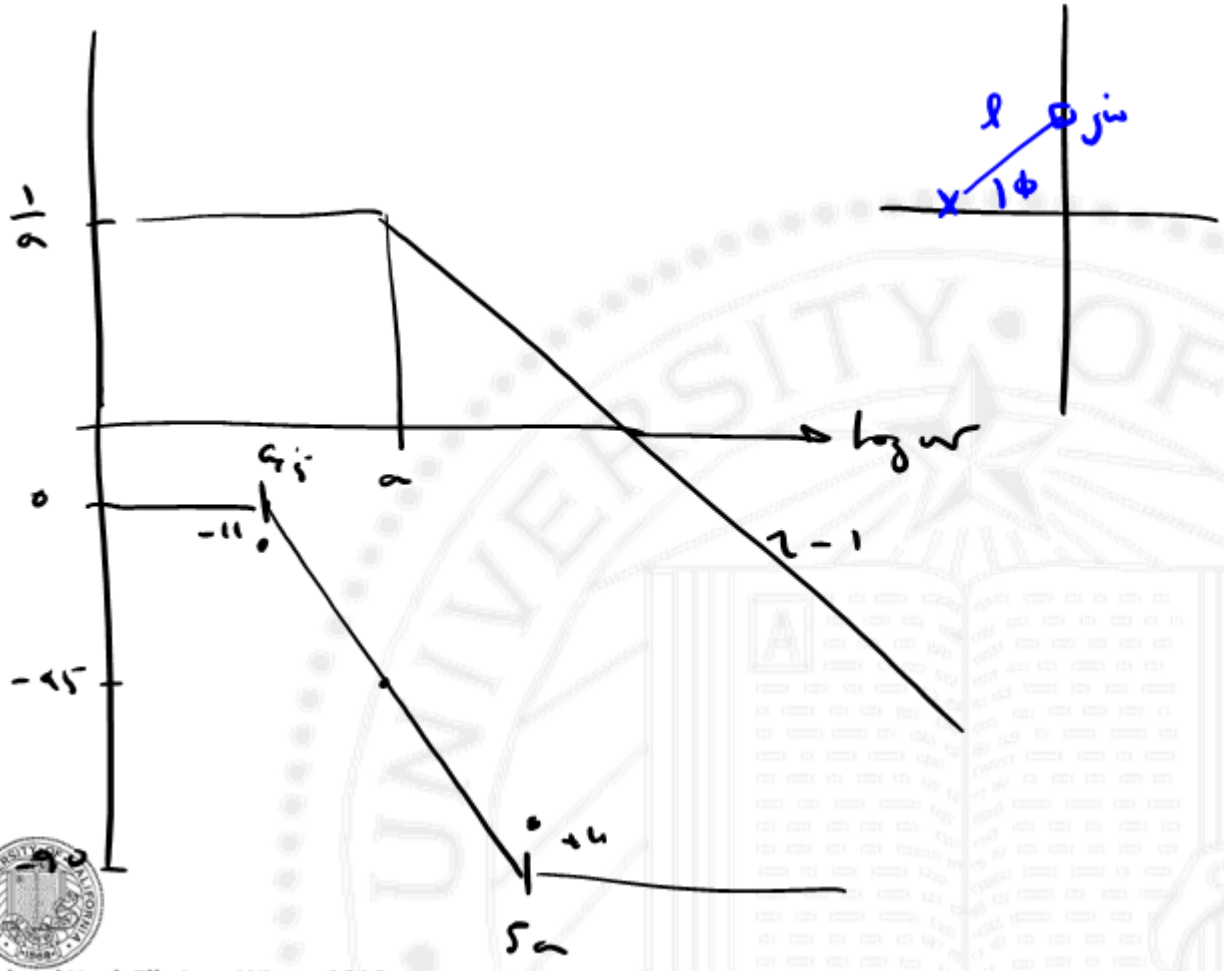
$G(s) = s + a$

@ $\omega = 0$	1:1	a	$\angle$	0
@ $\omega = a$	1:1	$a\sqrt{2}$	$\angle$	45
@ $\omega = \infty$	1:1	∞	$\angle$	90



