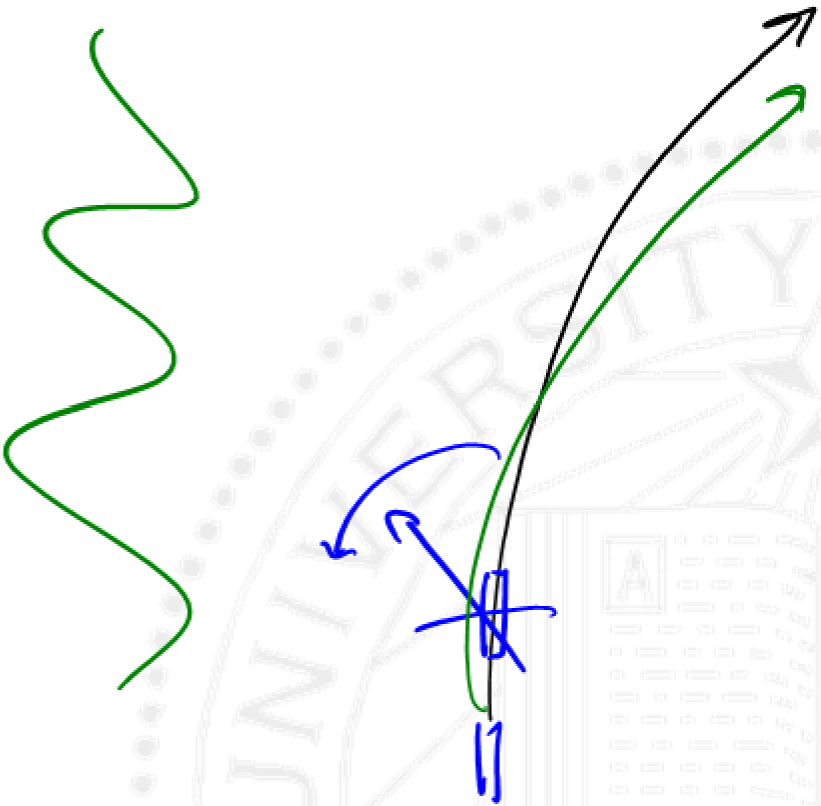
CMPE-242
Applied Feedback Control

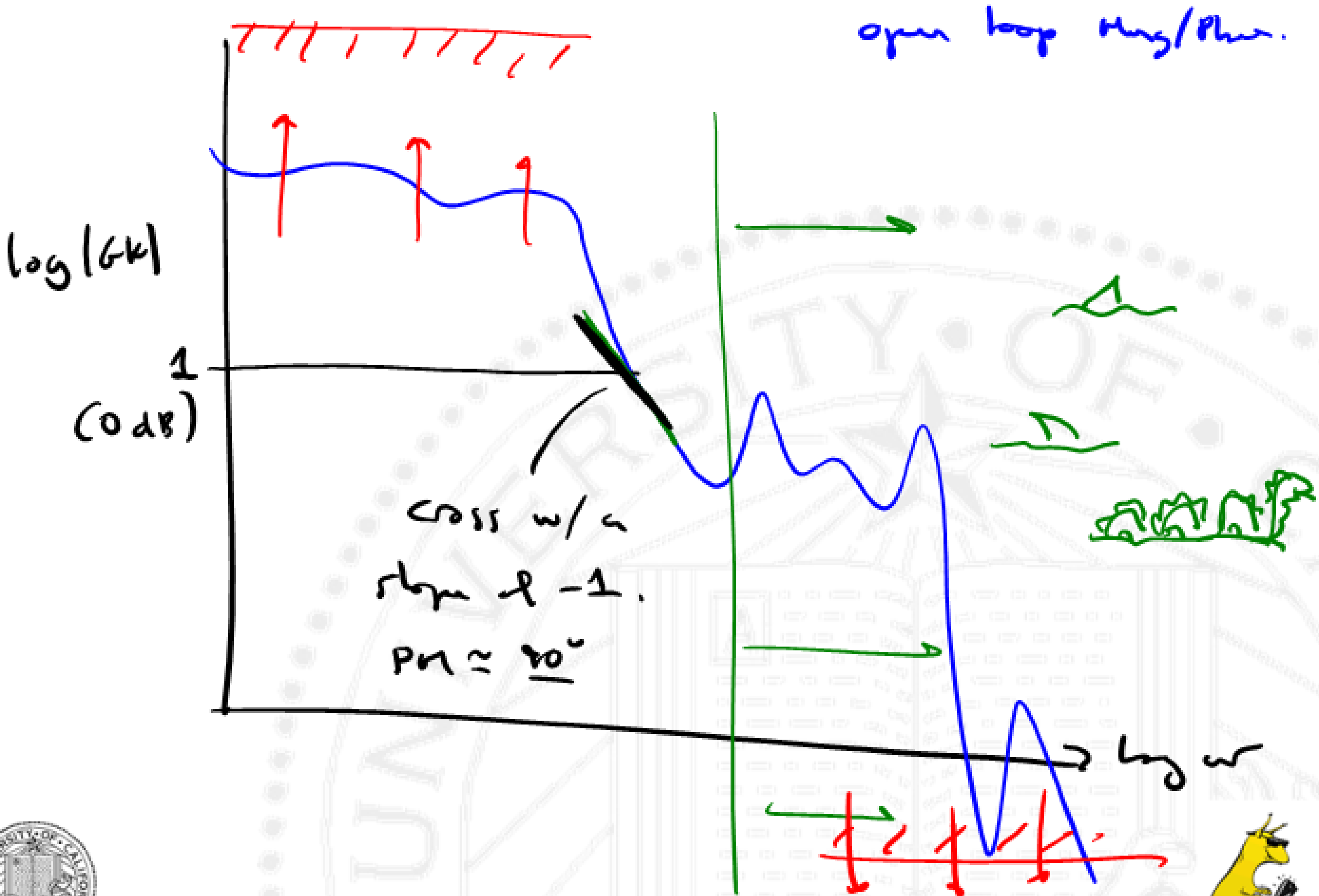
Gabriel Hugh Elkaim

Winter 2016



rlocus(-sys)

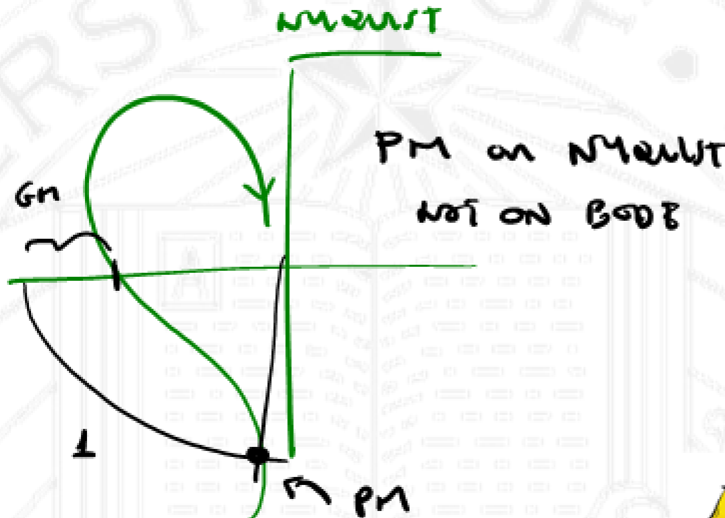
sys = tf(-[num], den)

