CMPE-242
Applied Feedback Control

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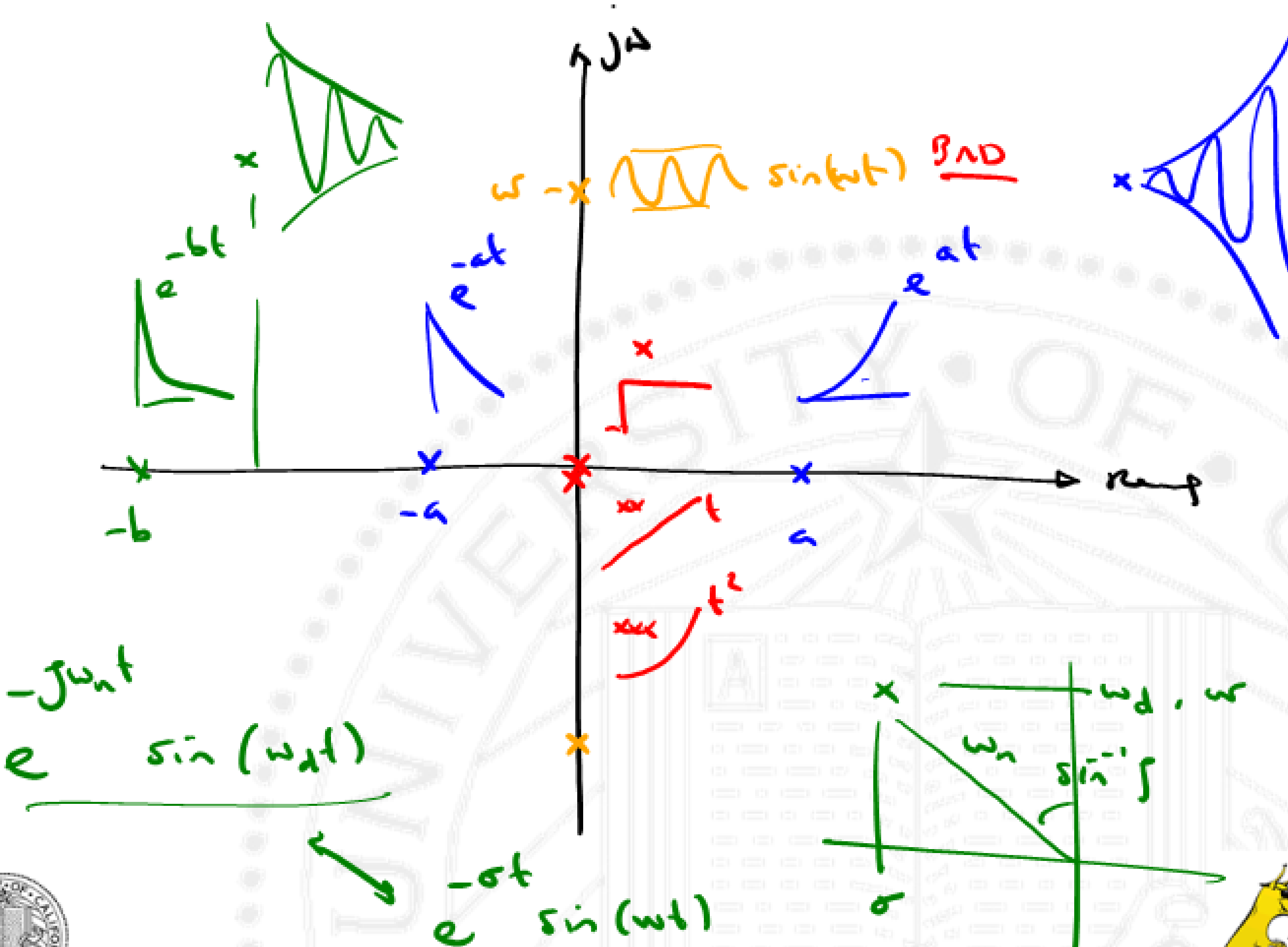
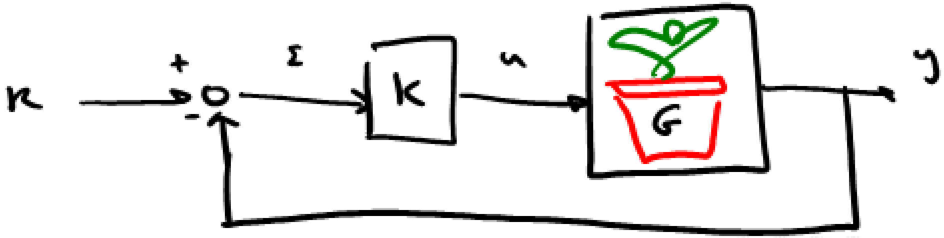
Winter 2016

Announcements:

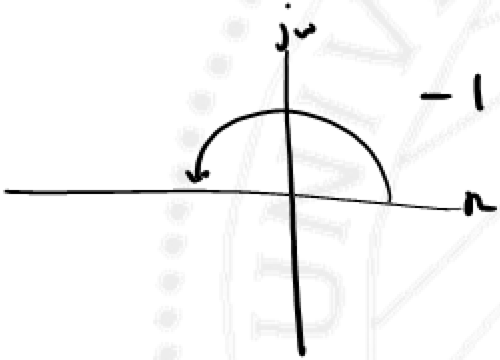
HW # 2 is posted.


jω
ω
sin(ωt)
BND
e^{-bt}
e^{-at}
e^{at}
-b
-a
a
Re
t
t^2
$e^{-\zeta\omega_n t}\sin(\omega_d t)$
$e^{-\sigma t}\sin(\omega t)$
ω_d, ω
ω_n
$\sin^{-1}\zeta$
σ



$$\frac{Y}{R} = \frac{GK}{[1+GK]} \leftarrow \Delta(s) - \text{characteristic equation}$$

Root locus: $\Delta(s) = 0 \rightarrow 1 + GK = 0 \;\therefore\; \boxed{GK = -1}$