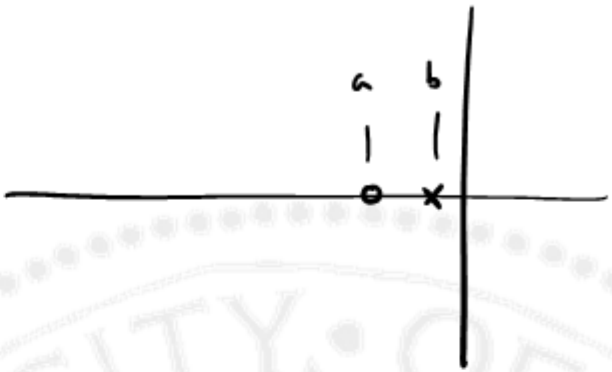


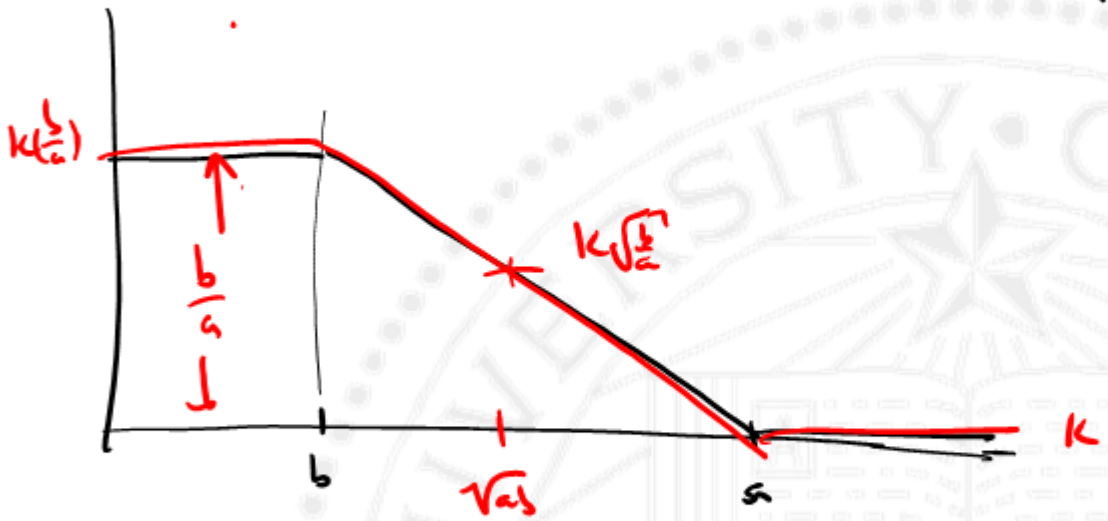
MATCHED INFLECTION

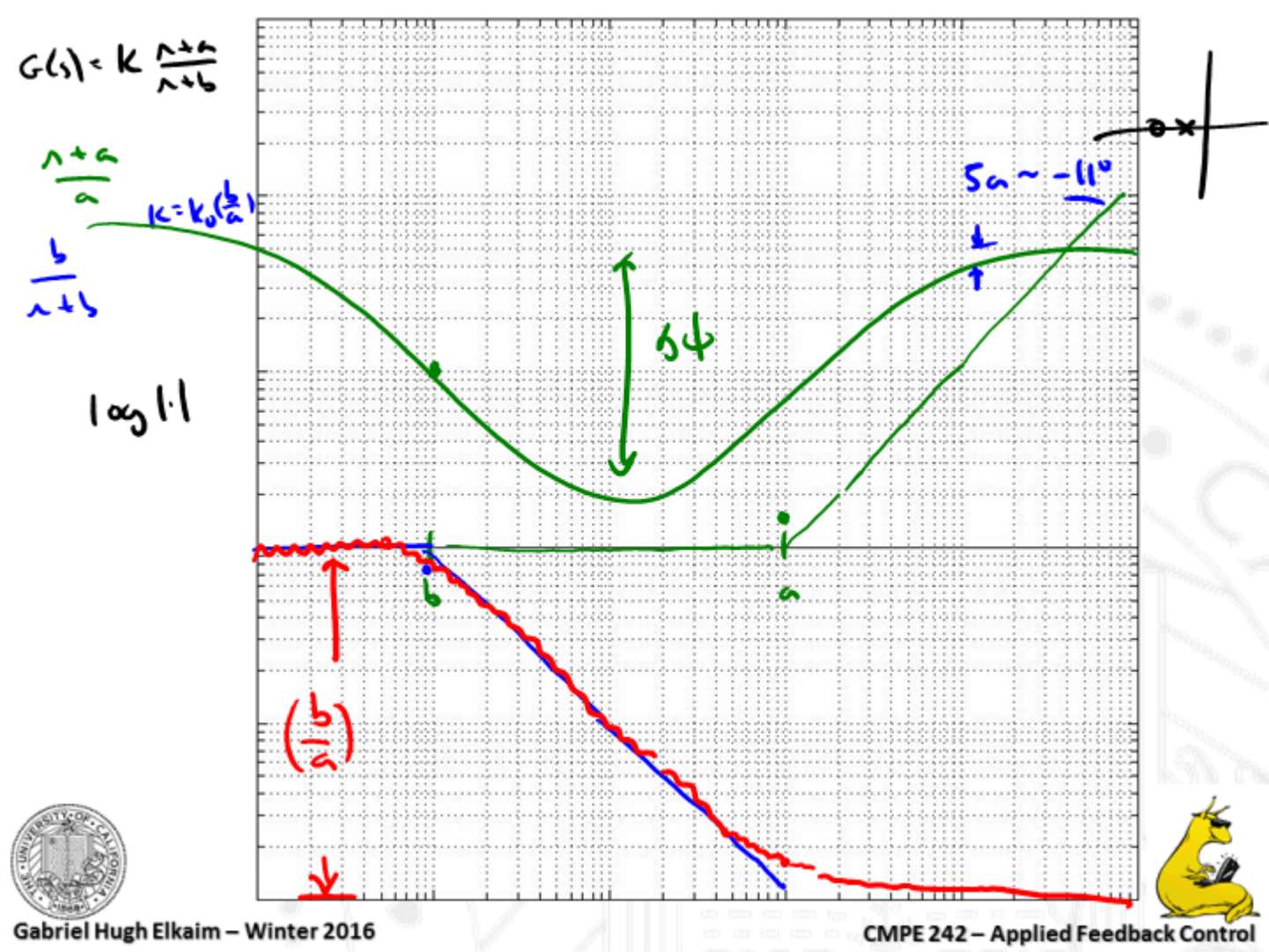


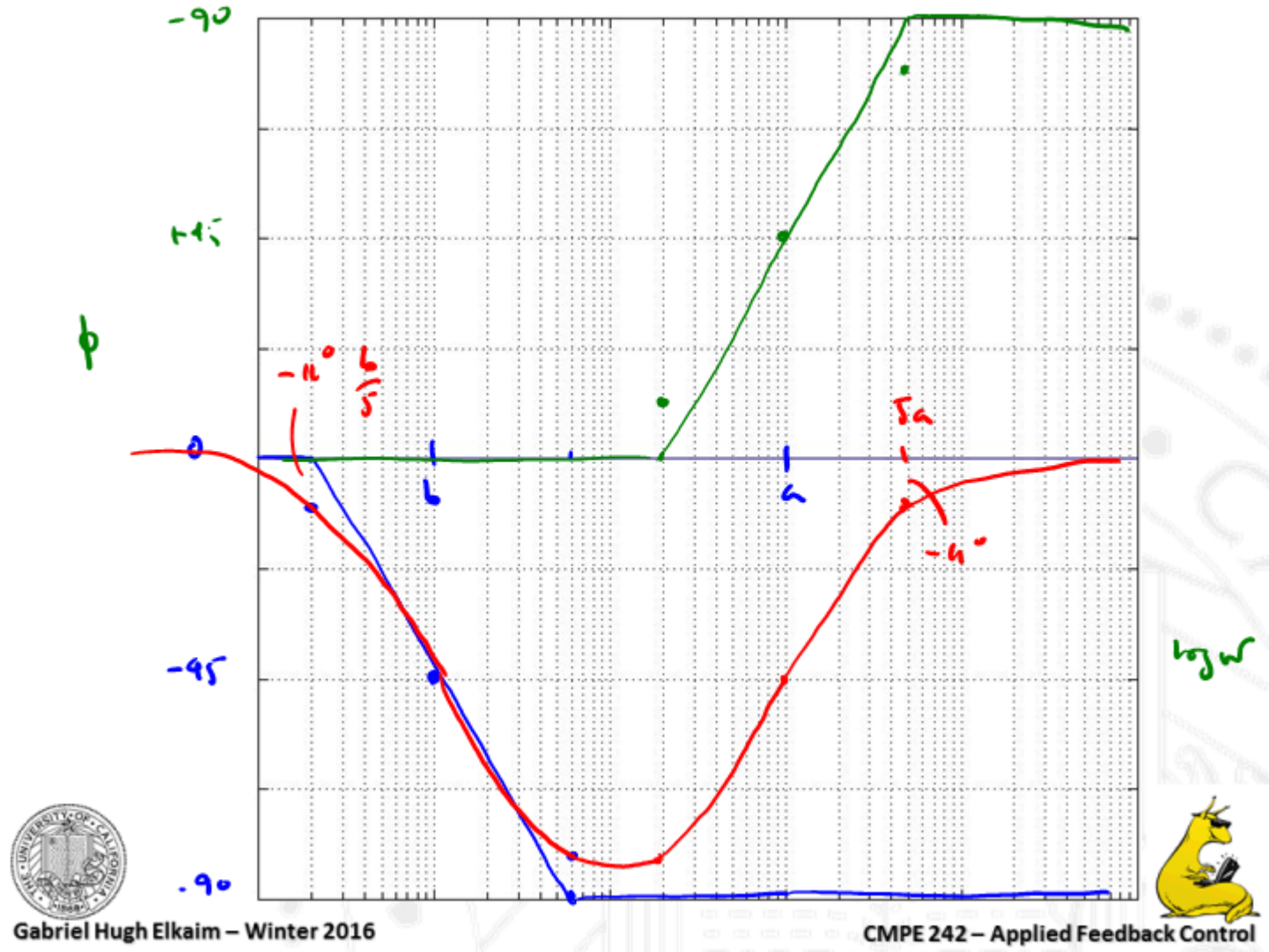
LAG $a > b$

$K \frac{s+a}{s+b}$

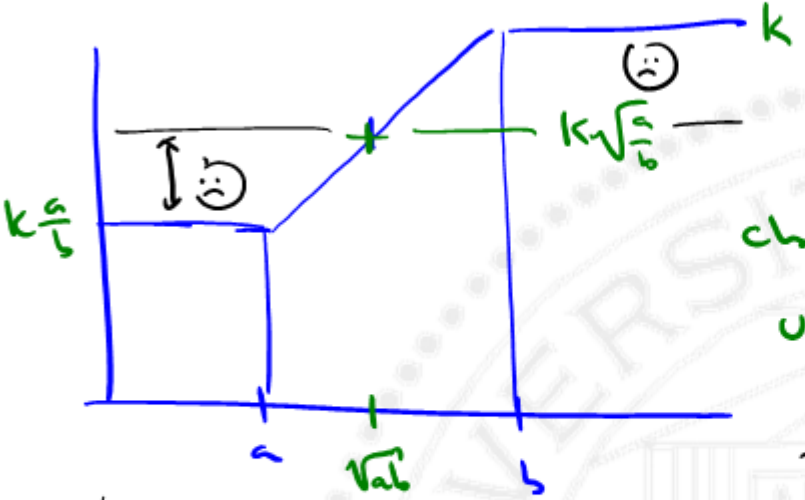


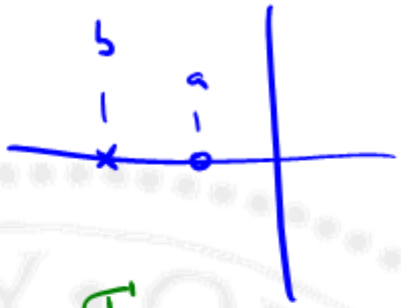



$G(s) = K \frac{s+a}{s+b}$
$\frac{s+a}{a}$
$\frac{b}{s+b}$
$K = K_0(\frac{b}{a})$
$\log|\cdot|$
$\delta\phi$
$\delta a \sim -11°$
$(\frac{b}{a})$
b
a