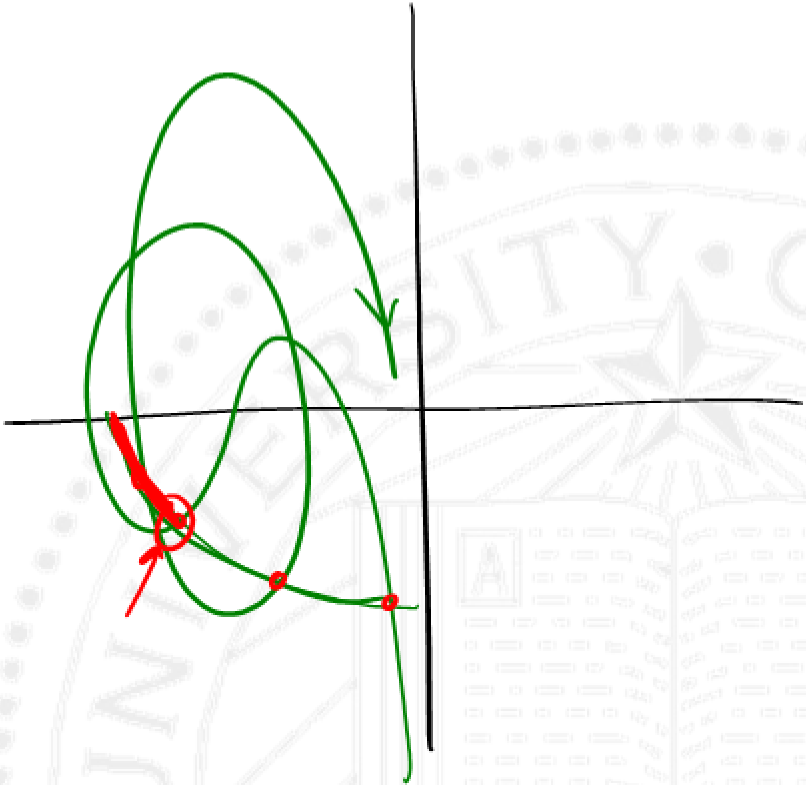

open loop Mag/Phase.
log |GK|
1
(0 dB)
cross w/ a
slope of -1.
PM ≈ 90°
log ω

BODE GAIN/PHASE THEOREM

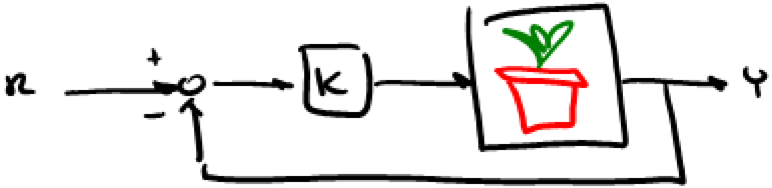
Simple systems → slope ‖ ↔ ∡ φ

Slope of $-1 \approx \phi = -90°$





	Root Locus	Bode
Stability	C.L.H.P λ's	GM/PM
Transient	Pole locations	BW ω_c $\quad \zeta \simeq PM/100$.
Tracking	—	$\lvert GK \rvert$ including DC
Sensitivity / Robustness	—	GM/PM
Control Effort	— (~K)	— ω_{x_0} ← LQ(x)



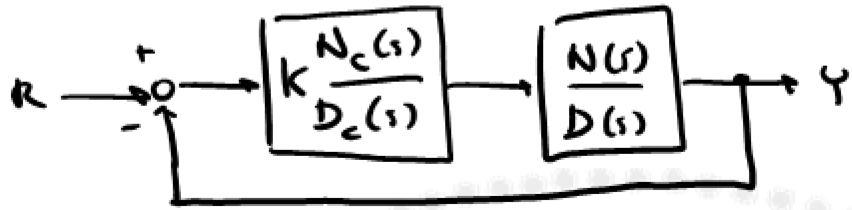
$$\frac{Y}{R} = \frac{GK}{\boxed{1+GK}} \leftarrow \Delta(s)$$

Root Locus : $R = 0$ "Regulator"

Bode : $R = \sim\sim$ "Tracker"

LQR : $J = \int_0^\infty (y^2 + u^2)\, dt$

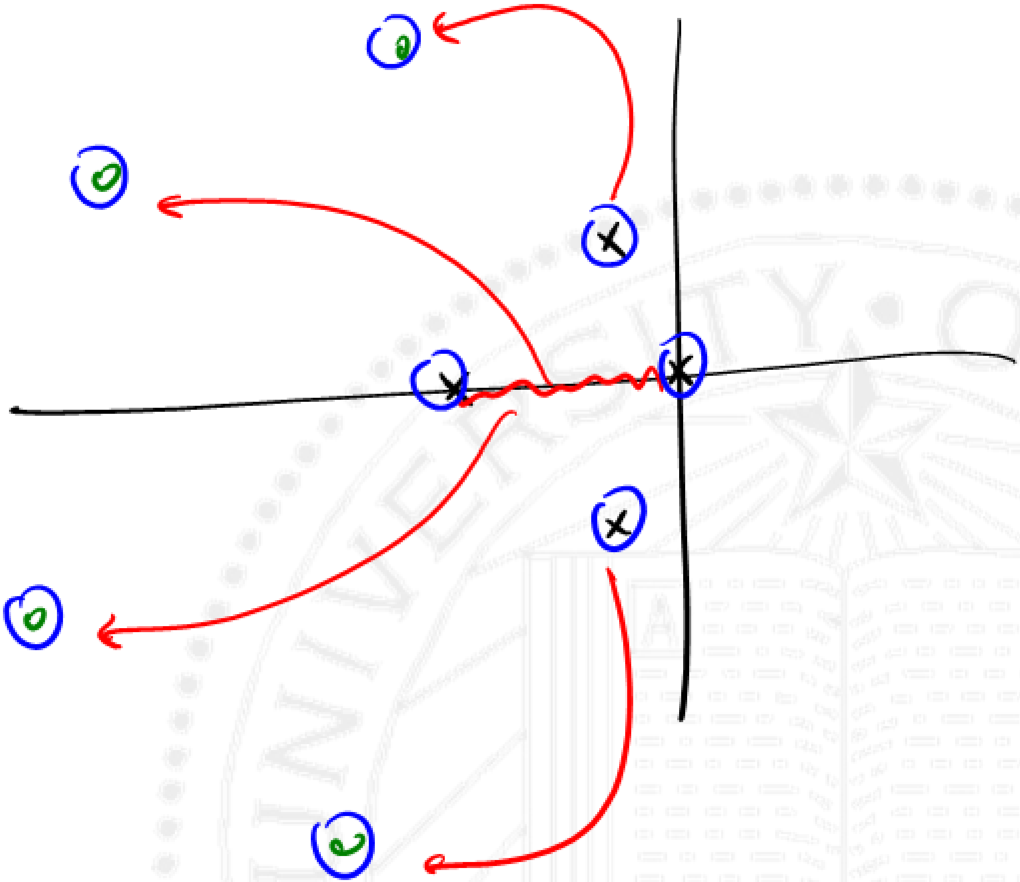
Root locus



$$\frac{Y}{R} = \frac{k\frac{N_c N}{D_c D}}{1 + k\frac{N_c N}{D_c D}} = \frac{k N_c N}{\boxed{\underbrace{D_c D}_{\text{open loop poles of the system}} + \underbrace{k N_c N}_{\text{open loop zeros of the system}}}} \leftarrow D_{cl}(s)$$