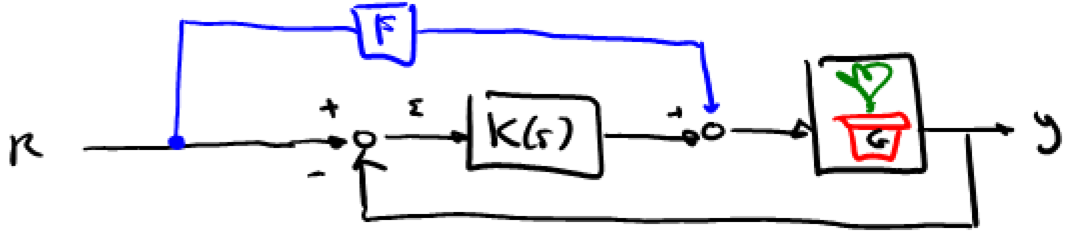


$$-1 = 1e^{\pm \pi j}$$

-1 complex #: $|\cdot| = 1$

$\angle = \pm 180^\circ$

$$K(s) = K_0 \frac{N_k(s)}{D_k(s)}$$



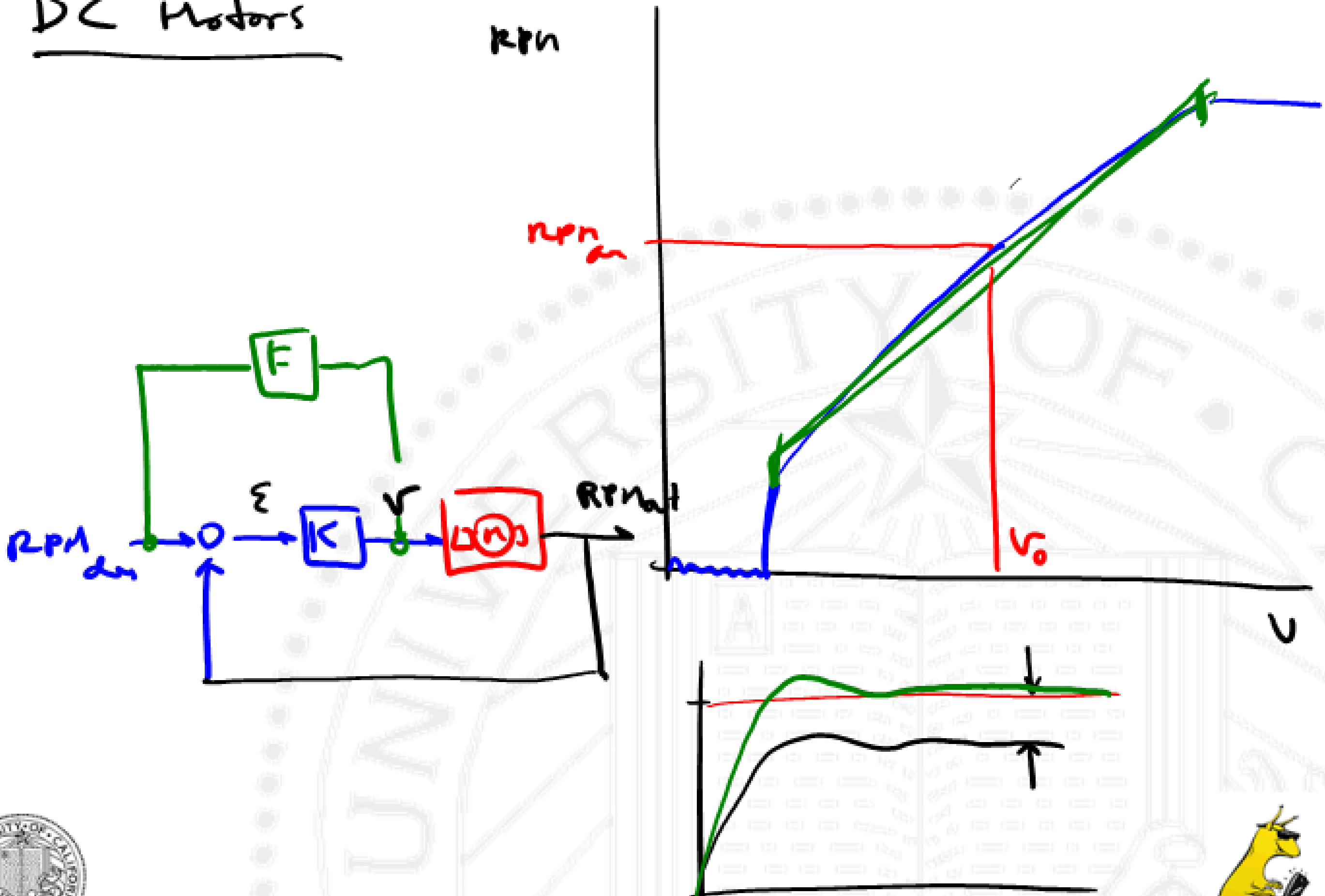
$$F = G^{-1}$$

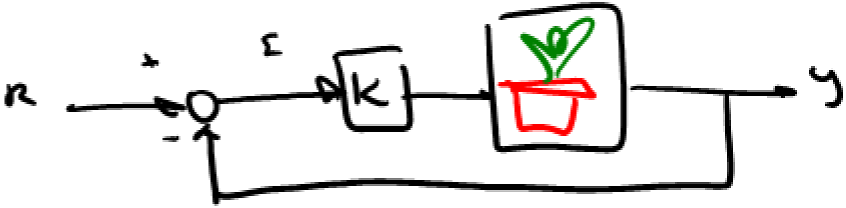
$$K(s) = \frac{N_K}{D_K} \qquad G(s) = \frac{N_G}{D_G} \qquad \frac{Y}{R} = \frac{N_K N_G}{N_K N_G + D_G D_K} \leftarrow \Delta(s)$$

$$\frac{Y}{R} = \frac{N_G N_K + F N_G D_K}{N_G N_K + D_G D_K} \leftarrow \Delta(s)$$

NO PLANT INVERSION

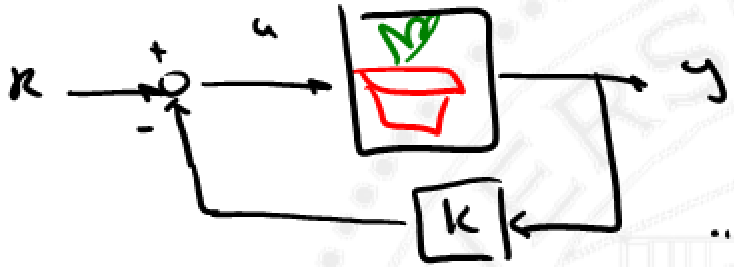
DC Motors





$$\frac{GK}{1+GK \leftarrow \Delta(s)}$$

"FULL AUTOPILOT"

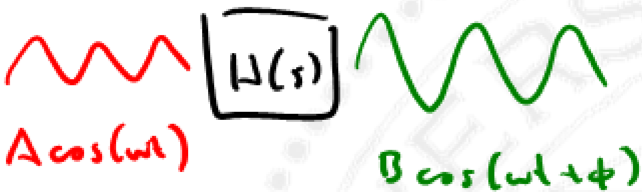


$$\frac{G}{1+GK \leftarrow \Delta(s)}$$

"STABILITY AUGMENTATION SYSTEM"

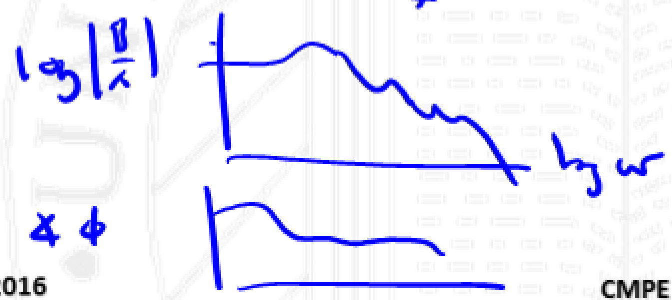
<u>Root Locus</u> — move the poles of $\Delta(s)$ to get good "transient" response.

$H(s) \leftarrow$ transfer function



$H(j\omega) \to$ complex number $<$ $|\cdot|$, $\measuredangle$

<u>Bode</u>



$$\frac{Y}{R} \simeq 1 \quad \text{for some range of } j\omega$$

"TRACKING RESPONSE"

STABILITY

BIBO — "Bounded Input / Bounded Output"

$|u(t)| \le M < \infty$ $\longrightarrow$ $\boxed{H(s)}$ $\longrightarrow$ $|y(t)| < N < \infty$

$$H(s) = k \frac{\prod (s - z_i)}{\prod (s - p_i)}$$

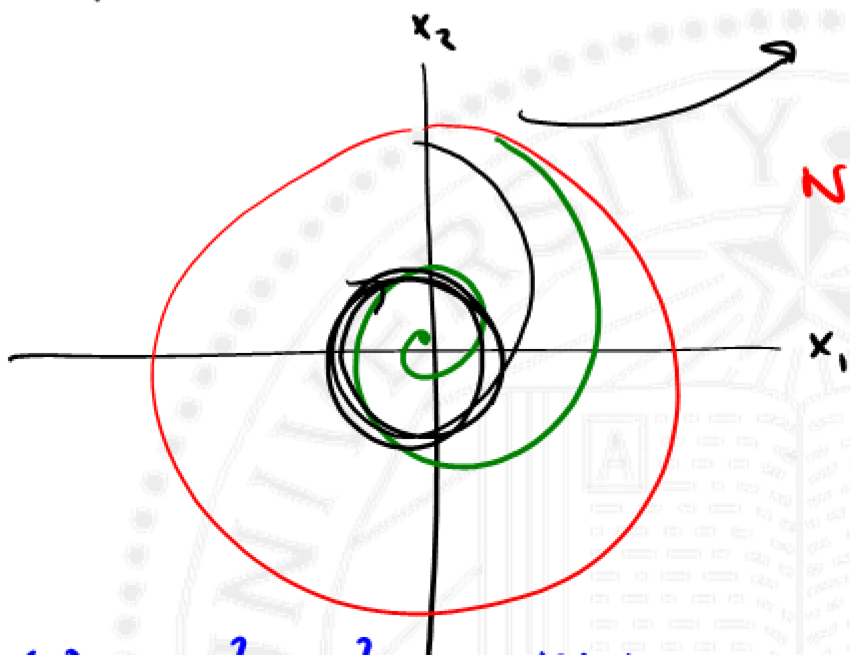
All poles in the LHP

$$\boxed{R_e(p_i) \le 0 \ \forall \ i}$$

non-repeated root on jω-axis

Lyapunov Stability

$|\underline{x}_k| < M$ $\qquad$ $|\underline{x}(t)| < N \ \forall t$ then stable

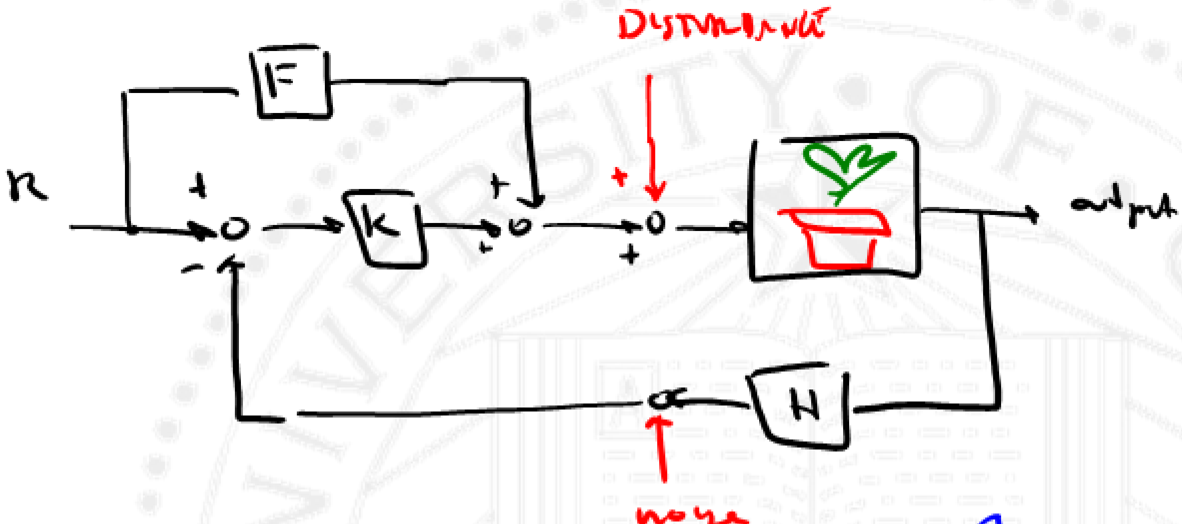


$N = \phi$ Asymptotically Stable

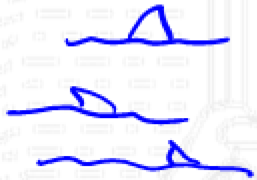
$V(x) = x_1^2 + x_2^2$ $\qquad$ $\frac{dV(x)}{dt} < \phi$.