-90
-45
0
-45
-90
ϕ
-11°
-4°
log ω

$L(s) = K\frac{s+a}{s+b}$ $b > a$

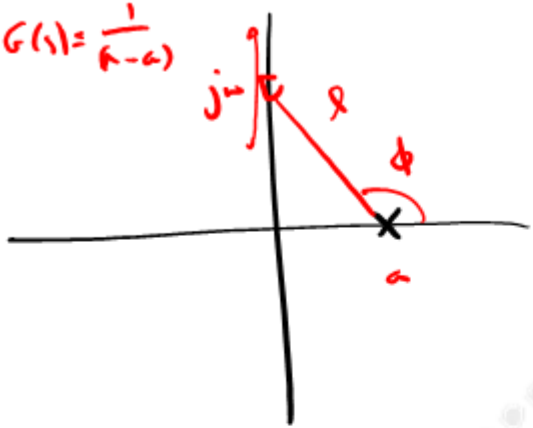




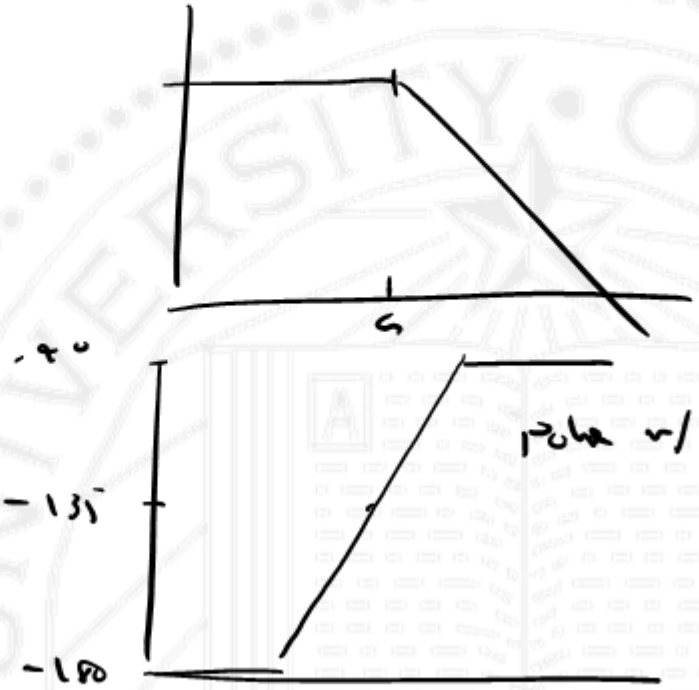
choose $K = \sqrt{\frac{b}{a}}$

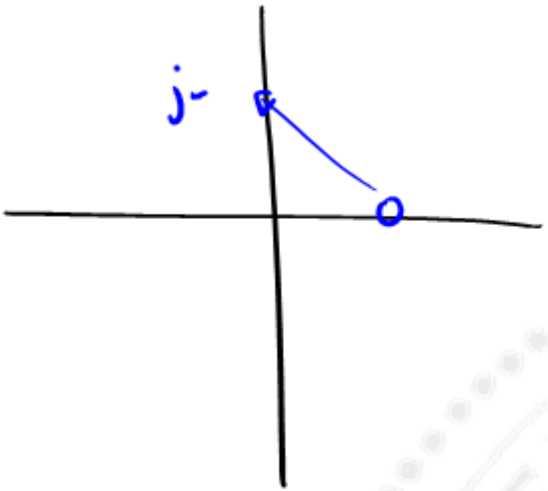
unity gain @ $\omega = \sqrt{ab}$

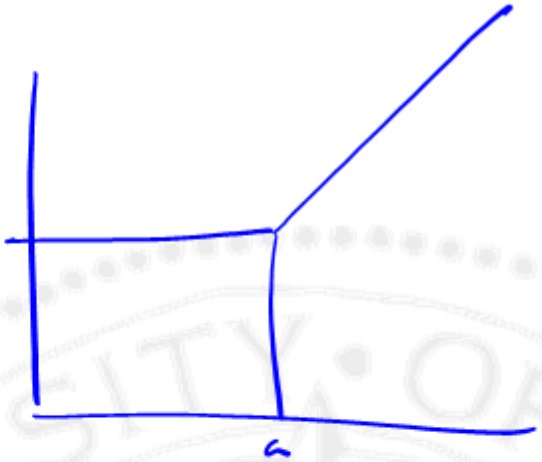
$\frac{b}{a}$	$\Delta\phi_{pm}$
4	36°
10	55°
25	68°
50	72°

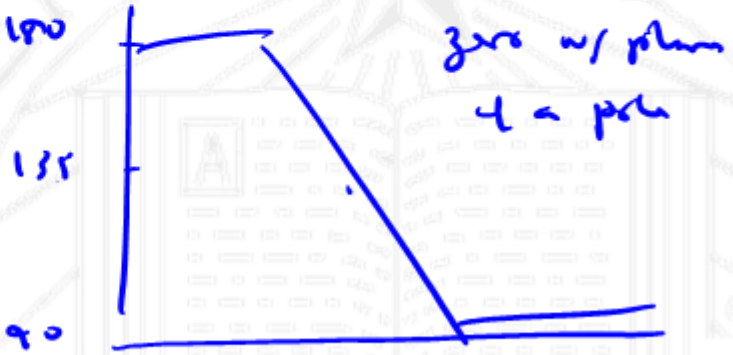


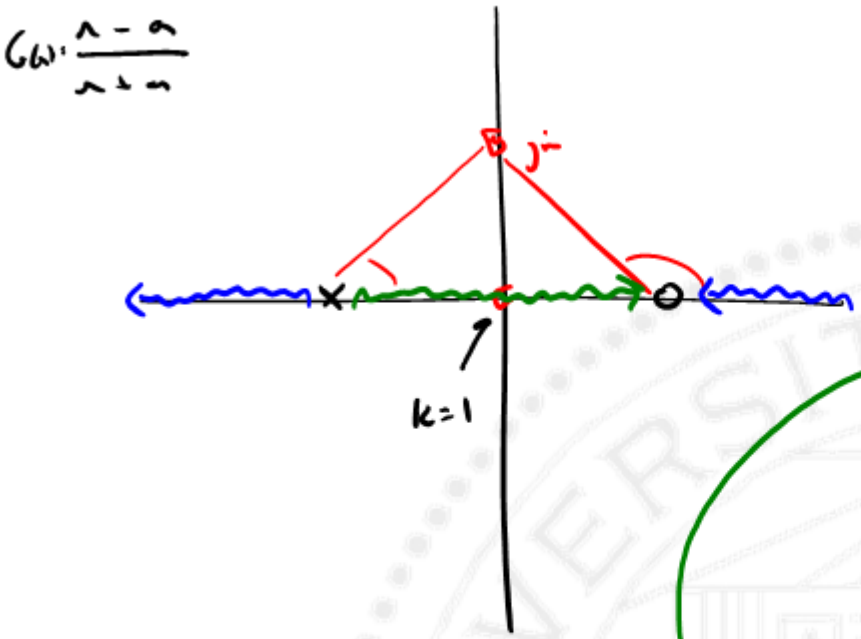
$\omega \approx 0$	1:1	$\frac{1}{a}$	∠	-180
$\omega = a$	1:1	$\frac{1}{\sqrt{2}a}$	∠	-135
$\omega = \infty$	1:1	$\frac{1}{\omega}$	∠	-90

