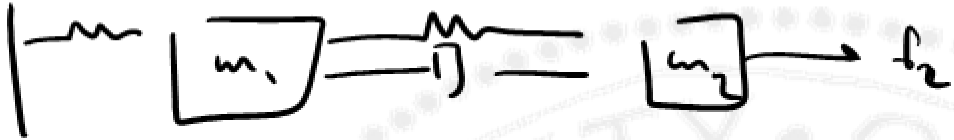
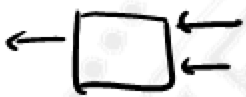
CMPE-242
Applied Feedback Control

Gabriel Hugh Elkaim

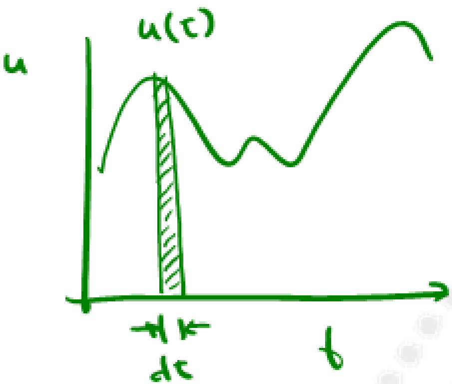
Winter 2016

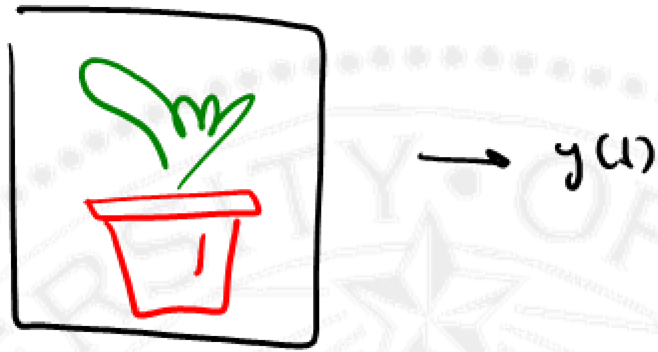
Questions?

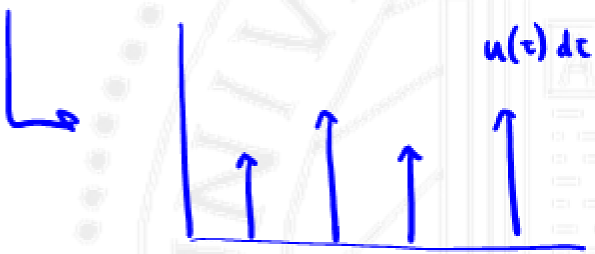


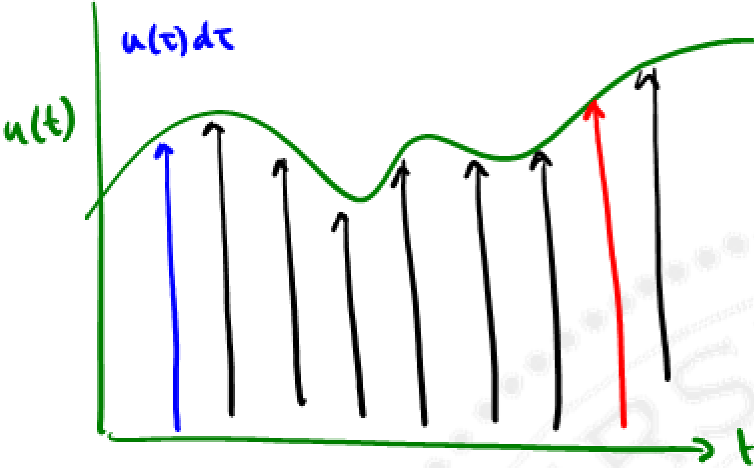


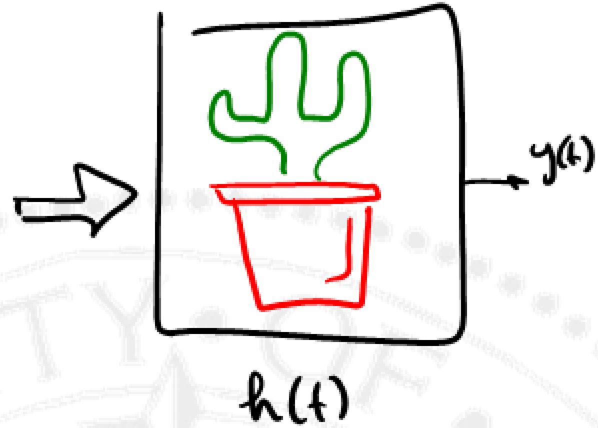
Impulse response $\triangleq h(t)$ w/ $F_a = 1$.