Root locus – TRANSIENT "KICK"

BODE – $H(j\omega) <^{|H|}_{\phi}$ "TRACKING"



BE WARY OF LARGE GAINS



(1) STABILITY — all poles in LHP, Lyapunov, BIBO

(2) Overshoot
Rise Time
Steady State
Damping
} TRANSIENT SPEC'S — ROOT LOCUS

(3) Bandwidth
Disturbance Rejection
Steady State Error
} TRACKING RESPONSE — BODE

(4) Gain Margin
Phase Margin
} ROBUSTNESS SPEC'S — NYQUIST

(5) Control Effort vs. Performance } cost trade-offs

LQR $J = \int y^2 + u^2$

"simulation"

x — position

$\frac{dx}{dt}$ — velocity

x^5

$\frac{d^2x}{dt}$ — acceleration

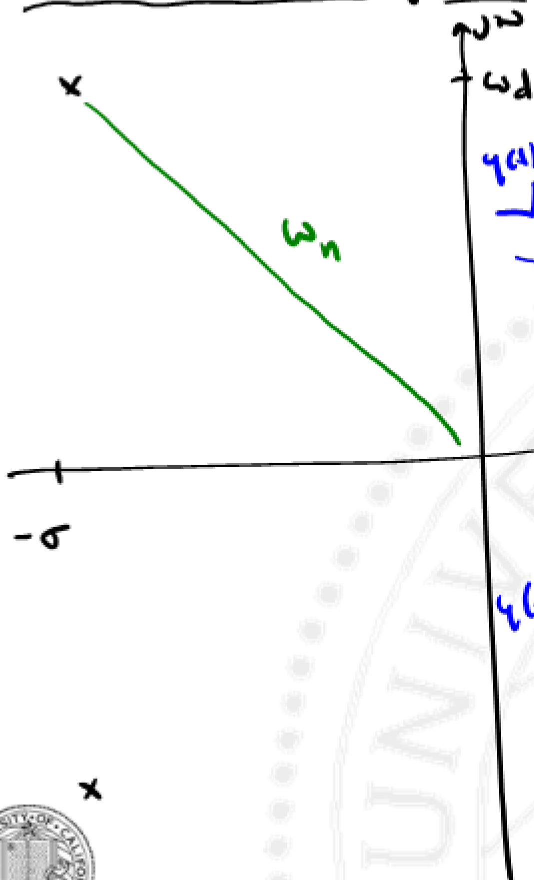
$\frac{d^3x}{dt}$ — jerk — "ride quality"

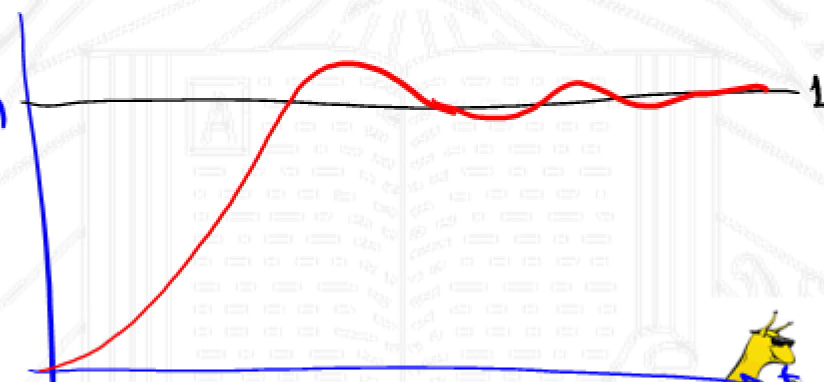
$\frac{d^4x}{dt}$ — snap — "comfort / fatigue"

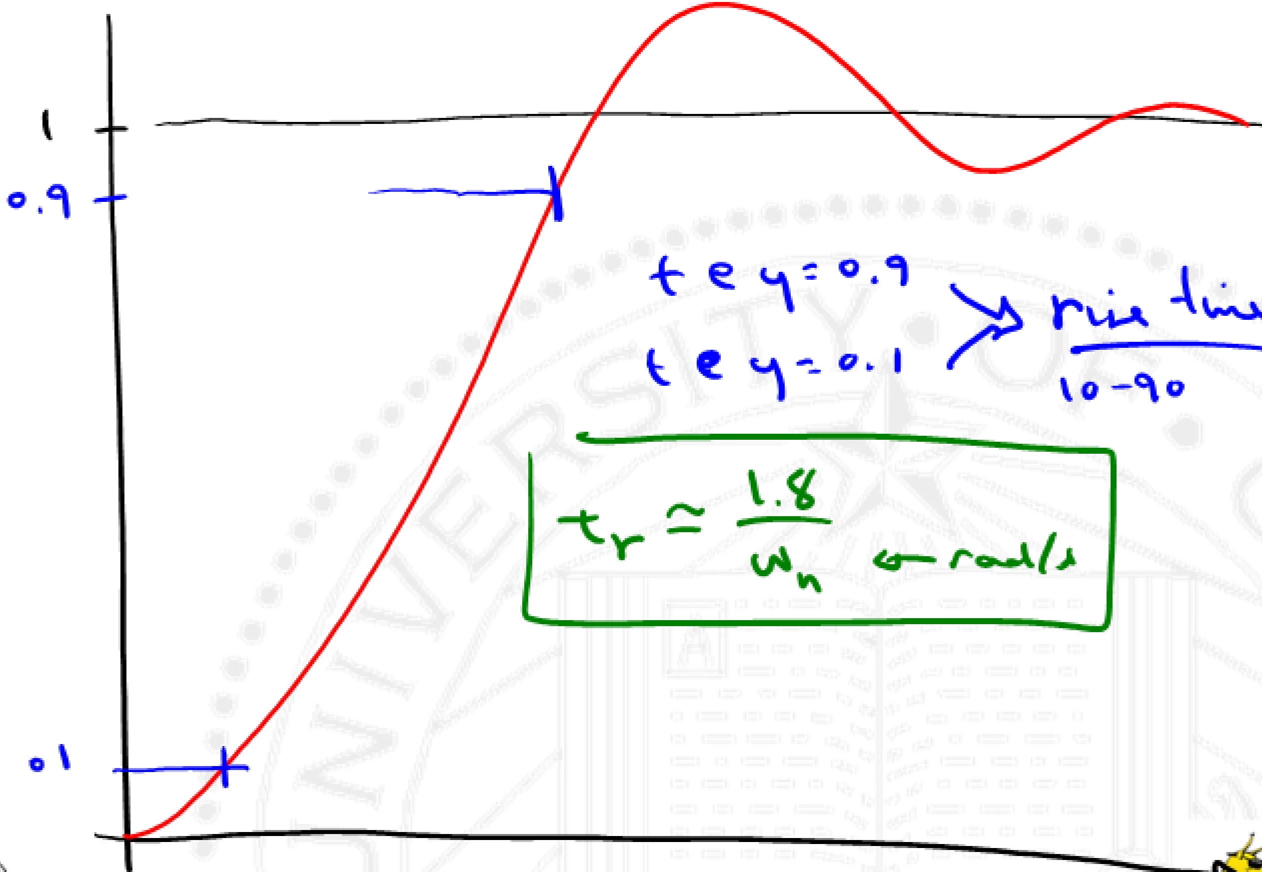
Performance Spec's

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \text{DC gain} = 1$$

$$\frac{y(t)}{\text{step}} = 1 - e^{-\zeta\omega_n t}\left[\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right]$$