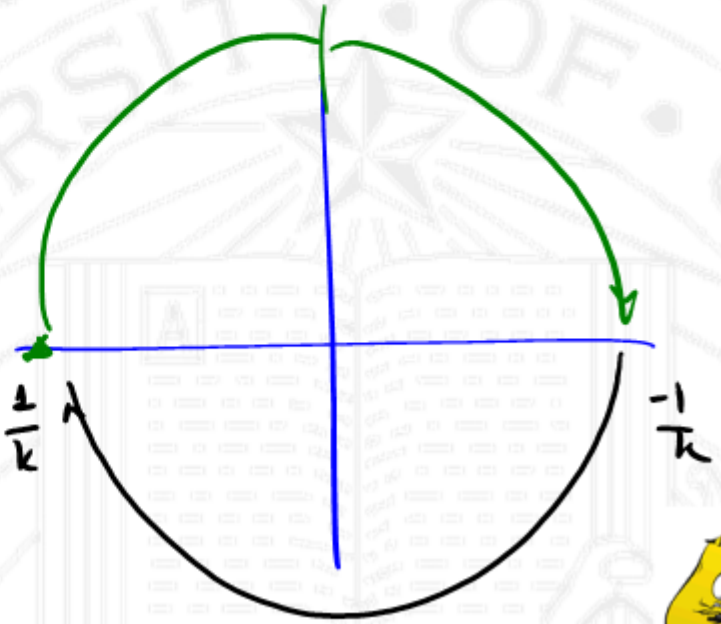


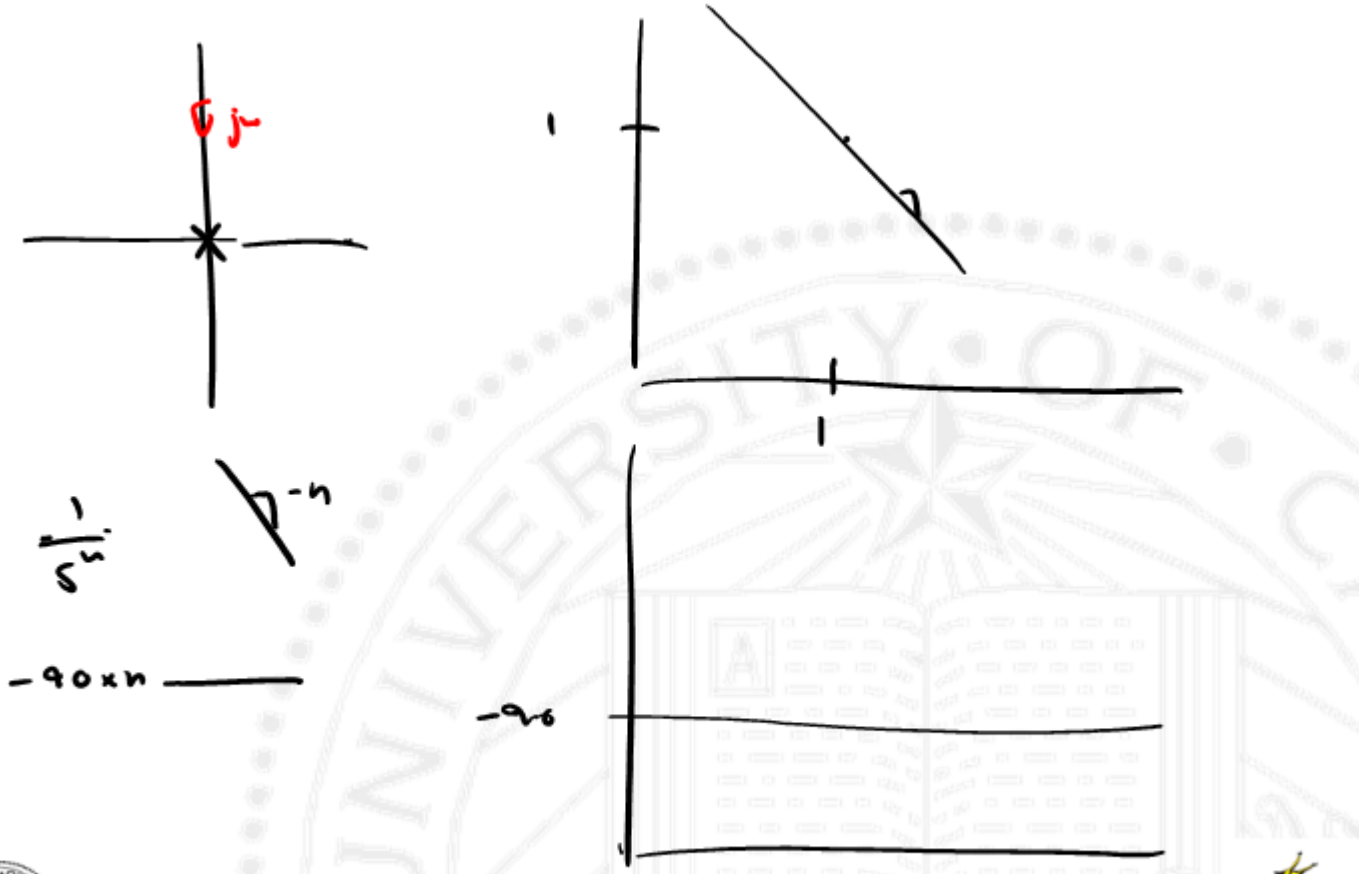

j-
O


a


180
135
90
zero w/ phase
4 a pole

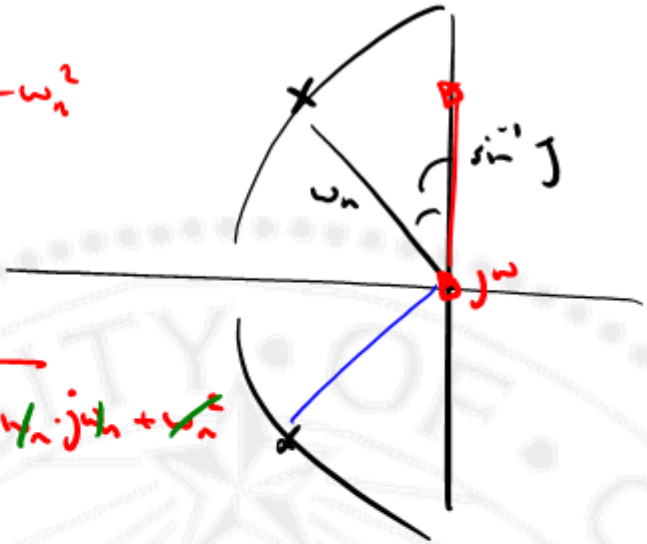

k=1


1/k
-1/k


jω
1
1
-90xn
-90

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$(j\omega_n)^2 = -\omega_n^2$

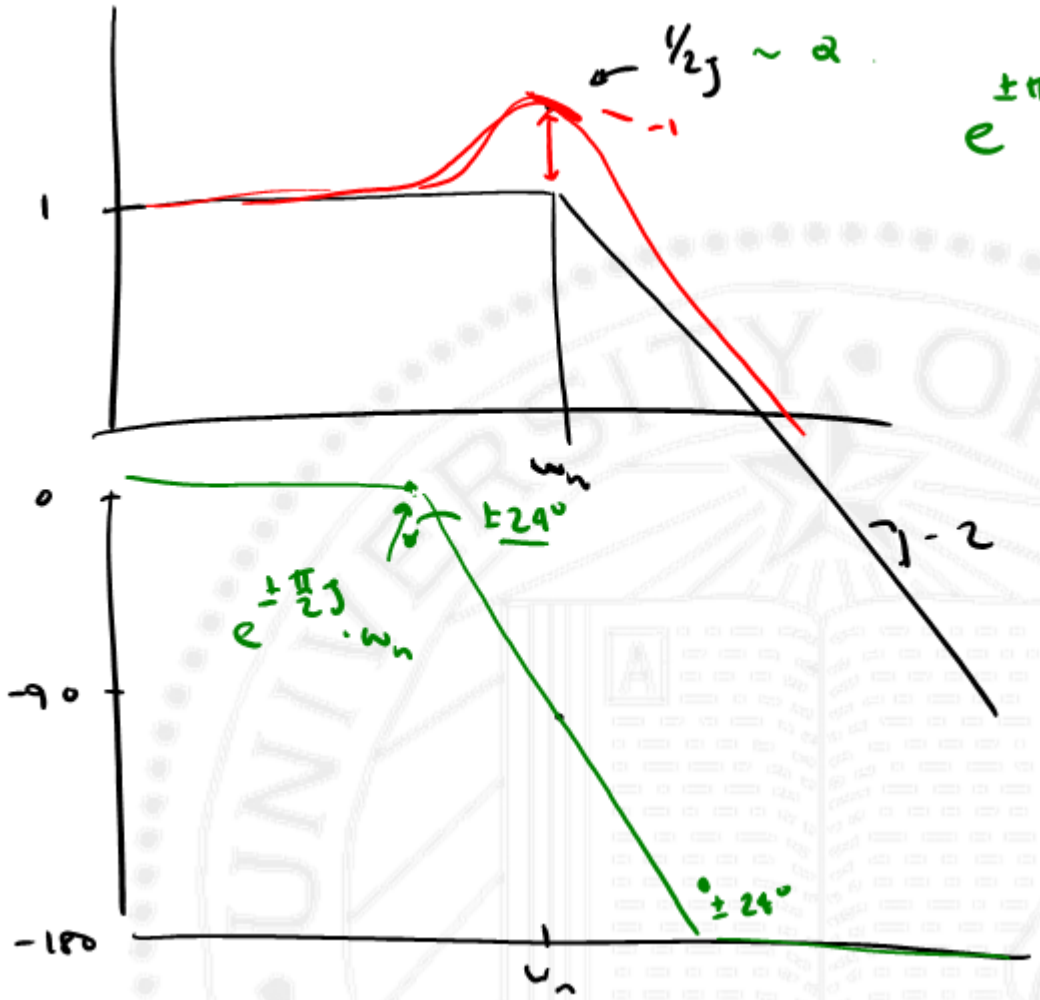


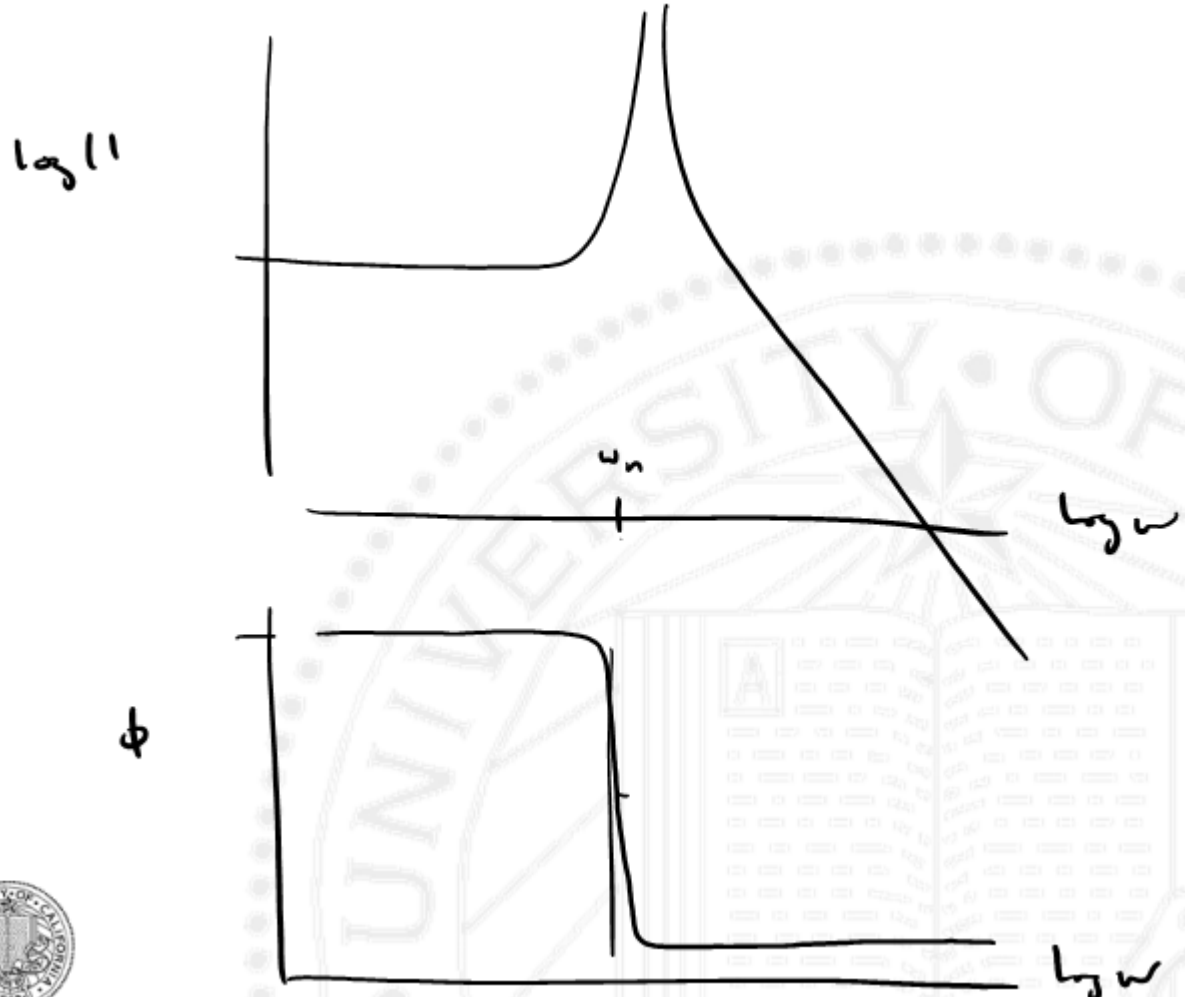
@ $\omega = 0$ $\quad |\cdot| = 1 \quad \angle = 0°$