START @ OPEN LOOP POLES $\longrightarrow$ OPEN LOOP ZEROS

$$k \longrightarrow 0 \longrightarrow \infty$$



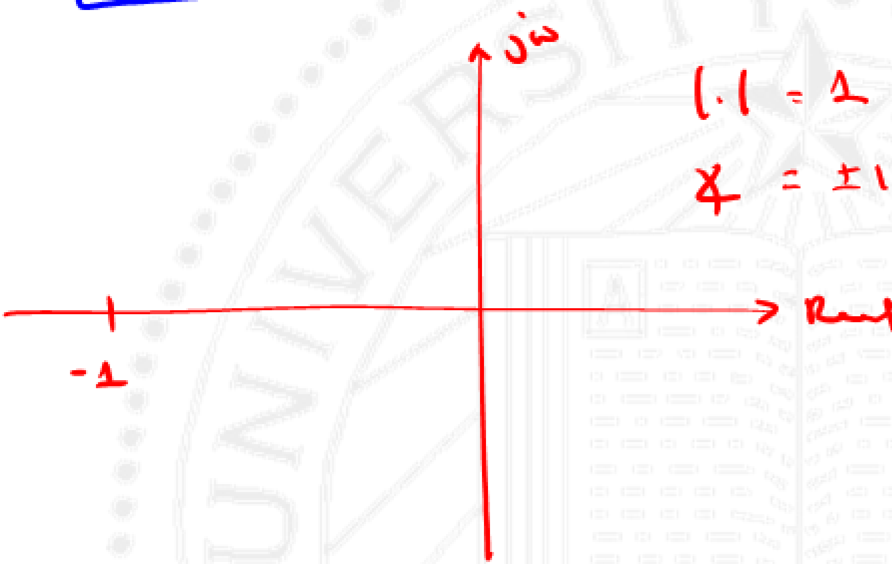
$$D_c D + K N_c N = 0 \longrightarrow 1 + \frac{K N_c N}{D_c D} = 0$$

$$\boxed{K \frac{N_c N}{D_c D} = -1}$$

"Evan's Form"

$$|\cdot| = 1$$

$$\angle = \pm 180$$

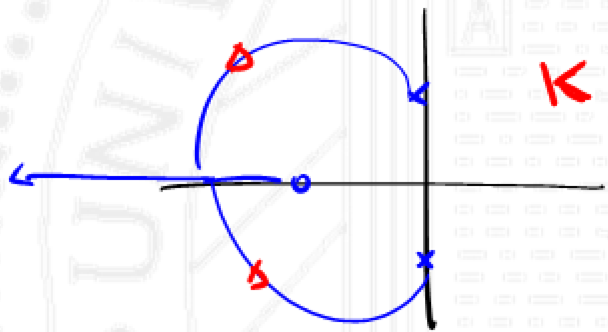


$$F(s) = \frac{N_c N}{D_c D}$$

Evaluate $F(s)\big|_{s=\bullet}$ $\quad |\cdot|$ $\quad \measuredangle$

Root locus

Find s, such that $F(s)\big|_s \to \measuredangle \pm 180^\circ$



$$K = \frac{1}{|F(s)|}\Bigg|_{s=\Delta}$$

Root Locus Rules

(1) locus starts @ open loop x's $\longrightarrow$ open loop o's

x $\longrightarrow$ o

(2) Real axis: To LEFT of ODD # of x's & o's.

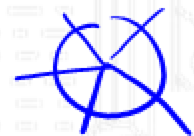
(3) Asymptotes: number of x's in excess of o's

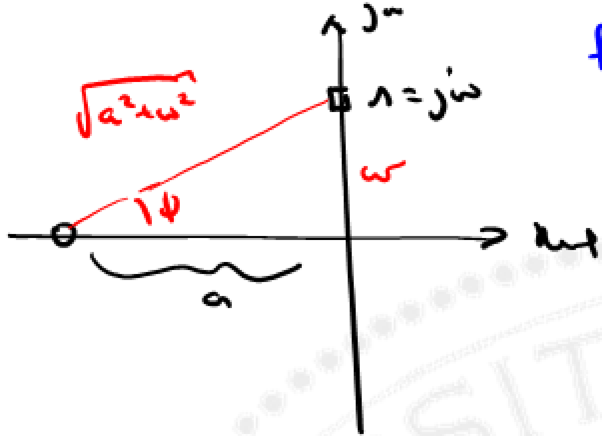
$$\alpha = \frac{\sum p_i - \sum z_i}{n-m}$$

$\measuredangle$ divide 360 into equal parts, symmetry holds

$$\measuredangle = \frac{-180}{n-m} + \frac{360(l-1)}{n-m} \qquad l = 1, 2, \ldots, n-m$$