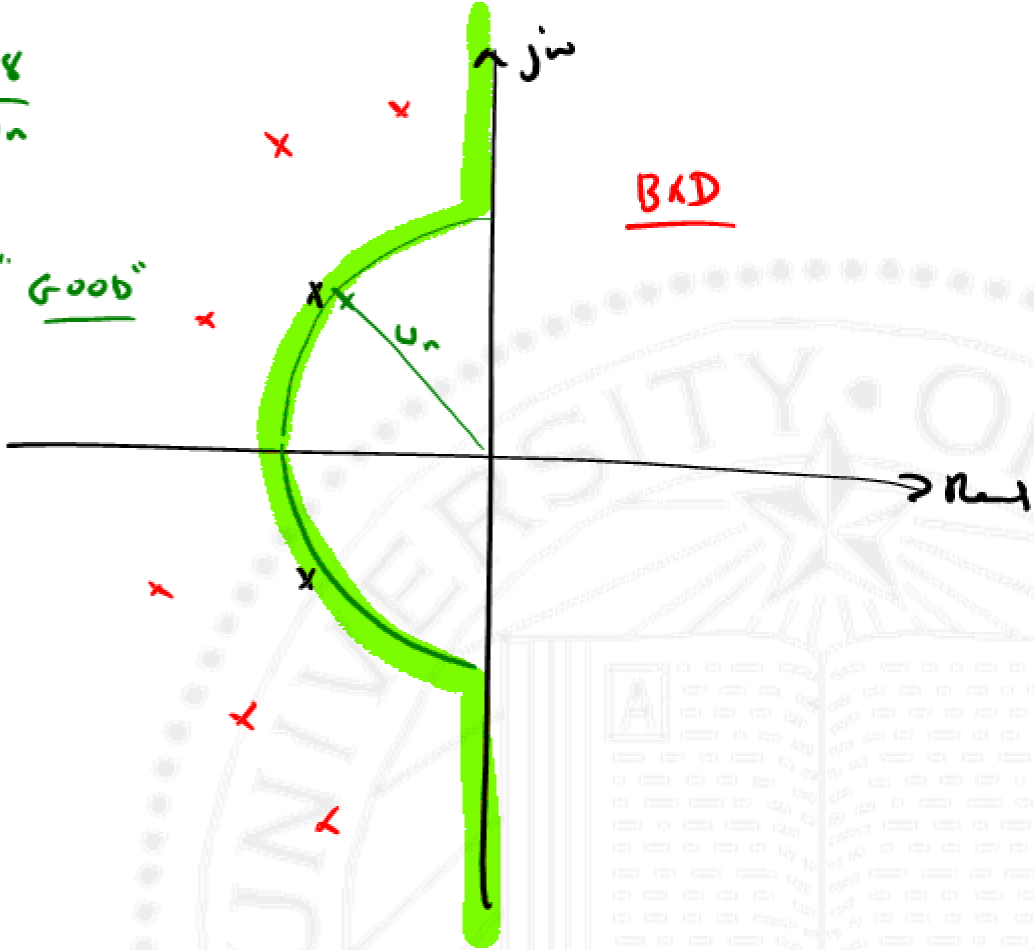


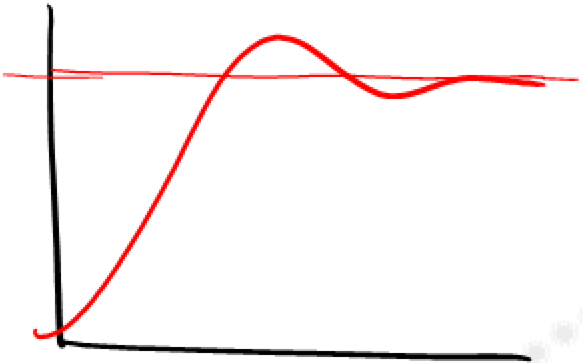


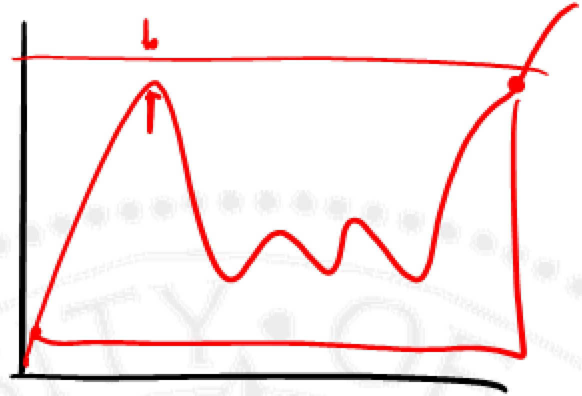
t @ $y = 0.9$
t @ $y = 0.1$ → $\underset{10-90}{\underline{\text{rise time}}}$

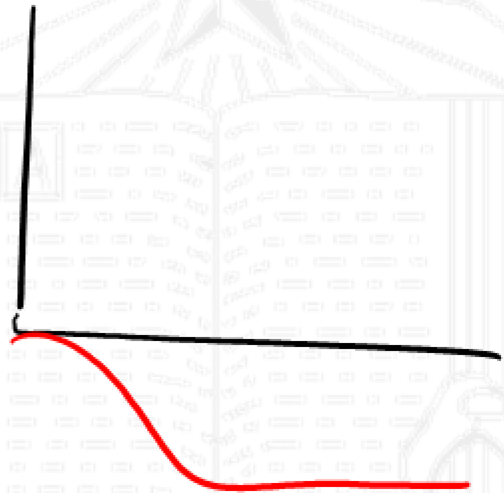
$$\boxed{t_r \simeq \frac{1.8}{\omega_n} \leftarrow \text{rad/s}}$$