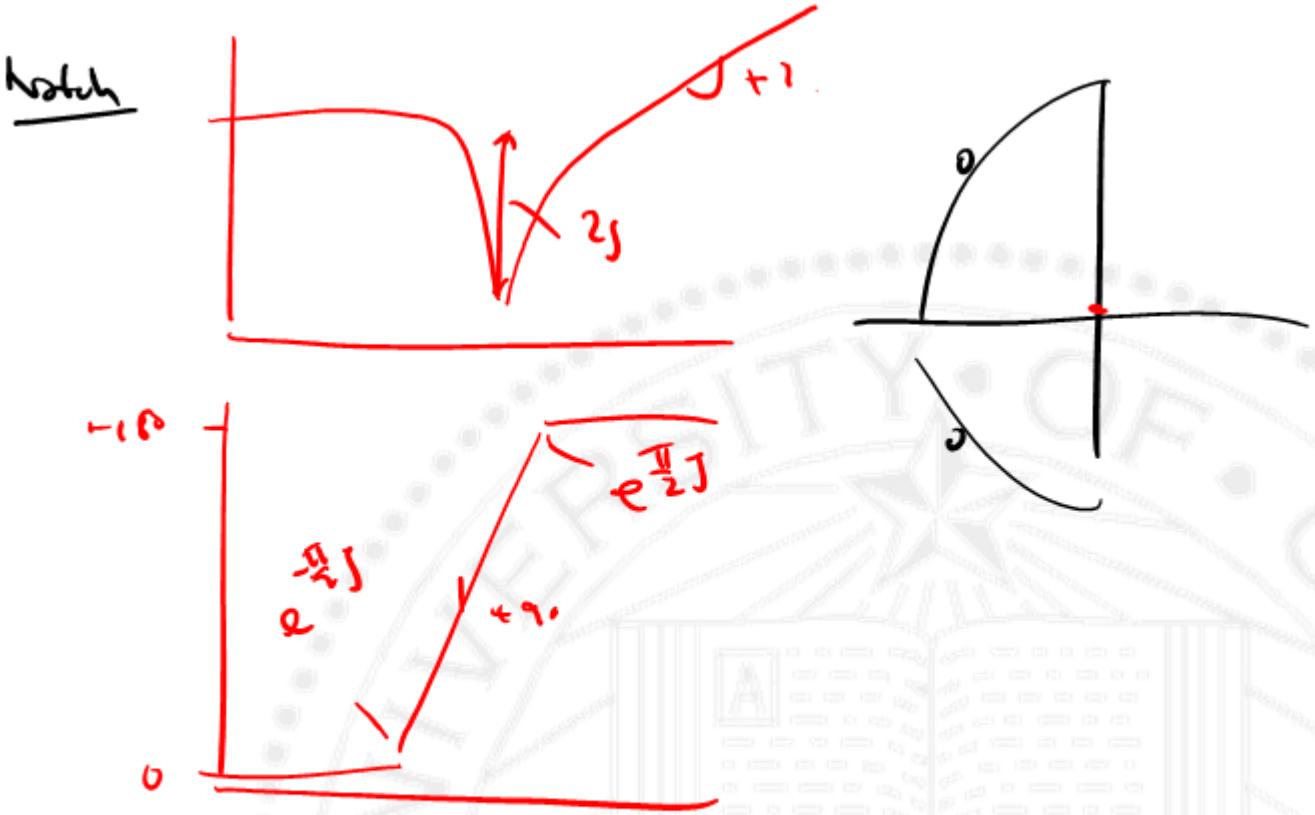
@ $\omega = \omega_n$ $\quad s = j\omega_n \rightarrow G(s) = \frac{\omega_n^2}{-\omega_n^2 + 2\zeta\omega_n \cdot j\omega_n + \omega_n^2}$

$|\cdot| = \frac{1}{2\zeta} \quad \angle = -90°$

@ $\omega = \infty$ $\quad |\cdot| = \frac{1}{\omega^2} \quad \angle = -180°$

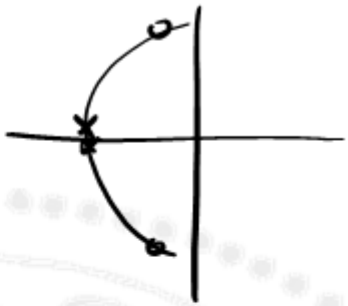

1
0
-90
-180
ω_n
ω_n
$\pm 24^\circ$
$\pm 24^\circ$
$\zeta - 2$
$e^{\pm \frac{\pi}{2} \zeta}$