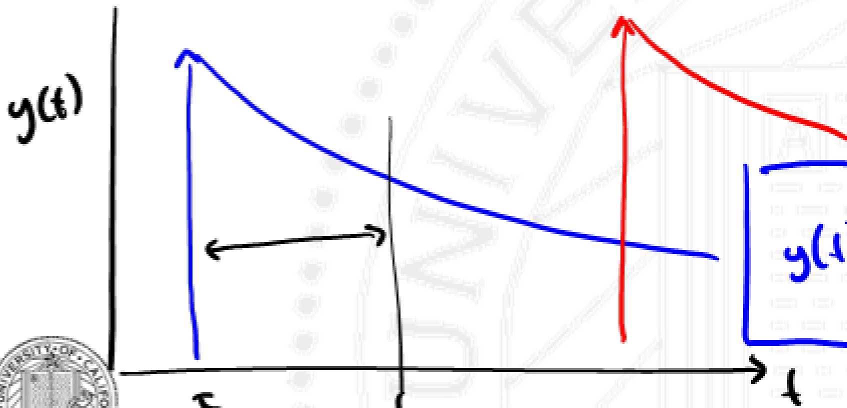












$y(t) = u(\tau)d\tau\, h(t-\tau)$

$\Sigma\, y(t)$ over all input impulses.

$$\boxed{y(t) = \int_0^t u(\tau)h(t-\tau)\,d\tau}$$

convolution integral

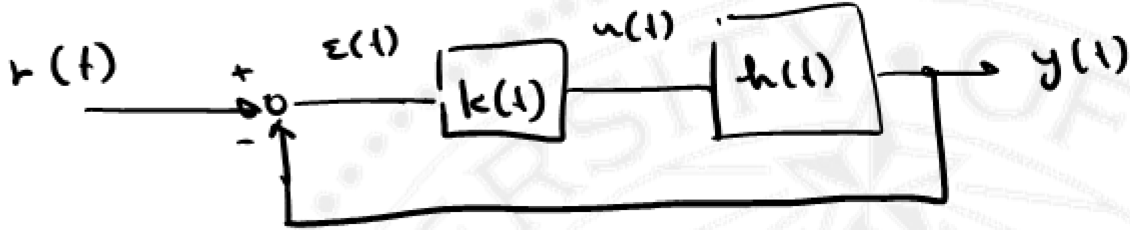
Convolution Integral

$$y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau$$

convolution

$$\downarrow$$

$$u(t) * h(t)$$



$$y(t) = h(t) * u(t)$$

$$u(t) = k(t) * \varepsilon(t)$$

$$\varepsilon(t) = r(t) - y(t)$$

$$u(t) = k(t) * [r(t) - y(t)]$$

$$y(t) = h(t) * k(t) * [r(t) - y(t)]$$

Laplace transform

$$\mathcal{L}\left[y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau \right]$$

$$\mathbb{Y}(s) = \mathbb{U}(s) \cdot \mathbb{H}(s)$$

$$\mathcal{L}\{*\} = \times$$

$$Y(s) = H(s) \cdot U(s)$$

$$U(s) = K(s) \cdot E(s)$$

$$E(s) = R(s) - Y(s)$$

$$Y(s) = H(s)\,K(s)\left[R(s) - Y(s)\right]$$

$$Y(s) + H(s)K(s)Y(s) = H(s)K(s)R(s)$$

$$\left[1 + H(s)K(s)\right]Y(s) = H(s)K(s)R(s)$$

$$Y(s) = \frac{H(s)K(s)}{\left[1 + H(s)K(s)\right]} R(s) \rightarrow \boxed{\frac{Y}{R} = \frac{KH}{1 + KH}}$$