(4) ϕ_d, ϕ_a use test point

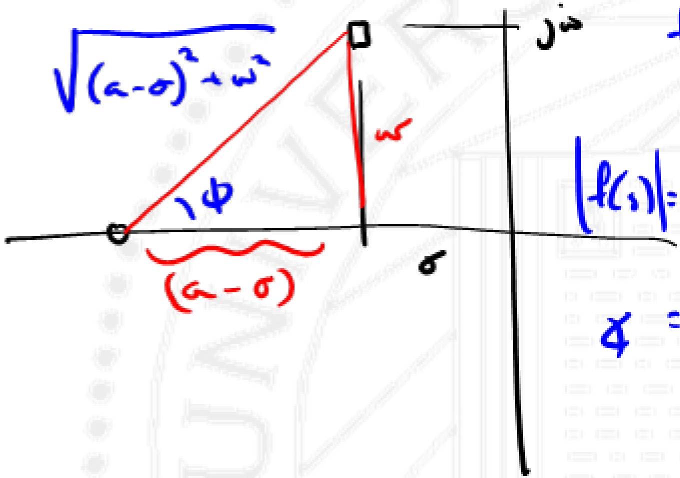




$$f(s) = s + a \Big|_{s = j\omega}$$

$$|f(s)| = \sqrt{a^2+\omega^2}$$

$$\measuredangle f(s) = \tan^{-1}\left(\frac{\omega}{a}\right)$$

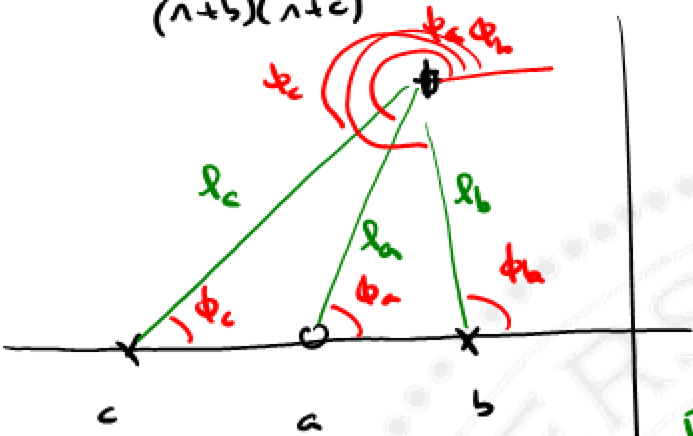


$$f(s) = s + a \Big|_{s = -\sigma + j\omega}$$

$$|f(s)| = \sqrt{(a-\sigma)^2+\omega^2}$$

$$\measuredangle = \tan^{-1}\left(\frac{\omega}{a-\sigma}\right)$$

$$F(s) = \frac{s+a}{(s+b)(s+c)}$$



$$F(s)\Big|_{s=\sigma+j\omega}$$

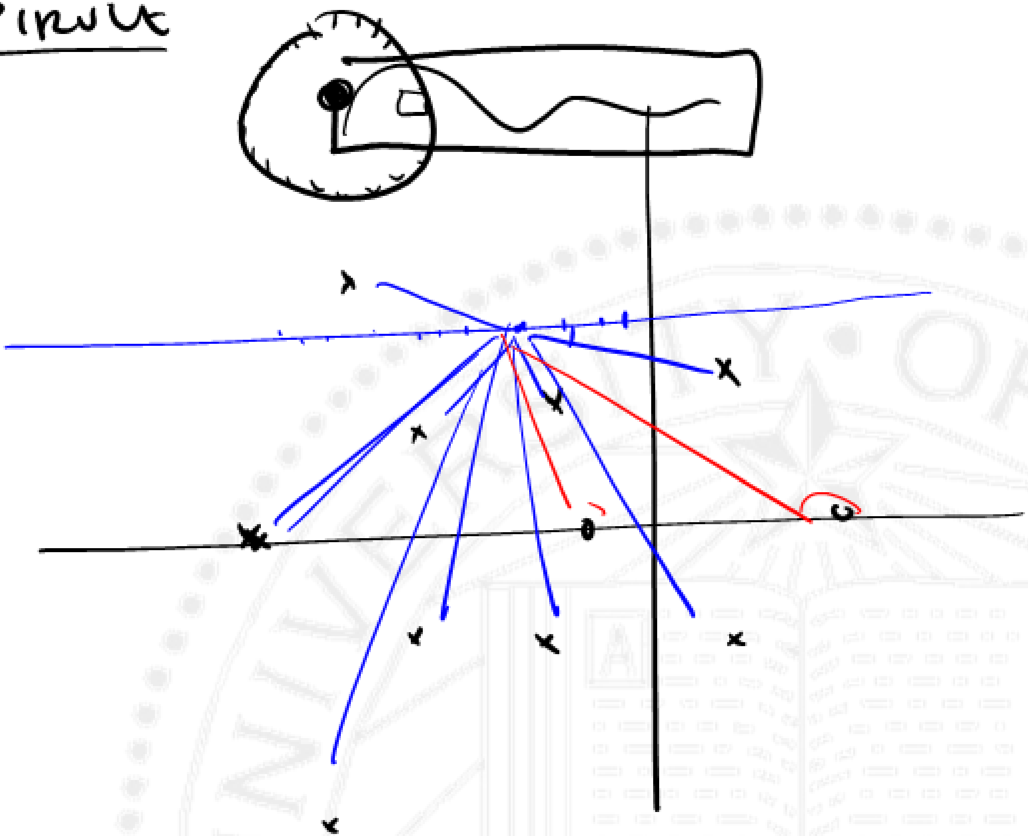
$$F(s) = \frac{f_a}{f_b f_c} = \frac{l_a e^{j\phi_a}}{l_b e^{j\phi_b}\, l_c e^{j\phi_c}}$$

$$F(s) = \left[\frac{l_a}{l_b l_c}\right] \cdot e^{j[\phi_a - \phi_b - \phi_c]}$$

$$|F(s)| = \frac{l_a}{l_b l_c}$$

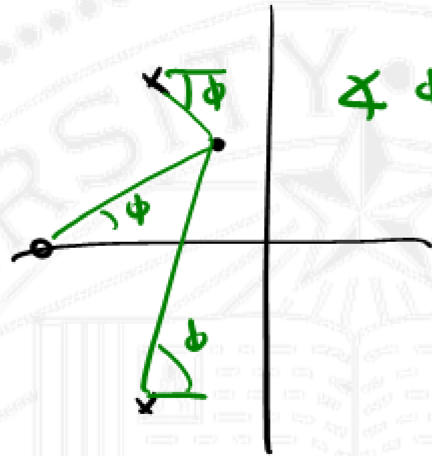
$$\measuredangle F(s) = \phi_a - \phi_b - \phi_c$$

SPINULE

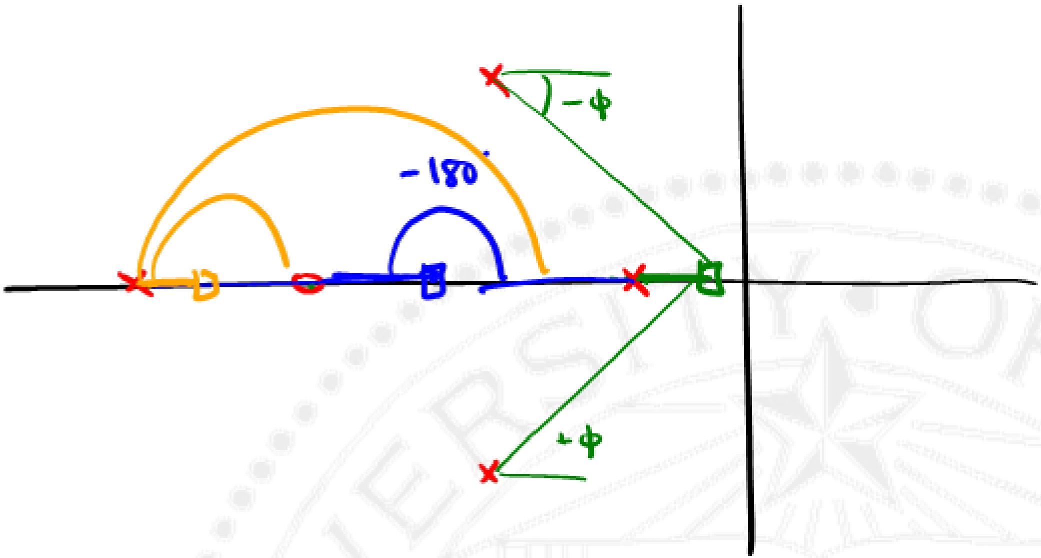


$$K\frac{N_c N}{D_c D} = -1 \qquad \measuredangle \frac{N_c N}{D_c D} = \pm 180^\circ$$