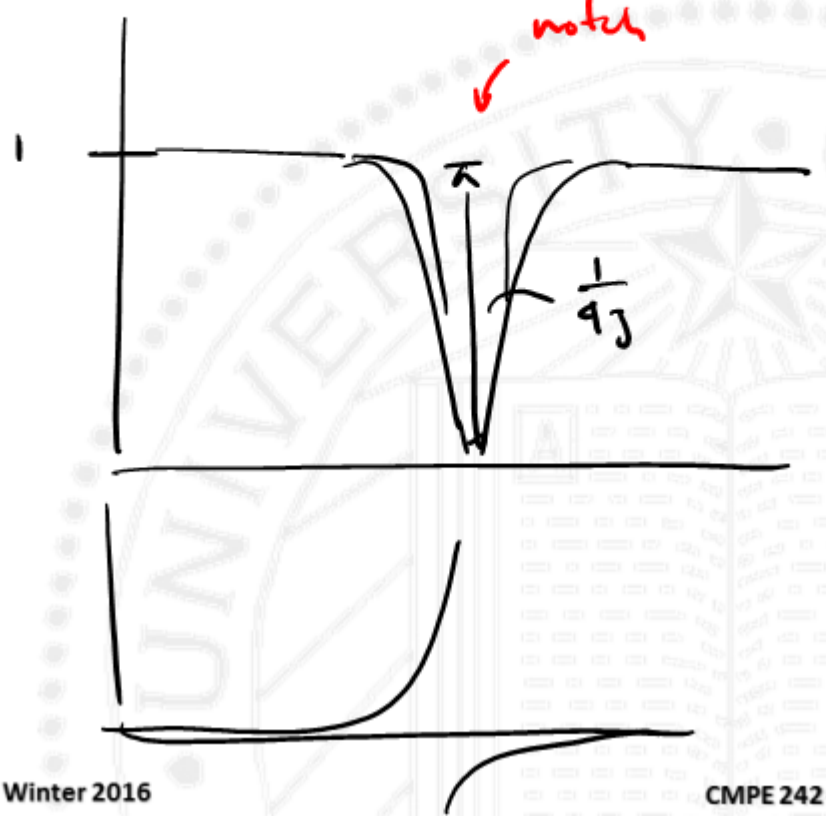

log|1|
ω_n
log ω
φ
log ω

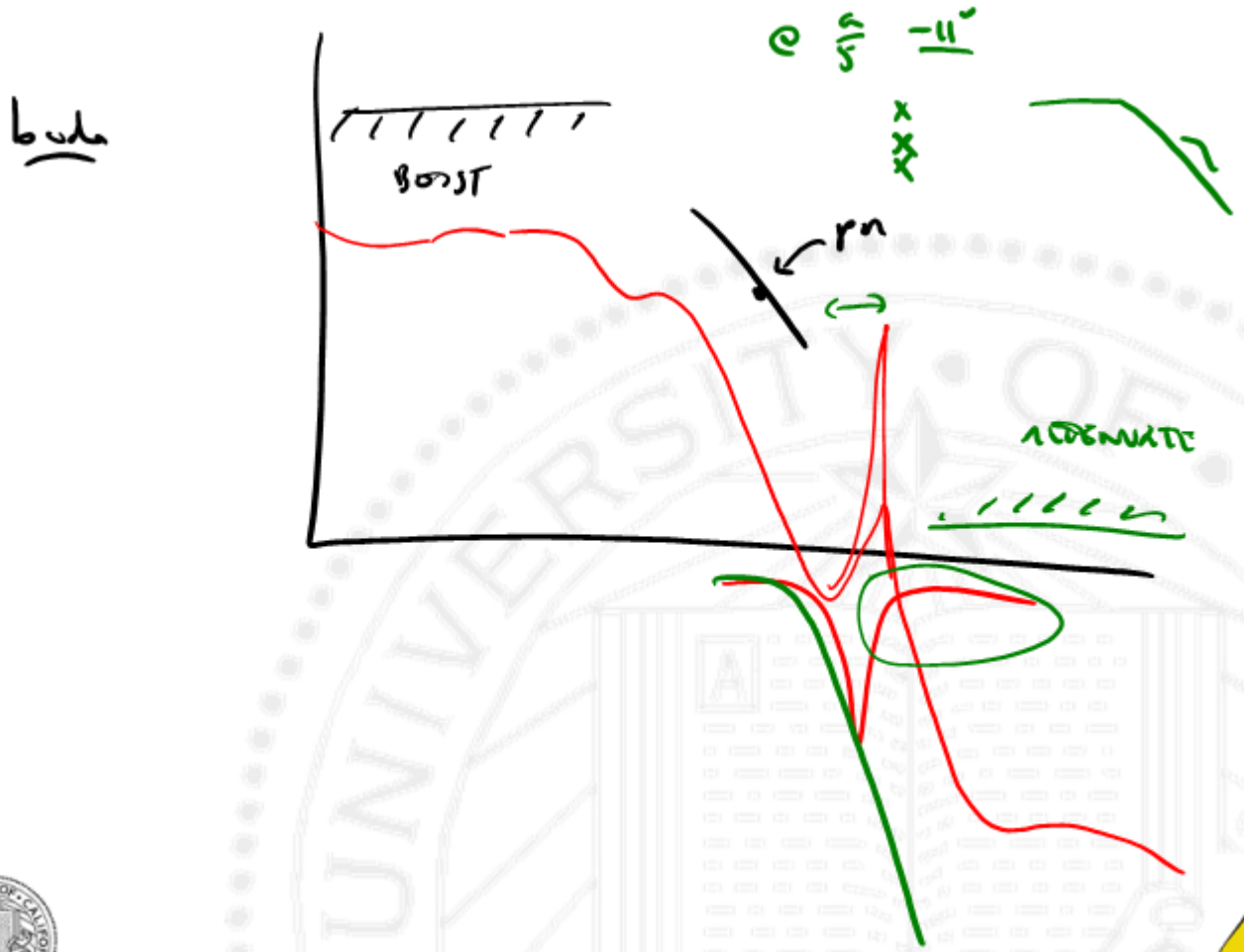

Notch
2J
J+1
-180
0
+90
0
J

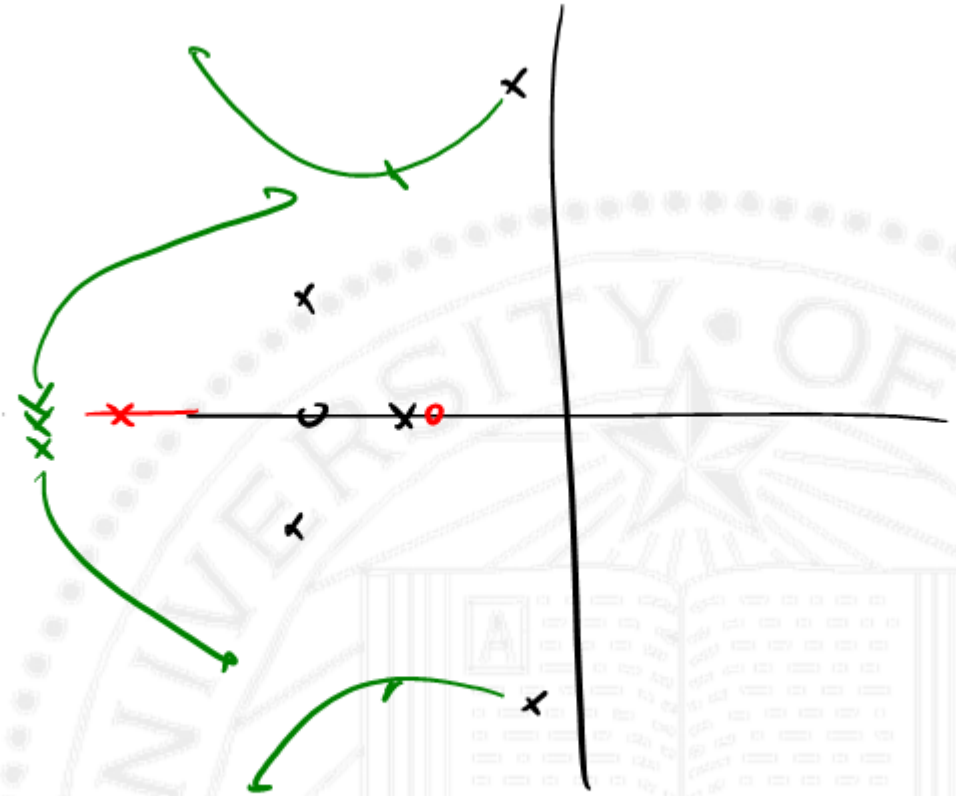
Notch

$$\frac{s^2 + 2\zeta\omega s + \omega^2}{(s+\omega)^2}$$






BOOST



Bode

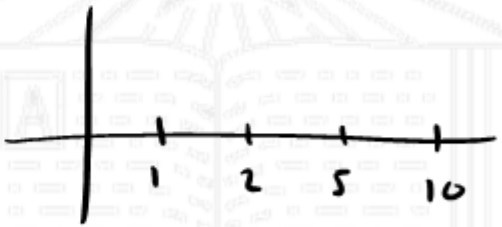
(1) $\frac{1}{\sqrt{2}}$ or $\sqrt{2}$ @ break point

(2) $\frac{1}{5}\omega$ 5ω for 4 $(e^{\frac{-\pi}{2}j}, e^{\frac{\pi}{2}j})$

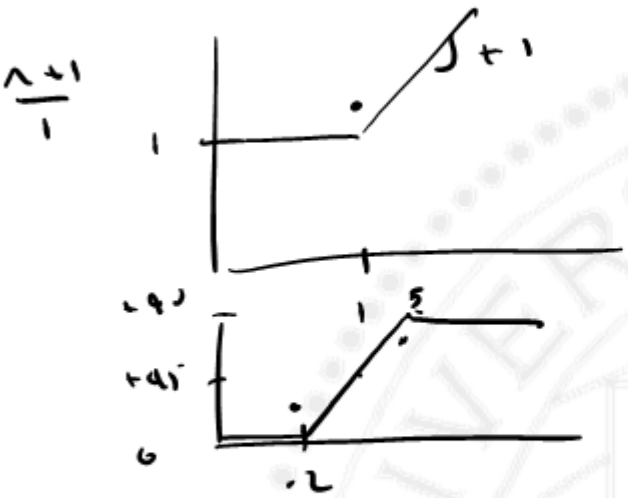
(3) error @ $\pm 12°$ for each pole/zero.