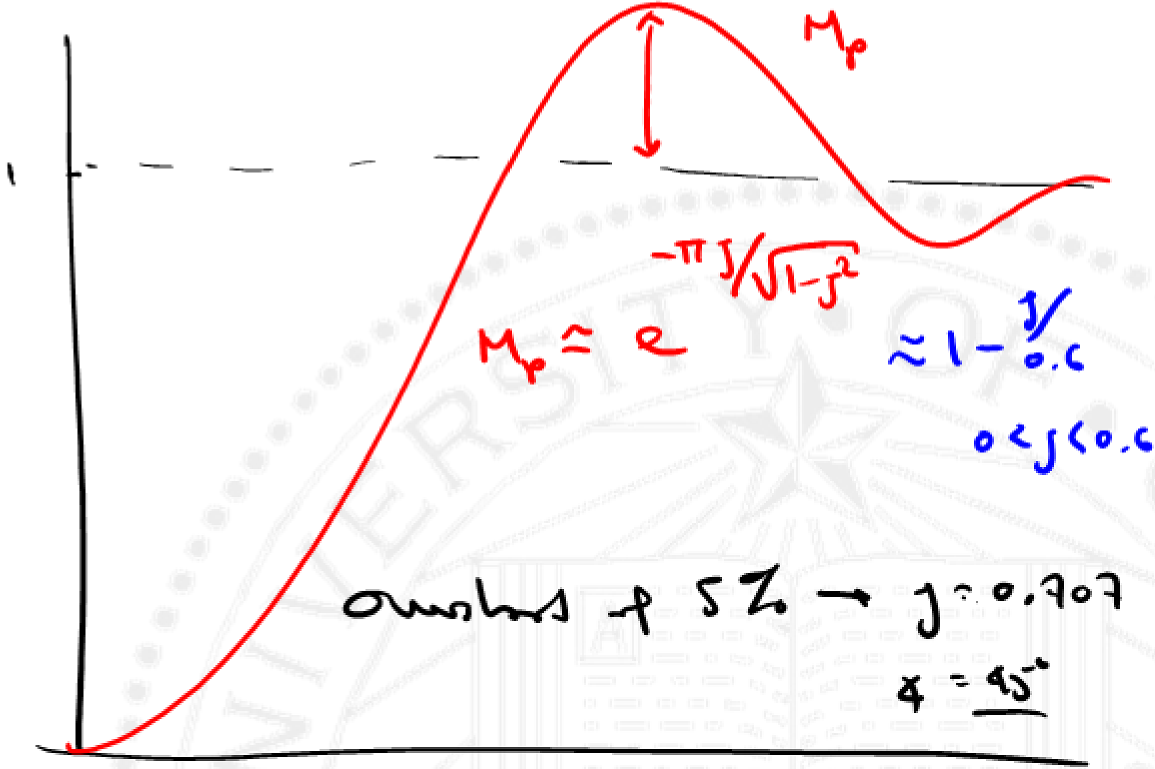
$T_r \approx \frac{1.8}{\omega_n}$

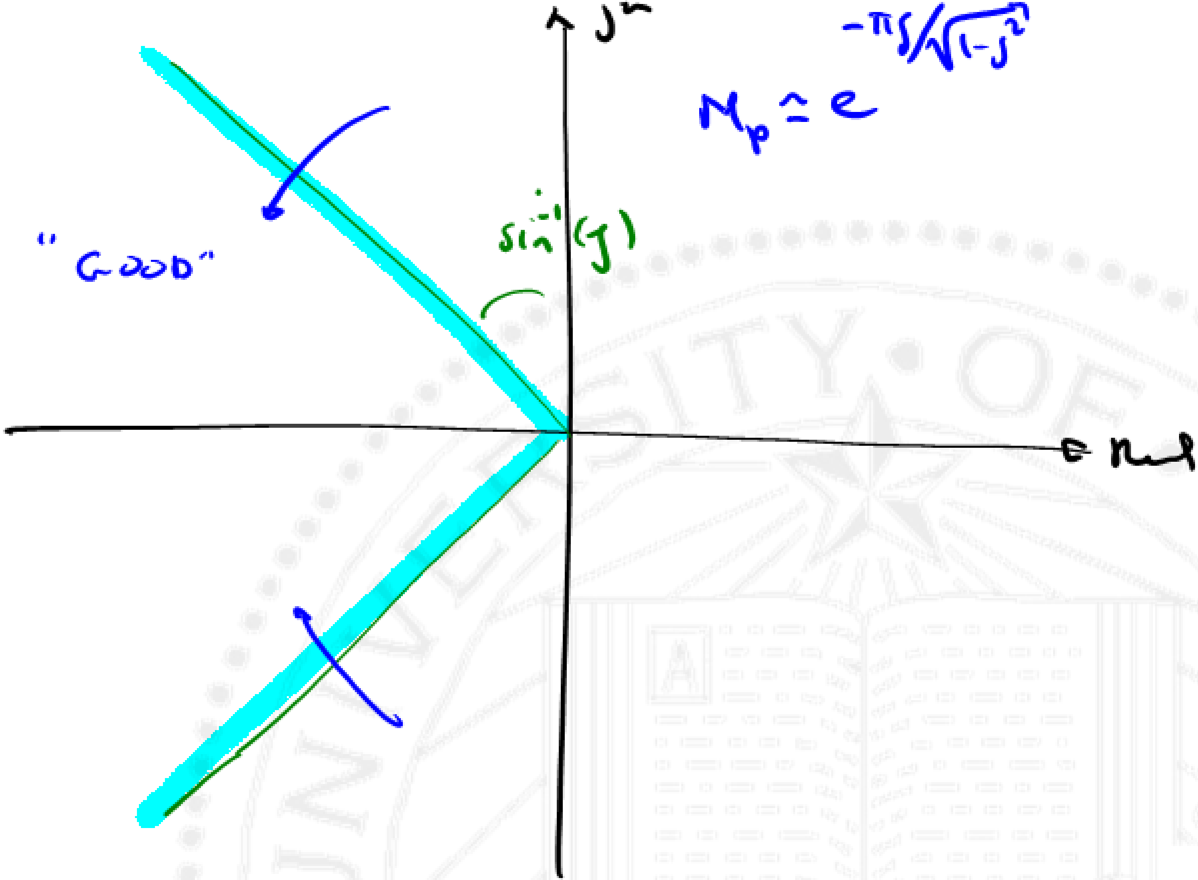




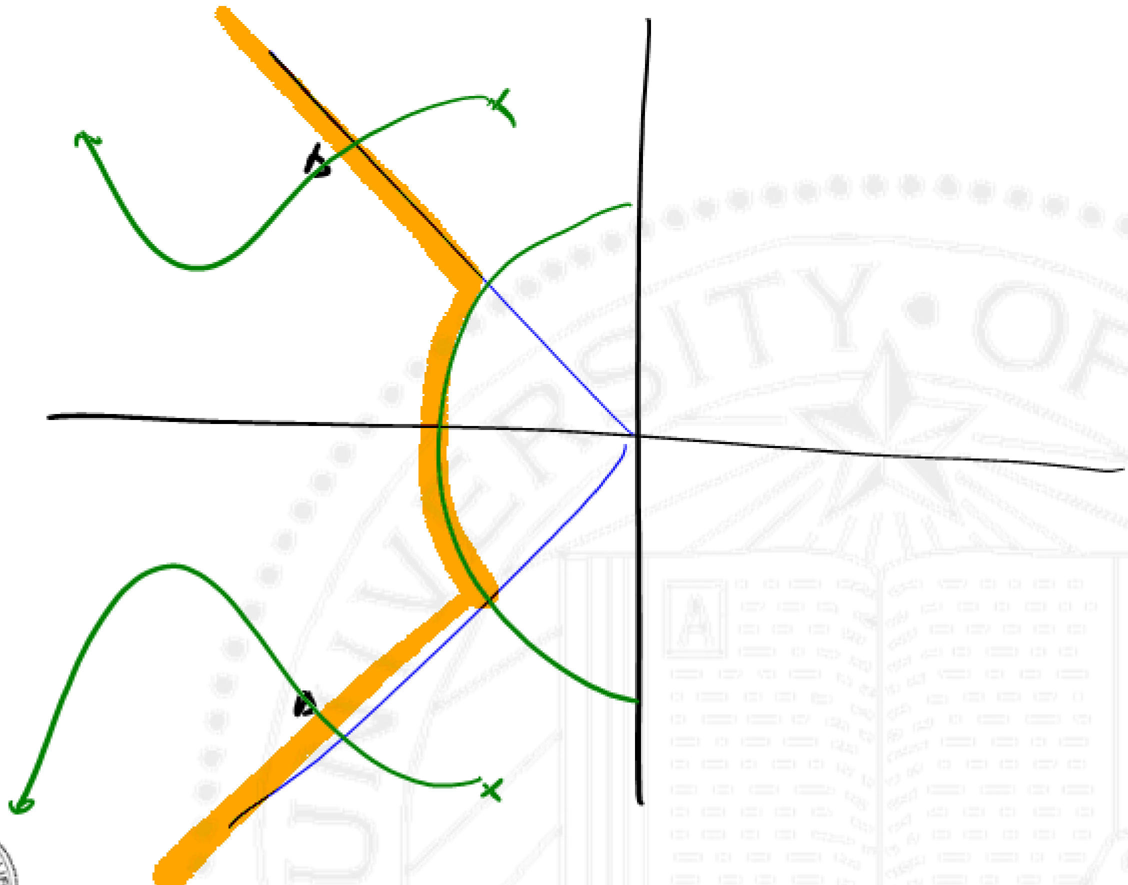


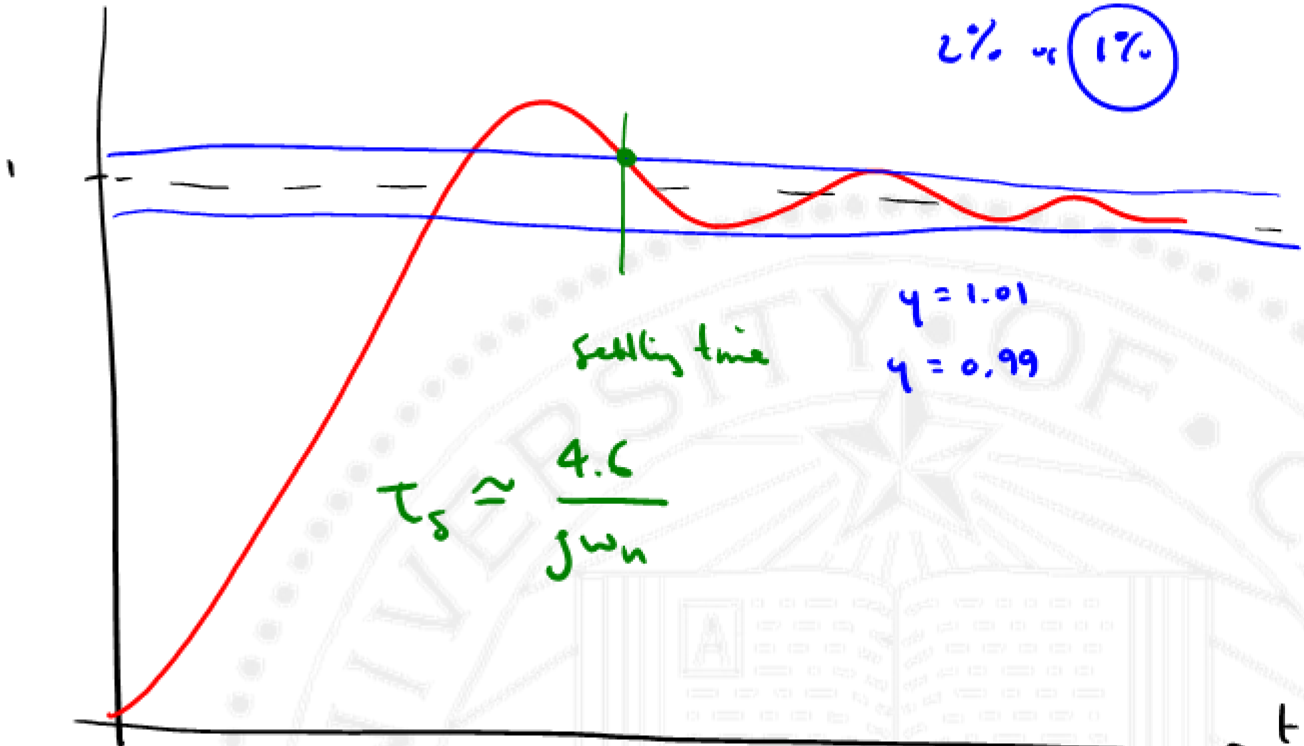



1
M_p
$M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
$\approx 1 - \zeta/0.6$
$0 < \zeta < 0.6$
overshoot of 5% → $\zeta = 0.707$
$\measuredangle = 45°$