$$GK = K_0 \frac{\Pi(s - z_i)}{\Pi(s - p_i)}$$



$\measuredangle\ \phi_o - \phi_x - \phi_x$



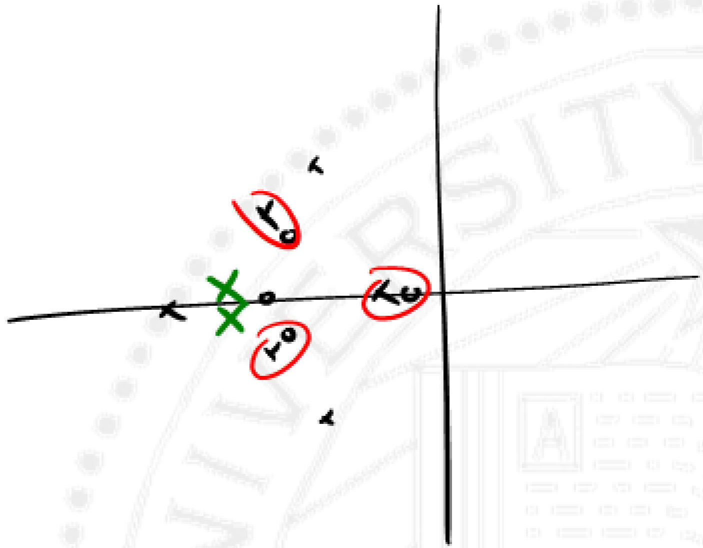
REAL AXIS: TO THE LEFT of an odd # of x's and o's ON THE LOCUS

Asymptotes

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m}$$

$$\measuredangle = \frac{180^\circ}{n-m} + \frac{360^\circ(l-1)}{n-m} \quad l = 1, 2, \ldots, n-m$$

$$(s-a_1)(s-a_2)(s-a_3) = s^3 + \underset{\Sigma}{(a_1+a_2+a_3)}s^2 + \dots \underset{\pi}{(a_1a_2a_3)}$$



$$\Delta = 1 + \frac{s^m + (\Sigma z_i) s^{m-1} + \dots}{s^n + (\Sigma p_i) s^{n-1} + \dots}$$

do long division

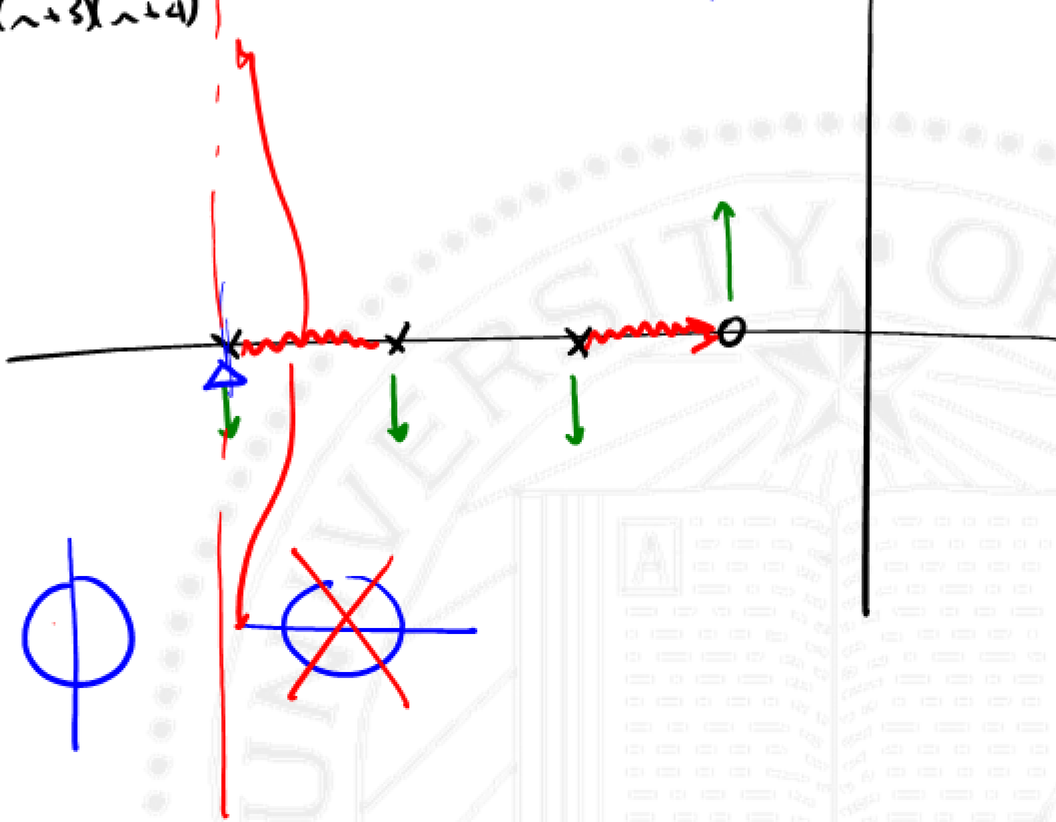
$$\Delta = 1 + \frac{k}{s^{(n-m)} + (\Sigma p_i - \Sigma z_i) s^{(n-m-1)} + \dots}$$