(4) $\log_{10}(2) \simeq 0.3$

$\log_{10}(5) \simeq .7$

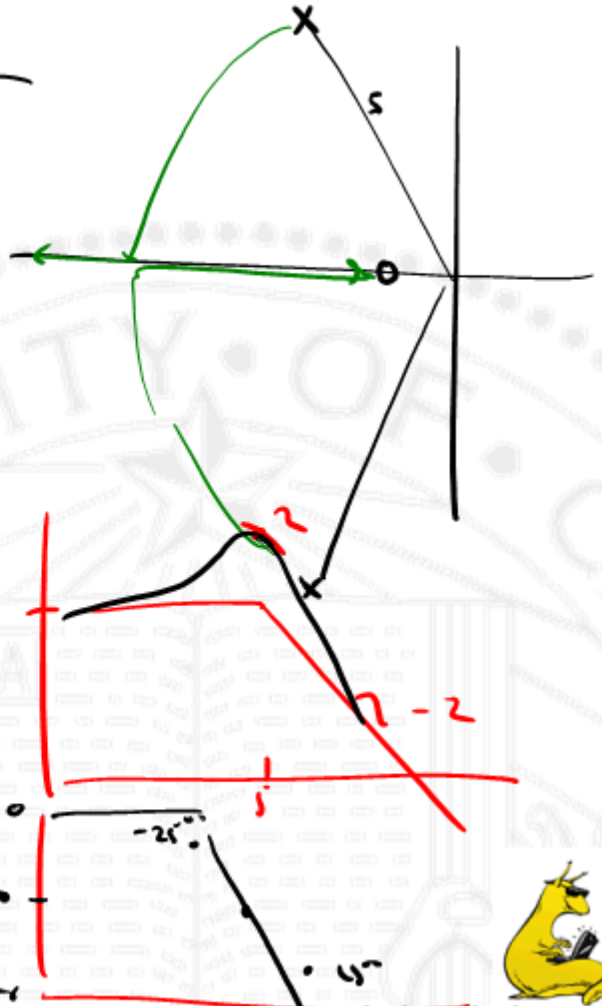


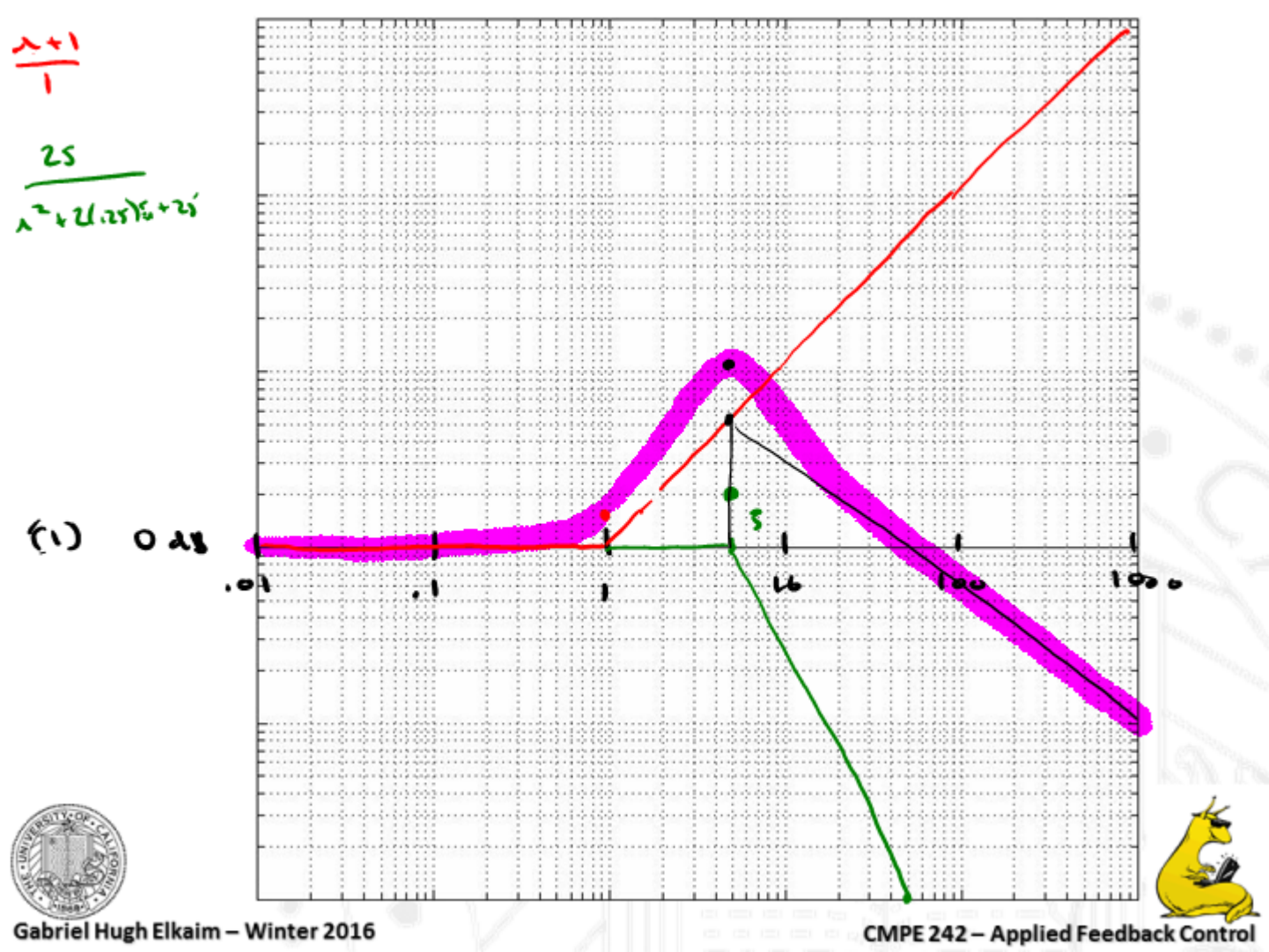
$$G(s) = \frac{(\lambda+1)\cdot s^2}{\lambda^2 + 2(0.25)s\lambda + s^2}$$