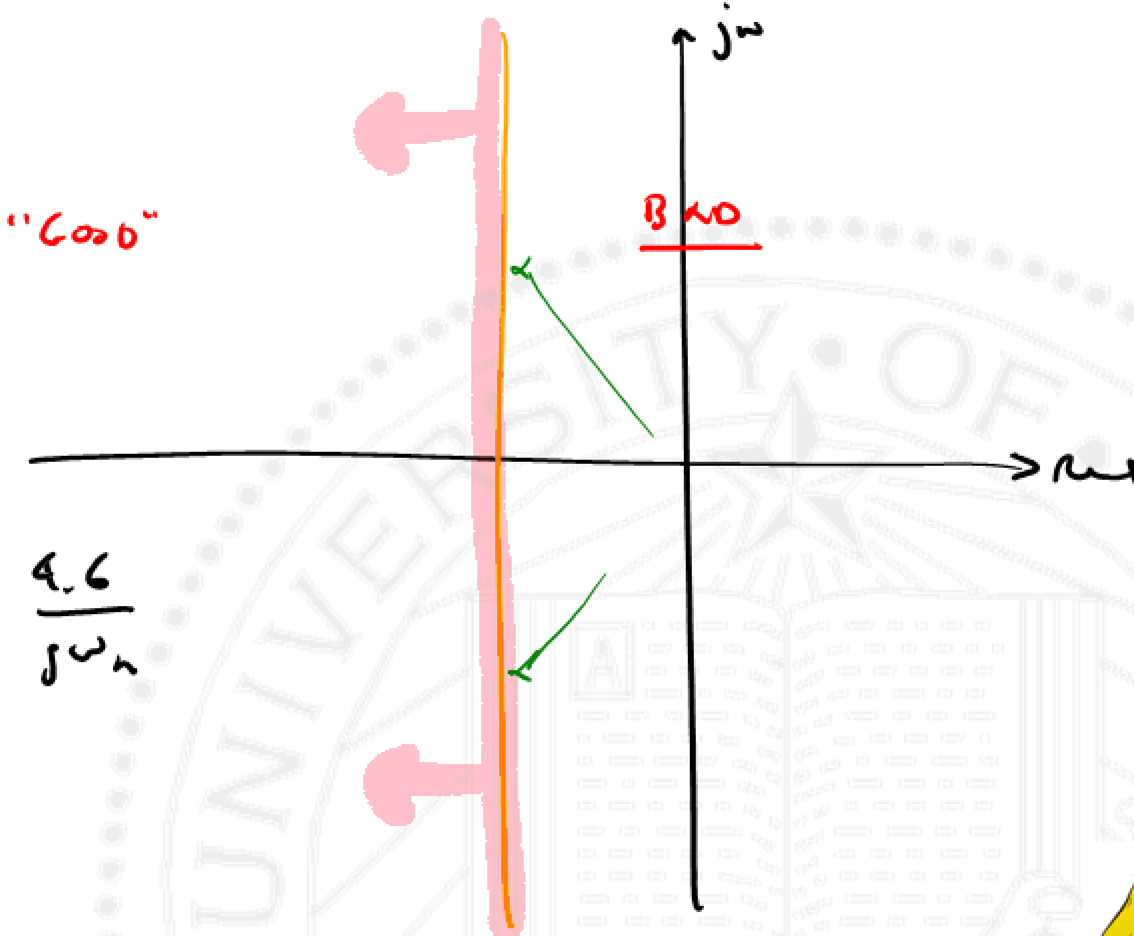


$$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

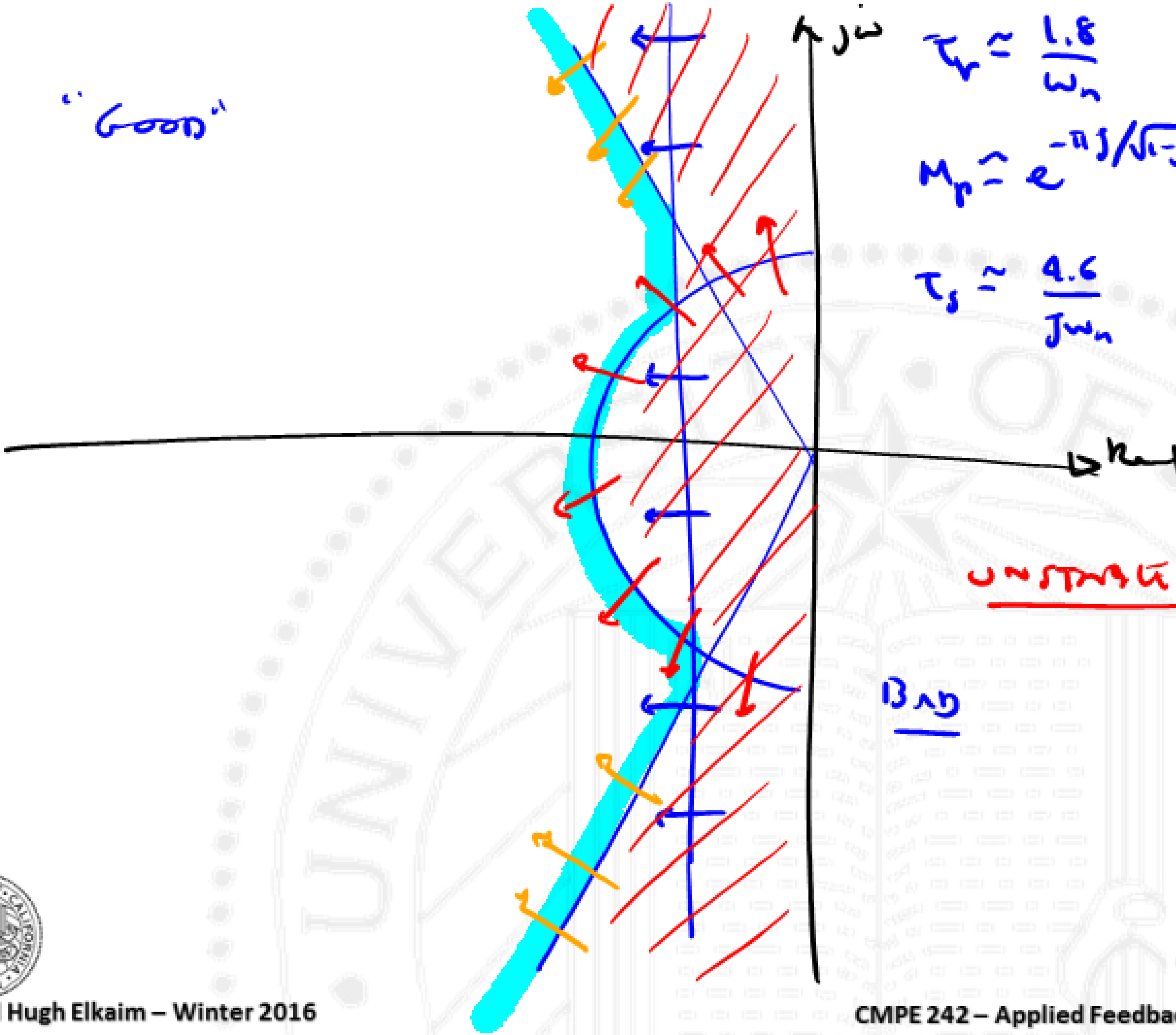




$$t_s \approx \frac{4.6}{\zeta \omega_n}$$



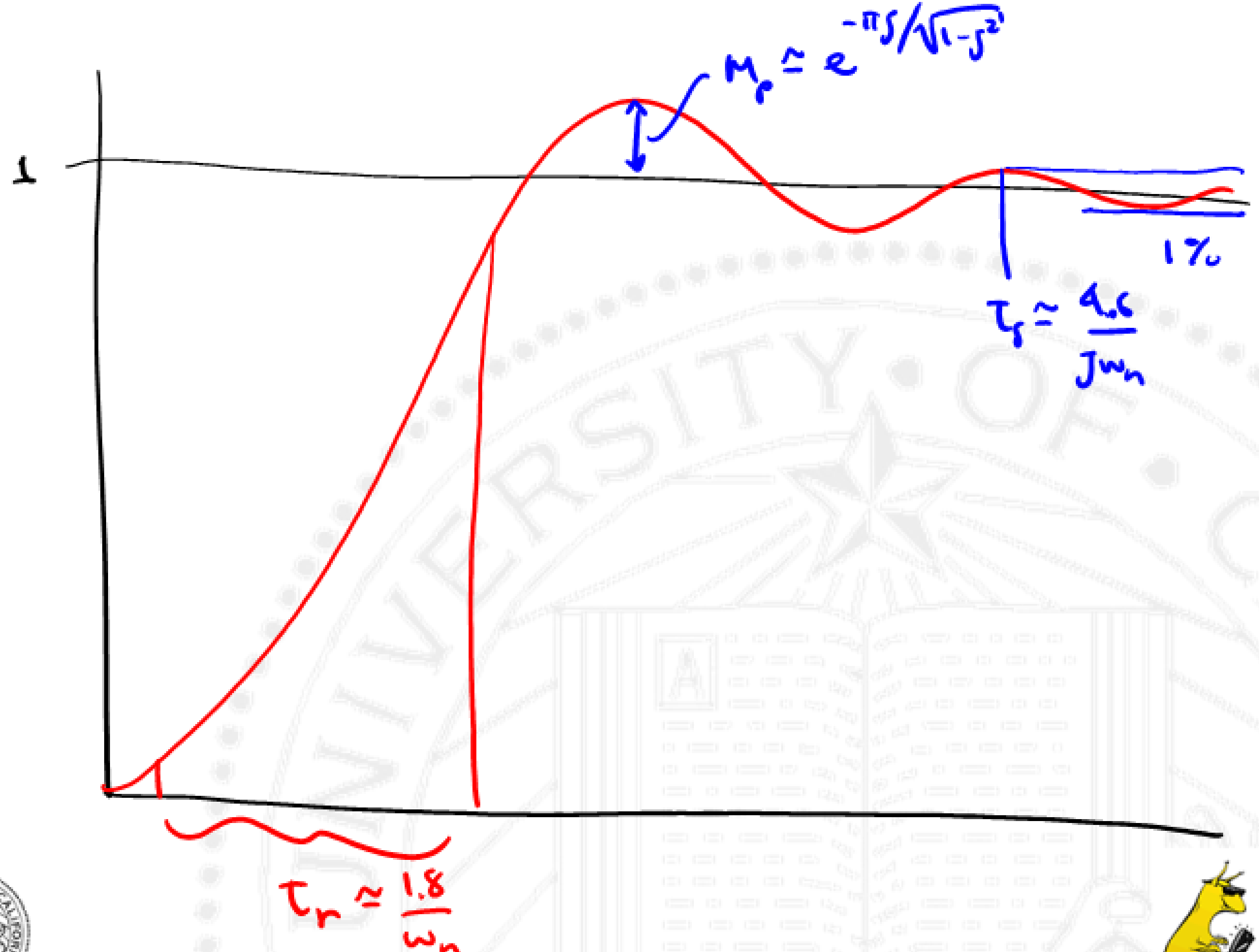
$$T_s \approx \frac{4.6}{\zeta\omega_n}$$

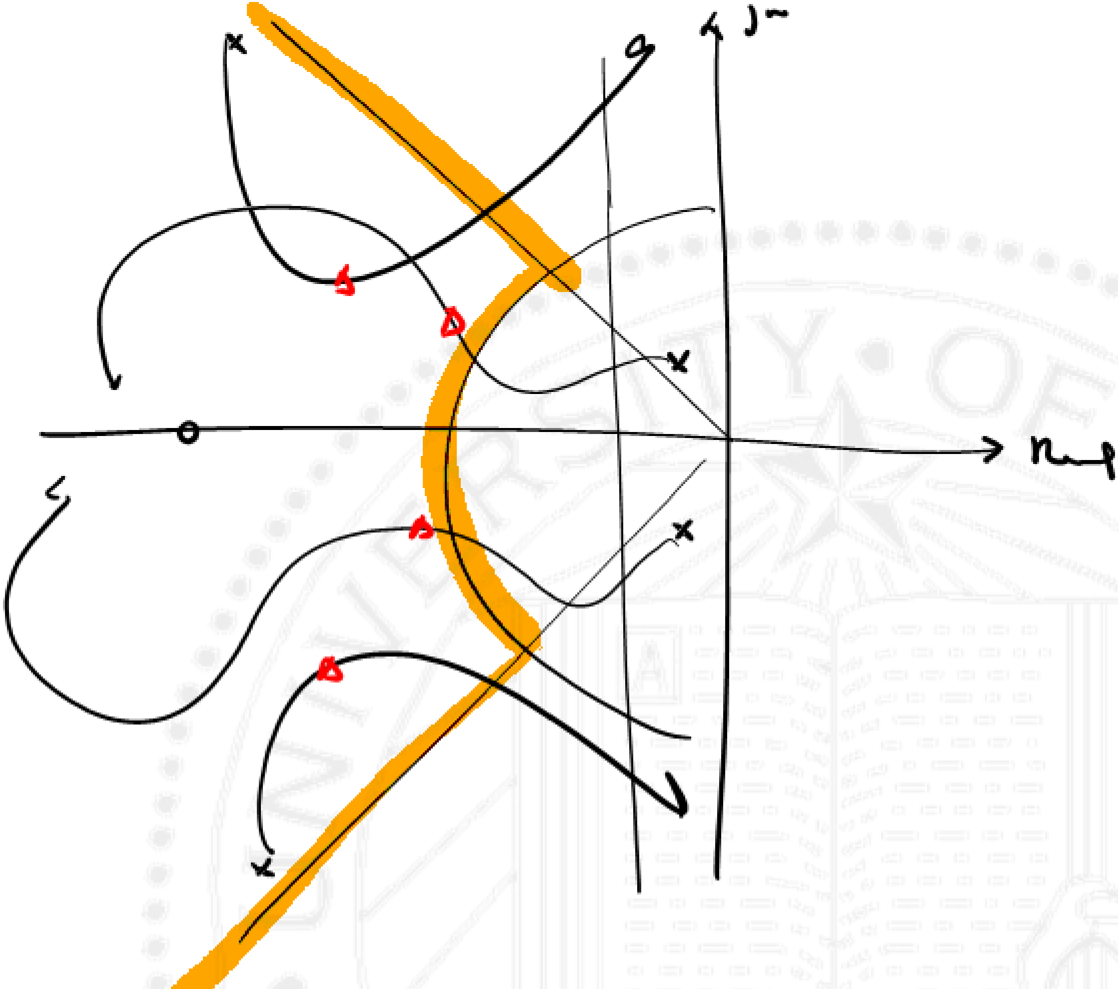