LAPLACE

$$\mathcal{L}\{y(t)\} \triangleq Y(s) = \int_0^\infty y(t)\, e^{-st}\, dt \qquad s \text{ is complex.}$$

initial condition

$$\mathcal{L}\{\dot{y}(t)\} = sY(s) - y(0)$$

$$\mathcal{L}\{\ddot{y}(t)\} = s^2 Y(s) - s\,y(0) - \dot{y}(0)$$

$$\mathcal{L}\{f = m\ddot{x} + b\dot{x} + kx\}$$

$$F = m[s^2 X - s x_0 - \dot{x}_0] + b[sX - x_0] + kX$$

$$F = [ms^2 + bs + k]X - (ms+b)\cancelto{0}{x_0} - m\cancelto{0}{\dot{x}_0}$$

$$\frac{X}{F} = \frac{1}{ms^2 + bs + k}$$

$$Y(s) = H(s)F(s) \longrightarrow \frac{F(s)}{ms^2+bs+k}$$

(with all i.c.'s = 0)

$$\mathcal{L}\{\delta(t)\} = \int_0^\infty \delta(t)e^{-st}\,dt = 1$$

$$\mathcal{L}\{\ \} = \int_0^\infty 1e^{-st}\,dt = -\frac{e^{-st}}{s}\Big|_0^\infty = 0 - \left(-\frac{1}{s}\right) = \frac{1}{s}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^2\} = \frac{1}{s^3}$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$Y(s) = \underbrace{TF(s)}_{\text{"easy"}} R(s)$$

$$R(s) = \int_0^\infty r(t) e^{-st}\, dt \quad \text{— "easy"}$$

$$y(t) \rightarrow \mathcal{L}^{-1}\{Y(s)\} = y(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} e^{st}\, Y(s)\, ds$$

Method 0: $\mathcal{L}^{-1}\{\ \} = \frac{1}{2\pi j}\int \text{many}$ (never ever do this)

Method 1: Look it up in a table $\quad Y(s) = \frac{s}{(s+a)^2}$ #14

CRC HANDBOOK OF MATH...

$$y(t) = (1 - at)e^{-at}$$

Method 2: Break it up into parts that ARE in my table.

Partial Fraction Expansion

$$Y(s) = \frac{s^2 + as + a^2}{s(s+a)^2} = \frac{a}{s(s+a)} + \frac{s}{(s+a)^2}$$

Residues

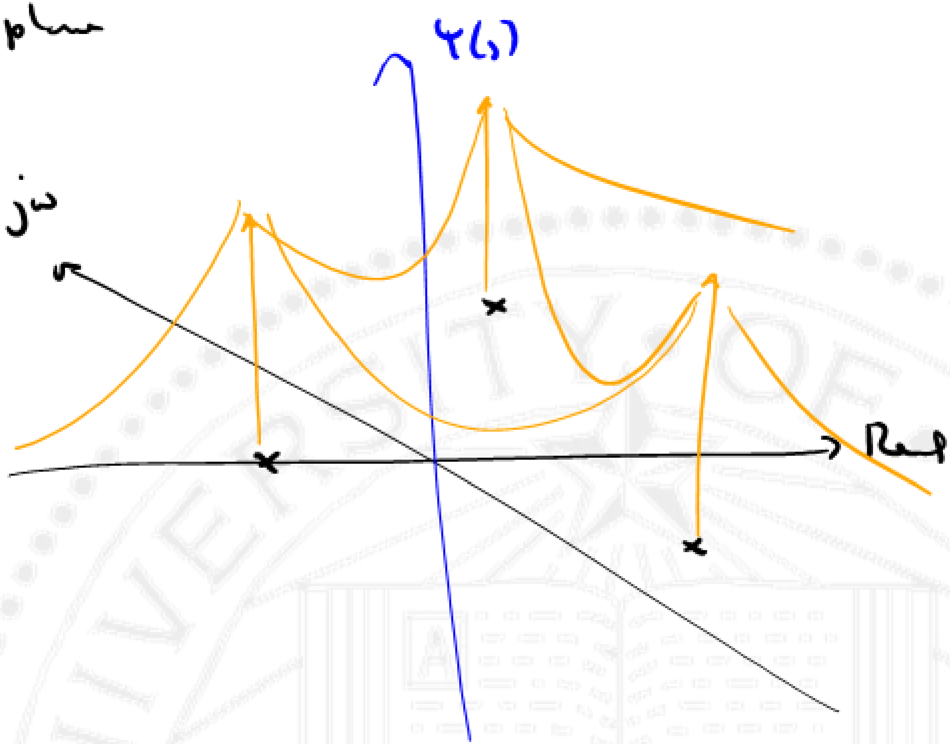
$$Y(s) = \frac{(s+d)(s+e)}{(s+a)(s+b)(s+c)} \longrightarrow \frac{A}{(s+a)} + \frac{B}{(s+b)} + \frac{C}{(s+c)}$$