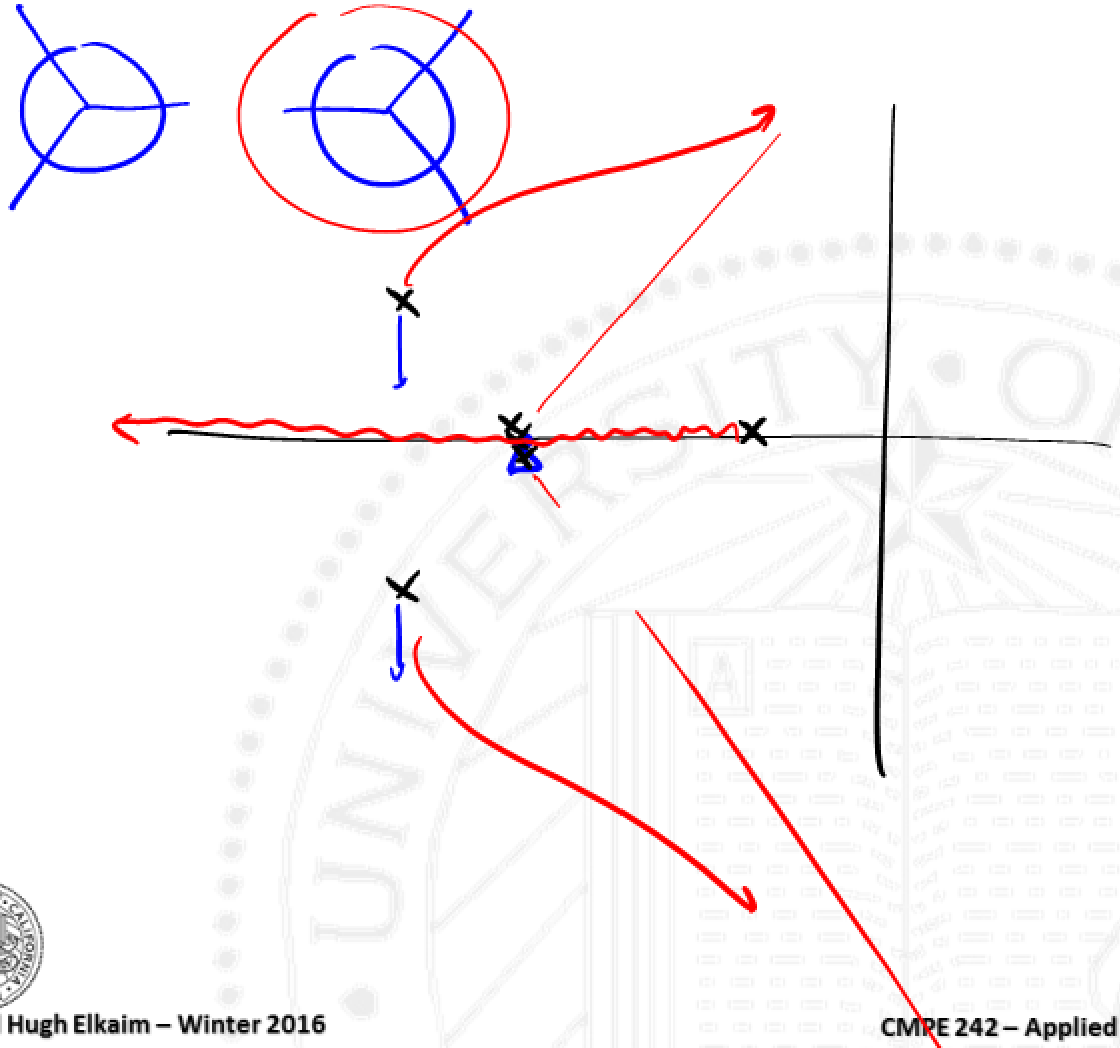
$$\frac{1}{(s-\alpha)^{n-m}} = \frac{1}{s^{(n-m)} + (n-m)\alpha}$$

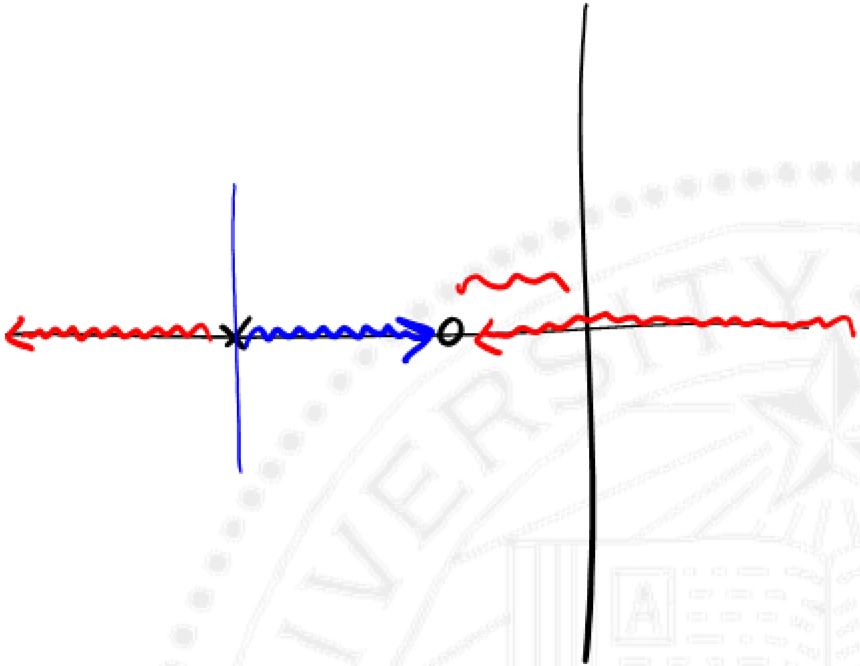
$$\boxed{\alpha = \frac{\Sigma p_i - \Sigma z_i}{n-m}}$$