$$\tau_r \simeq \frac{1.8}{\omega_n}$$

$$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$\tau_s \simeq \frac{4.6}{\zeta\omega_n}$$

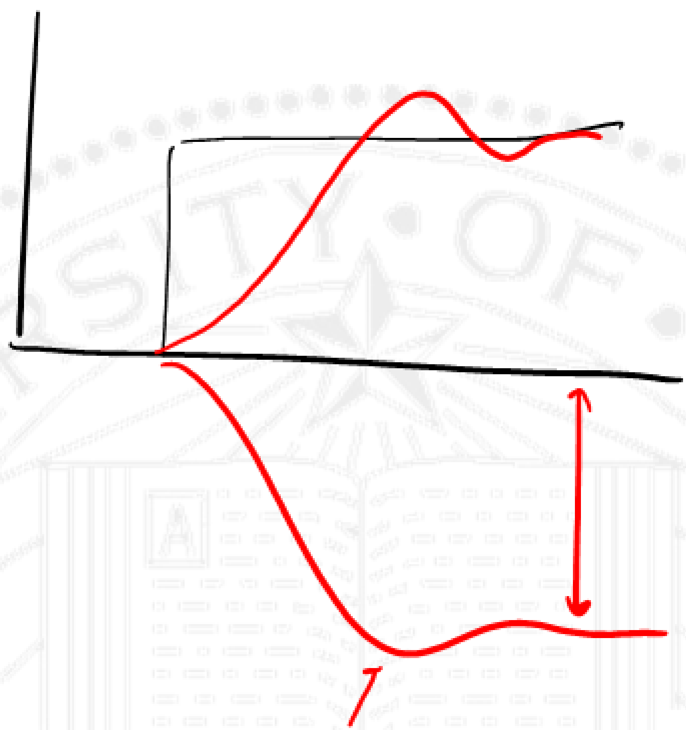

1
$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
1%
$t_s \simeq \frac{4.6}{\zeta\omega_n}$
$t_r \simeq \frac{1.8}{\omega_n}$