System Modes

Poles

$$y(t) = Ae^{-at} + Be^{-bt} + Ce^{-ct}$$

Y(s) — on a plane



Non-Repeated Roots

$$A = (s+a)Y(s)\Big|_{s=-a}$$

$$B = (s+b)Y(s)\Big|_{s=-b}$$

$$C = (s+c)Y(s)\Big|_{s=-c}$$

$$Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

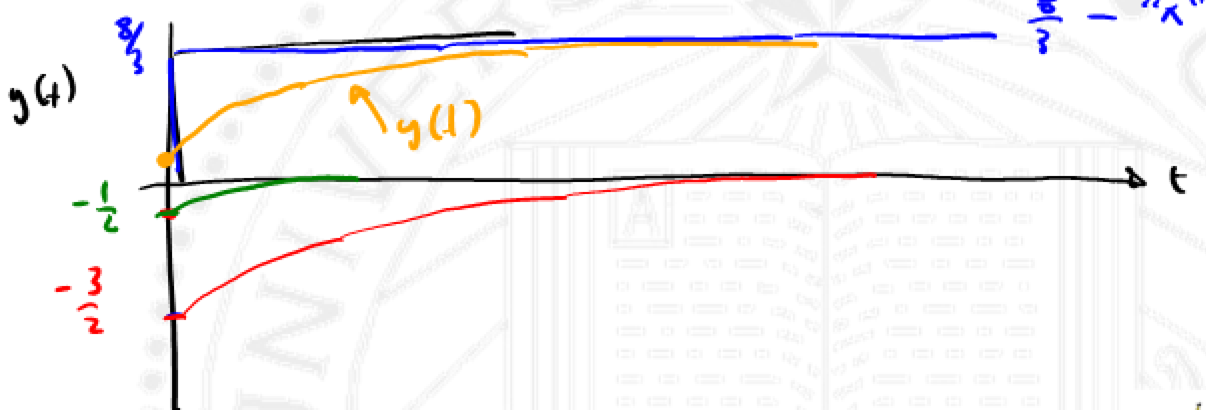
$$\underbrace{\frac{A}{s}}_{1} \quad \underbrace{\frac{B}{s+1}}_{e^{-t}} \quad \underbrace{\frac{C}{s+3}}_{e^{-3t}}$$

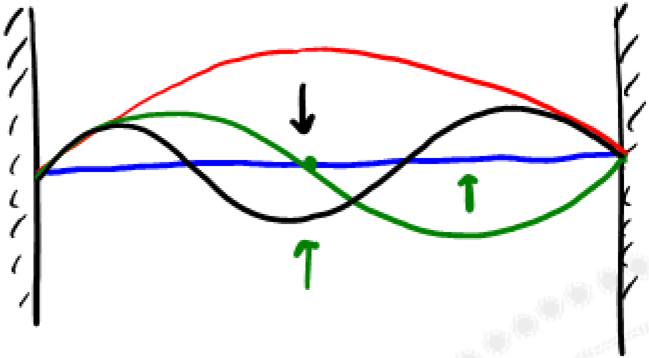
$$A = sY(s)\Big|_{s=0} = \frac{(s+2)(s+4)}{(s+1)(s+3)}\Big|_{s=0} = \frac{2\cdot 4}{1\cdot 3}$$

$$A = \frac{8}{3}$$

$$B = (s+1)Y(s)\Big|_{s=-1} = \frac{(s+2)(s+4)}{s(s+3)}\Big|_{s=-1} = \frac{1\cdot 3}{-1\cdot 2} = -\frac{3}{2} = B$$