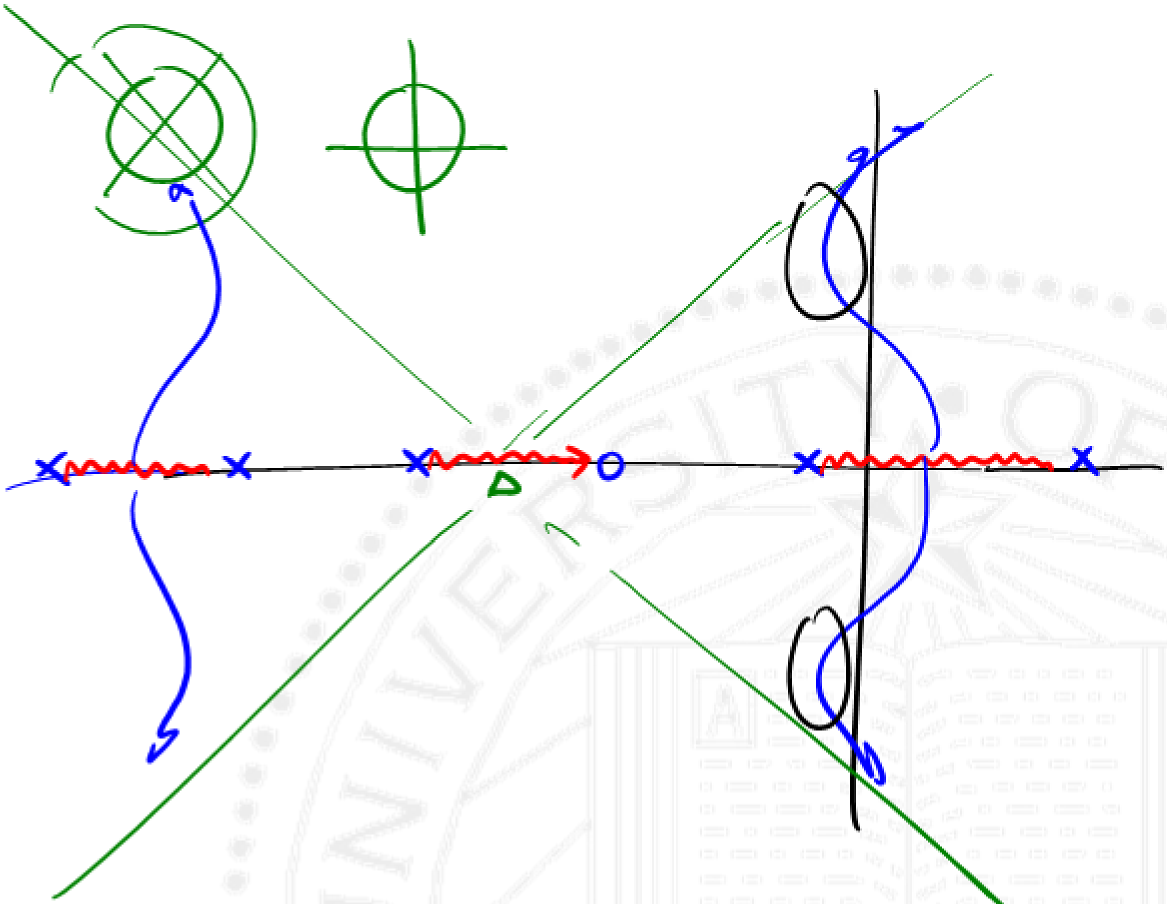
$$\frac{(s+1)}{(s+2)(s+3)(s+4)}$$

$$\alpha = \frac{-2-3-4-(-1)}{3-1} = -4$$





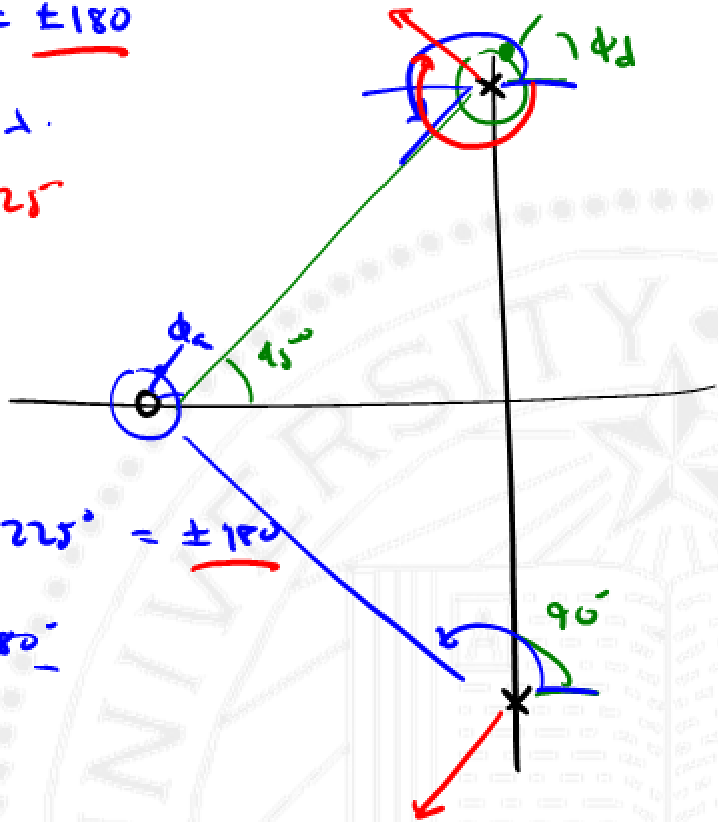




$45 - 90 - \phi_d = \pm 180°$

solve for ϕ_d.