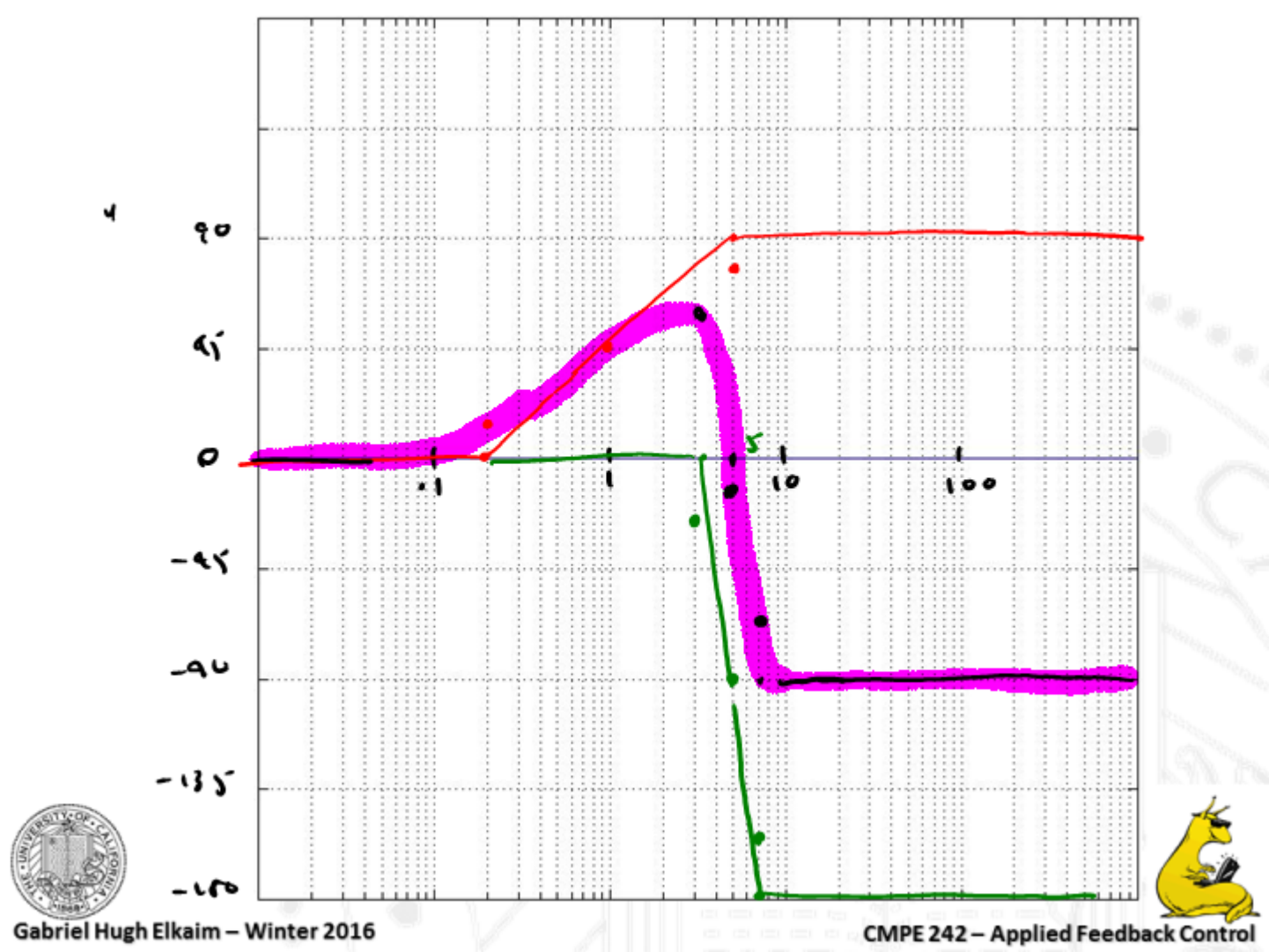


$\frac{1}{2s}$

$$\frac{s^2}{\lambda^2 + 2(.25)s\lambda + s^2}$$




.01
.1
1
10
100
1000
5


90
0
-90
-135
-180
.1
1
10
100

$$G_F(s) = \frac{10^2}{s^2 + 2(.2)10s + 10^2}$$

$\zeta = .2 \quad \omega_n = 10 \quad \frac{1}{2\zeta} = 2.5$

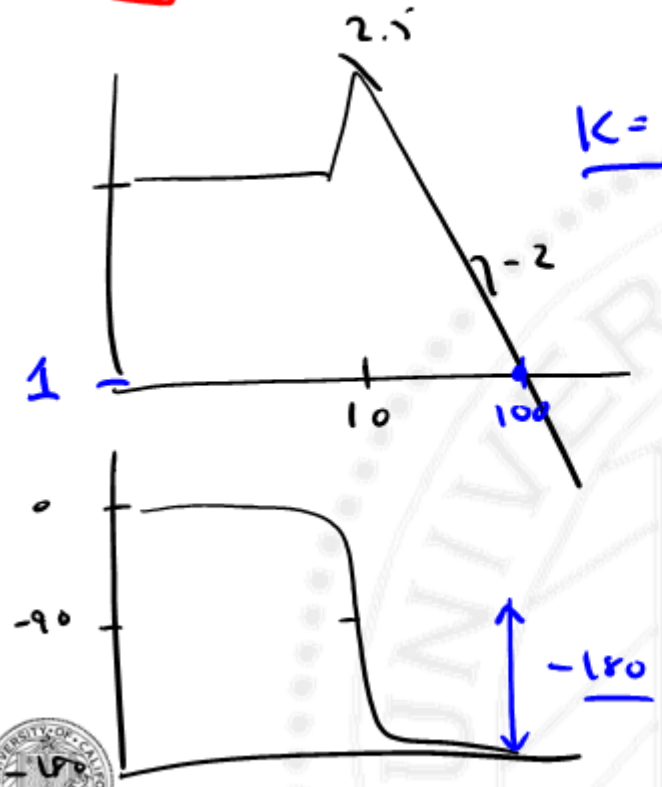
$\omega_{180} = 100$ rad/sec

$\phi_{pm} \geq 50°$

$\Sigma_{ss} < .1\%$

DC gain ≥ 100

$K = 100$

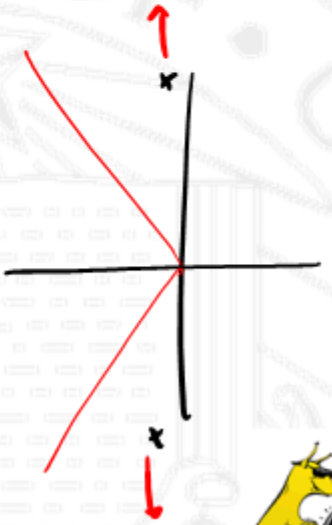


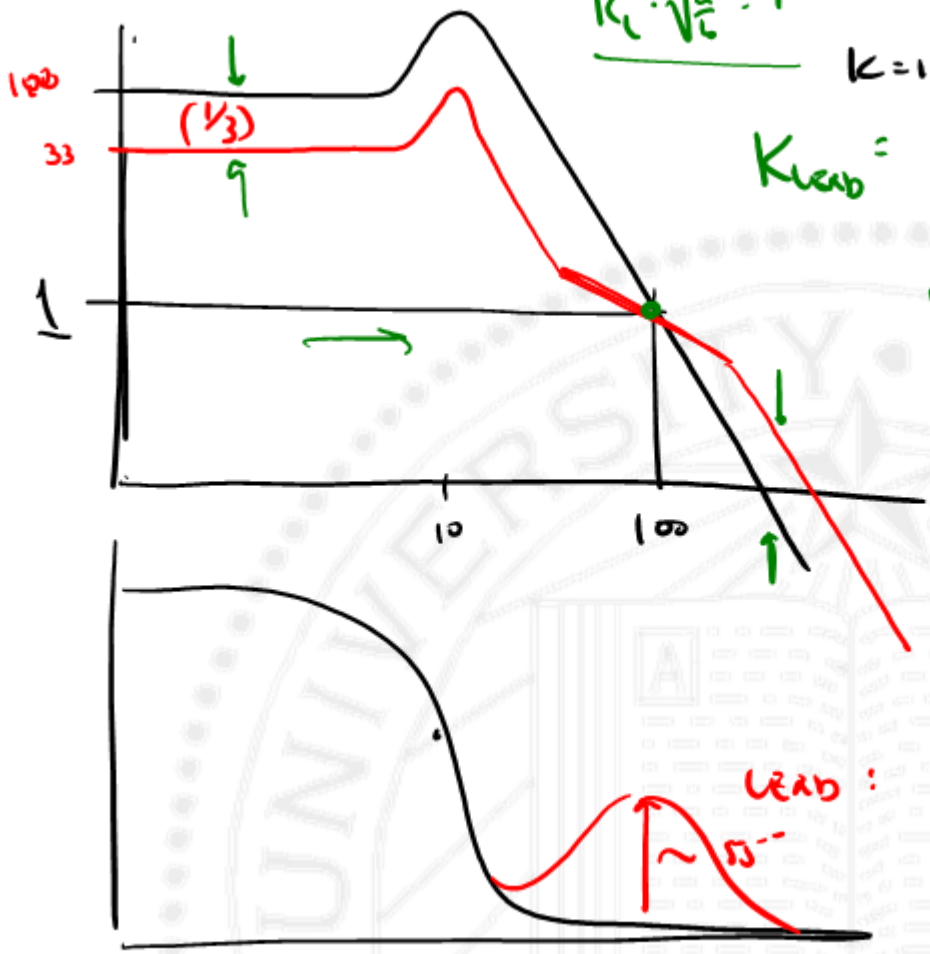
ADD LEAD

@ $\omega = 100$

$\sqrt{ab} = 10$

$\frac{b}{a} = 10$





$K_L \cdot \sqrt{\frac{a}{b}} = 1$

$K = 100$

$K_{lead} = K_L \frac{\wedge + b}{\wedge + a}$

$\sqrt{ab} = 100$

$\frac{b}{a} = 10 \quad a = \frac{100}{\sqrt{10}}$

$b = 10a$

$\sqrt{10}\, a = 100$

$\sqrt{ab} = 100$

lead: $\frac{b}{a} = 10$

~55°

$\sqrt{ab} = 100$ $\Rightarrow$ $\sqrt{a^2 \cdot 10} = 100$

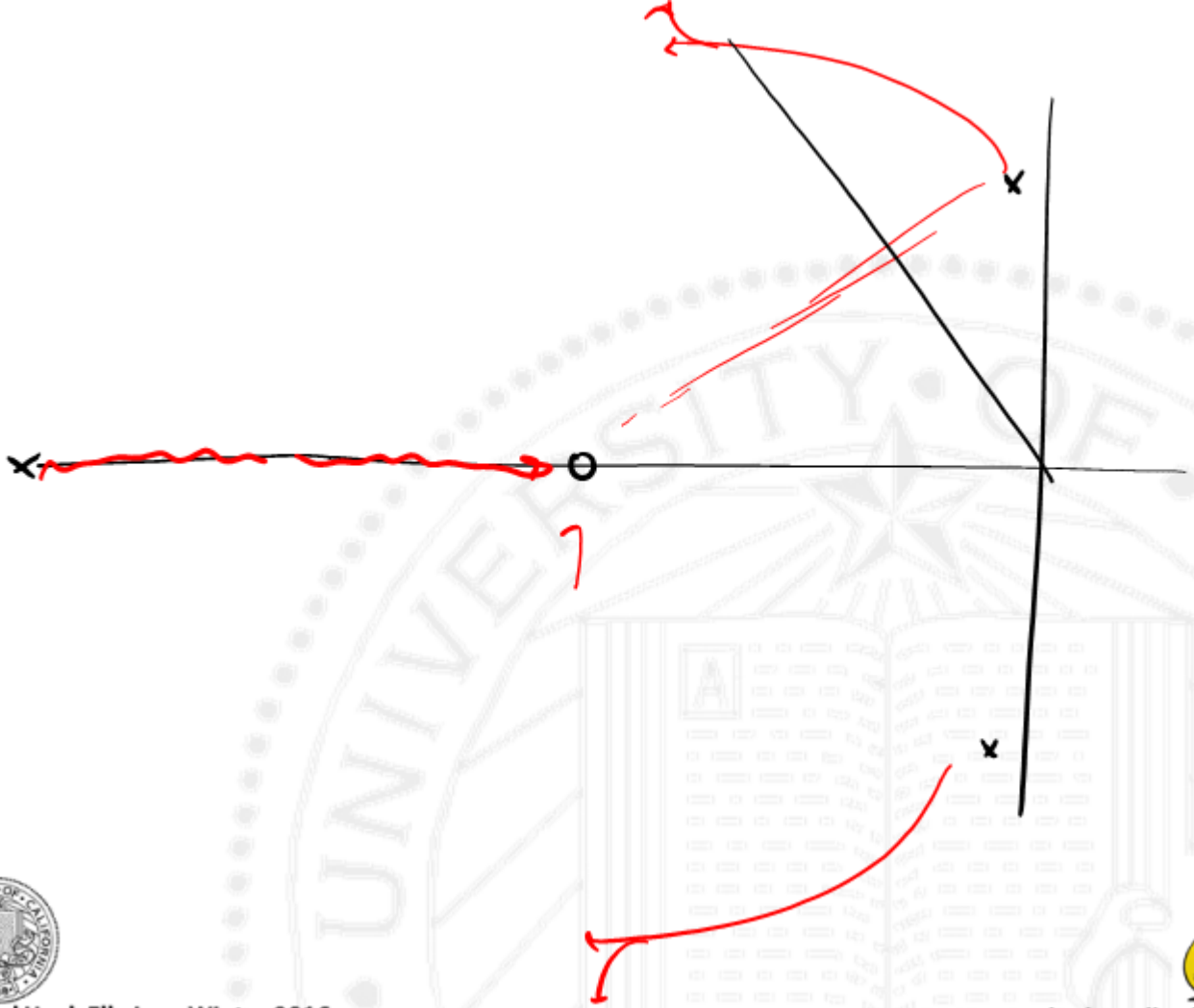
$\frac{b}{a} = 10$ $\quad b = 10a$ $\quad a\sqrt{10} = 100$ $\sim a \approx 31.6$

$b \approx 316$

$k_c\sqrt{\frac{b}{a}} = 1 \Rightarrow k_c = \sqrt{10}$

$$K = k_0 \cdot k_c(s) = 100 \cdot \sqrt{10}\,\frac{s + 31.6}{s + 316}$$

$k\frac{a}{b} = \frac{\sqrt{10}}{10} \approx .3$

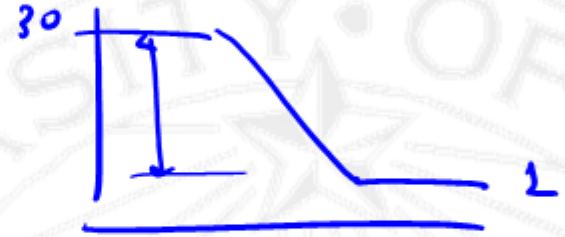


DC gain ~ 1 → K → 100 ÷ 3 → 33 want 1000

LEAD

LHC: $\frac{b}{a} = \frac{1000}{33} \approx \frac{b}{a} = 30$

$K_{LHC} = \frac{\lambda + 30a}{\lambda + a}$



$5(30a) \rightarrow -12^{\circ}$

$a \ll \frac{100}{60}$

$30\omega_z = 100$

$a = 1/3$

$$K(s) = 100\sqrt{10} \cdot \frac{s+33}{s+316} \cdot \frac{s+10}{(s+1/3)}$$