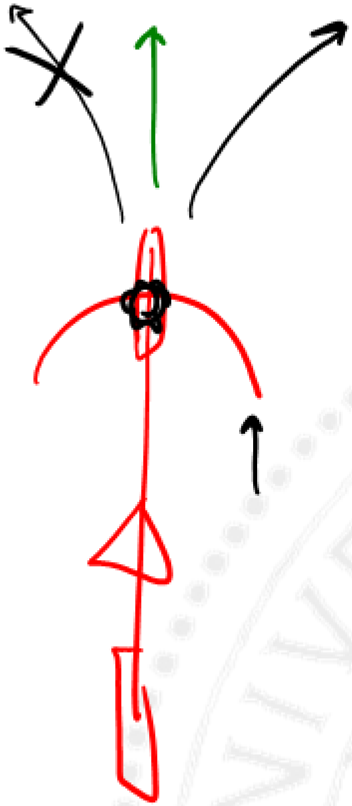


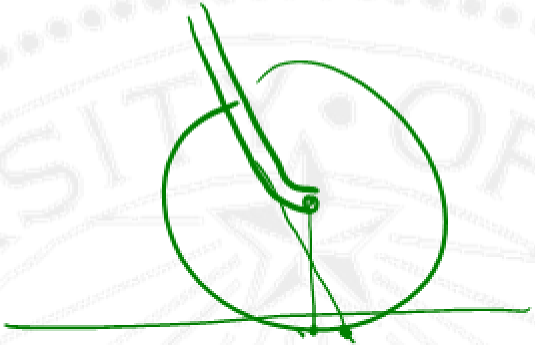
jω
Re

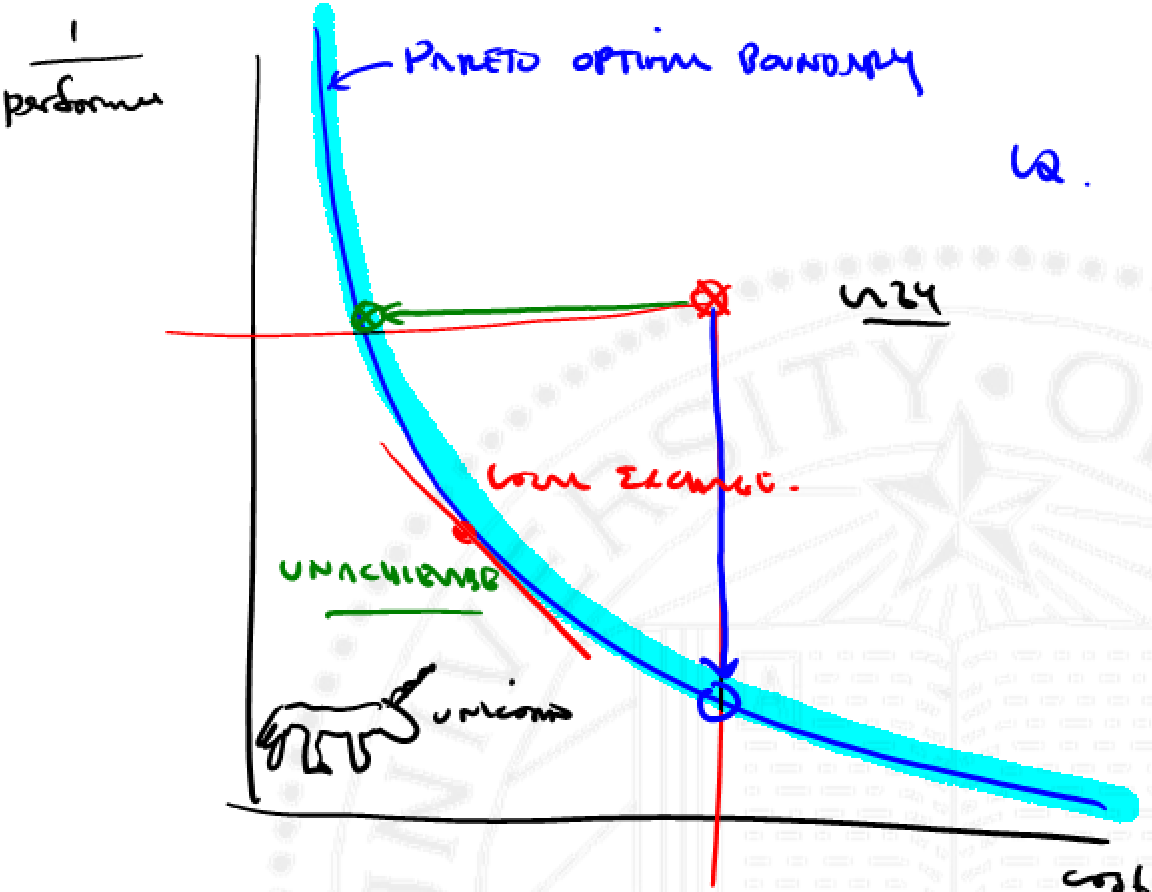
Poles of the system $\quad 1+GK=0 \;.\; - \Delta(s)$

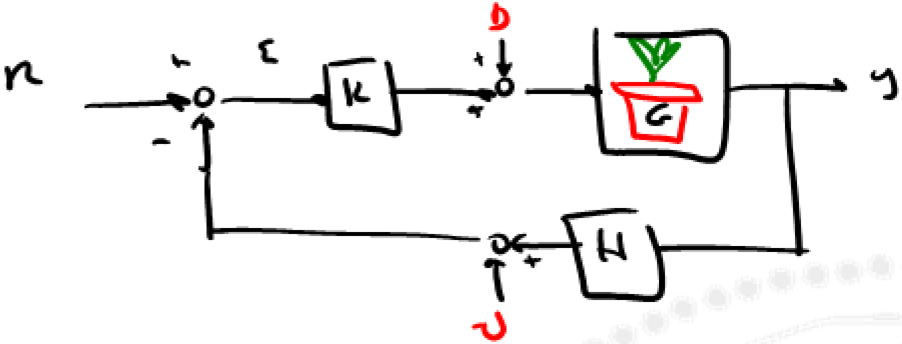
Zeros of the system








1
performance
PARETO OPTIMUM BOUNDARY
LQ.
UNACHIEVABLE
unicorn
cost



H = 1.

$$\frac{Y}{R} = \frac{GK}{1+GK} \quad — \Delta(s) = \phi$$

$$\frac{\varepsilon}{R} = \frac{1}{1+GK}$$

$$\frac{\varepsilon_{ss}}{R} < 0.1 \quad \text{"Tracking spec"}$$

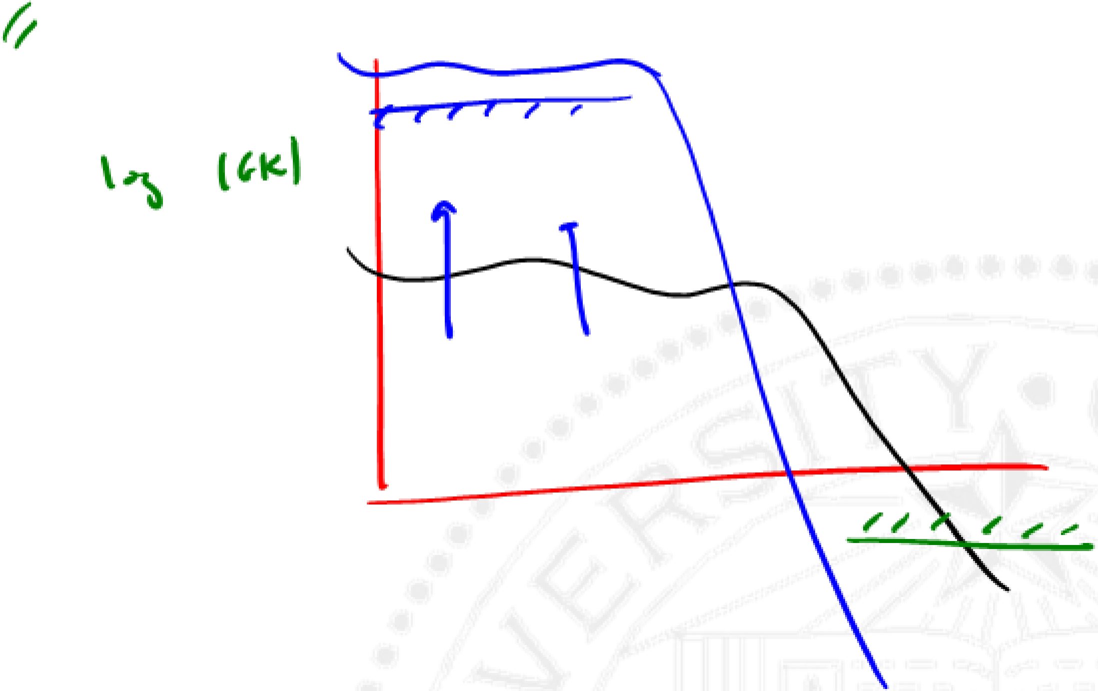
$$.1 > \frac{1}{1+GK} \rightarrow |1+GK| > 10 \quad |GK| > 11$$

$$\frac{Y}{R} = \frac{GK}{1+GK}$$ $GK \uparrow$ $|GK|$ BIG → TRACKING

$$\frac{E}{R} = \frac{1}{1+GK}$$ $GK \uparrow$ $|K|$ BIG DISTURBANCE REJECTION

$$\frac{Y}{D} = \frac{G}{1+GK}$$ $K \uparrow$

$$\frac{Y}{U} = \frac{GK}{1+GK}$$ $GK \downarrow$ GK small, good noise immunity


log |GK|