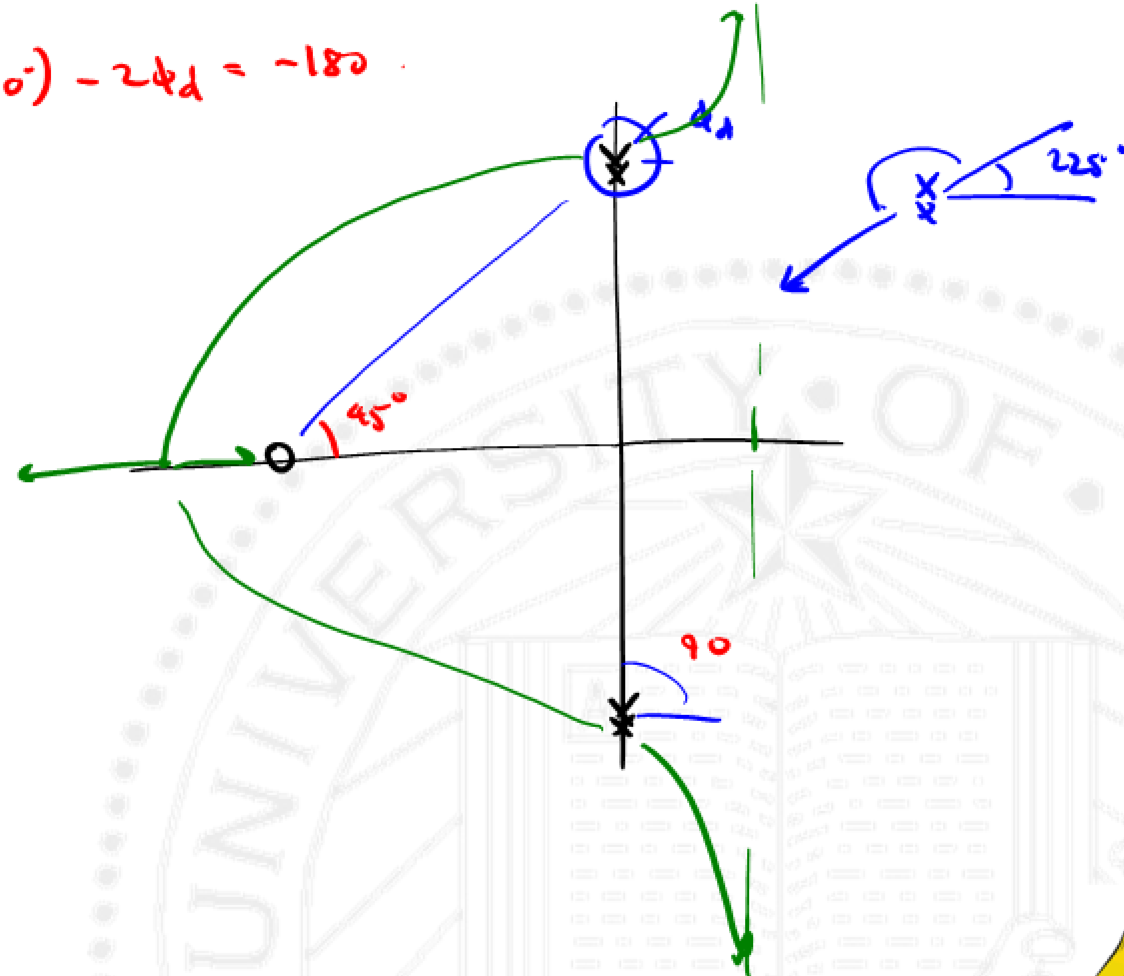
$\phi_d = -225°$



$\phi_a - 135° - 225° = \pm 180$

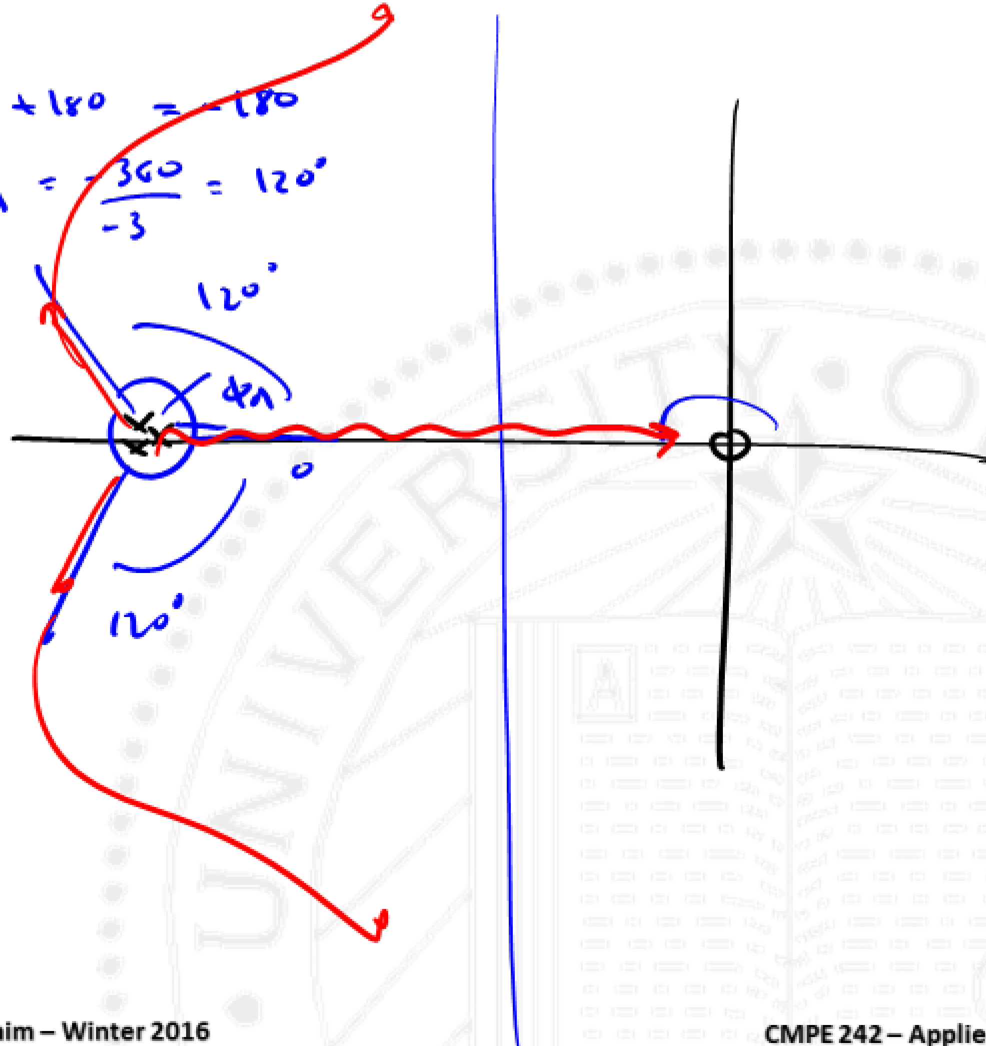
$\phi_a = 180°$

$$45 - (2 \cdot 90^\circ) - 2\phi_d = -180$$



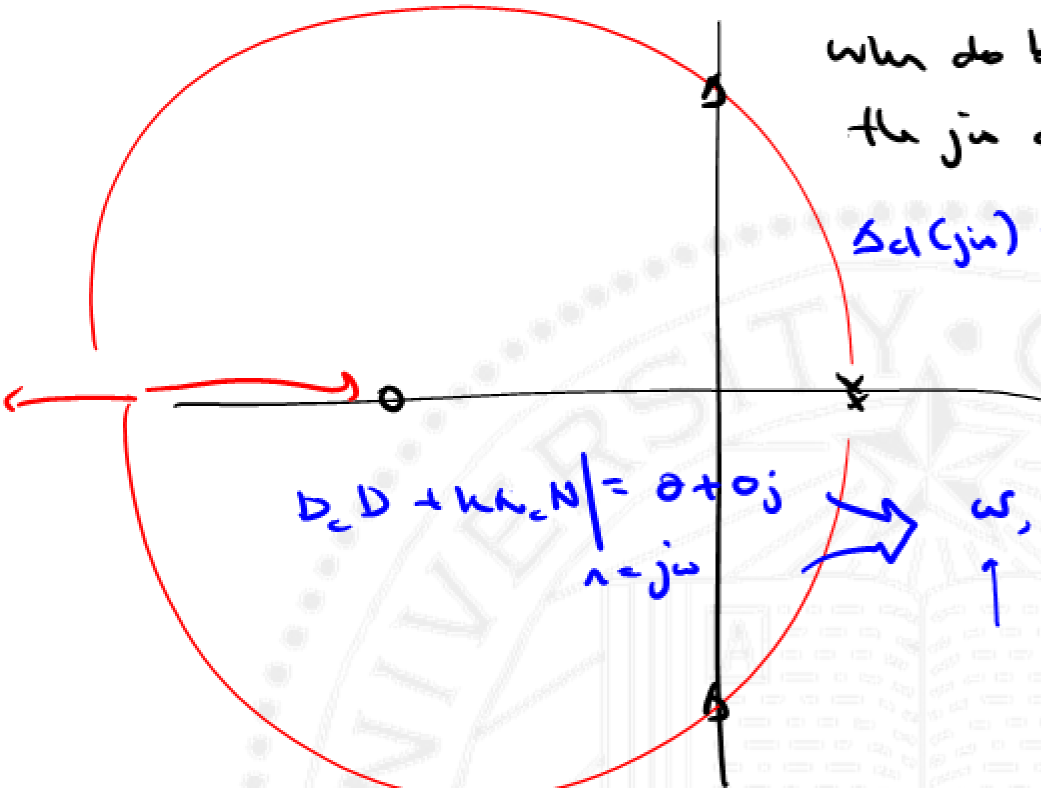
$$-3\phi_d + 180 = -180$$

$$\phi_d = \frac{-360}{-3} = 120^\circ$$



when do branches cross the $j\omega$ axis.

$\Delta_{cl}(j\omega) = \phi$.



$$D_c D + K K_c N \Big|_{s=j\omega} = 0 + 0j \Rightarrow \omega, K$$

Rule # 5

$j\omega$ crossing $\longrightarrow$ $\Delta_{cl}(j\omega) = 0 + 0j$

solve for ω; K.

Rule # 6

Break-in & Break-out points

$$\Delta_{cl}(s) = 1 + K\frac{b(s)}{a(s)}$$

$$b(s)\frac{da}{ds} + a(s)\frac{db}{ds} = \phi$$ solve for s.

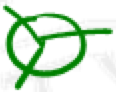
USE MATLAB.

"Root Locus Rules (180°)

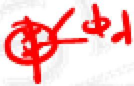
(1) x → o

(2) Real Axis: Left of odd #

(3) Asymptotes: $\alpha = \dfrac{\sum p_i - \sum z_i}{n-m}$



(4) ϕ_d, ϕ_a on test point



(5) $j\omega$ crossing: $\Delta_{cl}(j\omega) = 0 + 0j$ solve for ω, K.

(6) Break-in/Break-out: use MATLAB.