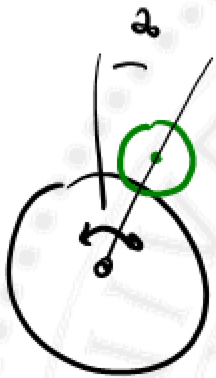
$$C = (s+3)Y(s)\Big|_{s=-3} = \frac{(s+2)(s+4)}{s(s+1)}\Big|_{s=-3} = \frac{-1}{6}$$





$\mathcal{O}$ observability – that which I can see / measure

$\mathcal{C}$ controllability – that which I can push on / move



Repeated Roots

$$Y(s) = \frac{s+3}{(s+1)(s+2)^2} = \frac{C_1}{s+1} + \frac{C_2}{s+2} + \frac{C_3}{(s+2)^2}$$

C_1 cover up.

$$\frac{C_1}{s+1} + \frac{C_2 s + C_3}{(s+2)^2}$$

easy

$$C_2 = \frac{d}{ds}\left[(s+2)^2 F(s)\right]\Big|_{s=-2}$$

$$C_3 = (s+2)^2 F(s)\Big|_{s=-2}$$

Complex Roots

$$F(s) = \frac{}{(s^2+as+b)} \longrightarrow \frac{c_1}{s+p_1} + \frac{c_1^*}{s+p_1^*} \longrightarrow \frac{[As+B]}{(s^2+as+b)}$$

$$m\dot{v} = f - bv$$

$$\underline{m\dot{v} + bv = f}$$

$$\mathcal{L}\{m\dot{v} + bv = f\} = m[sV - v_0] + bV = F$$

$$sV + \frac{b}{m}V = \frac{F}{m} + v_0$$

$$\left[s + \frac{b}{m}\right]V = \frac{F}{m} + v_0 \longrightarrow V = \frac{1/m}{\left[s + \frac{b}{m}\right]}F + \frac{1}{\left[s + \frac{b}{m}\right]}v_0$$

$F(s) =$  $= \frac{F_0}{s}$

$$V_s = \underbrace{\frac{1/m}{s\left[s + \frac{b}{m}\right]}}_{\#11} + \underbrace{\frac{1}{s + \frac{b}{m}}}_{\#7} v_0$$

$$\boxed{v(t) = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right) + v_0 e^{-\frac{b}{m}t}}$$

Final Value Theorem (FVT)

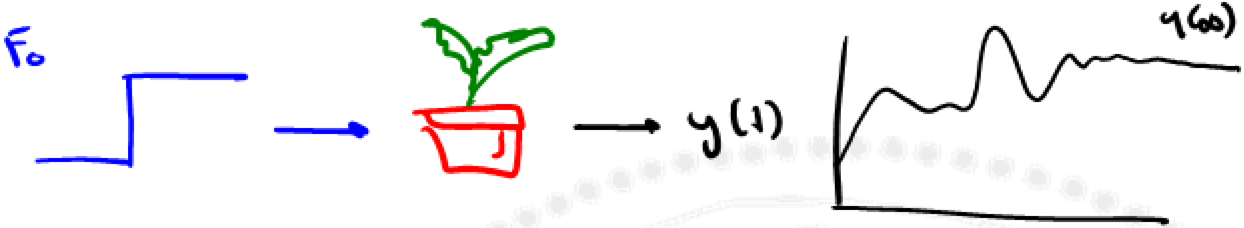
$$y(\infty) = \lim_{s \to 0} sY(s)$$

IF AND ONLY IF system is stable.

Initial Value Theorem (IVT)

$$y(0^+) = \lim_{s \to \infty} sY(s)$$

DC GAIN



$$\frac{y(\infty)}{F_0} \triangleq \text{DC GAIN}$$

$$y(\infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \cancel{s} H(s) \frac{F_0}{\cancel{s}}$$

DC gain

$$\text{DC gain} = \frac{\lim H(s) \cancel{F_0}}{\cancel{F_0}} = \boxed{H(0) \triangleq \text{DC GAIN}}$$

ONLY FOR STABLE SYSTEMS

$$Y(s) = \frac{N(s)}{D(s)}$$

← residues – "how much"

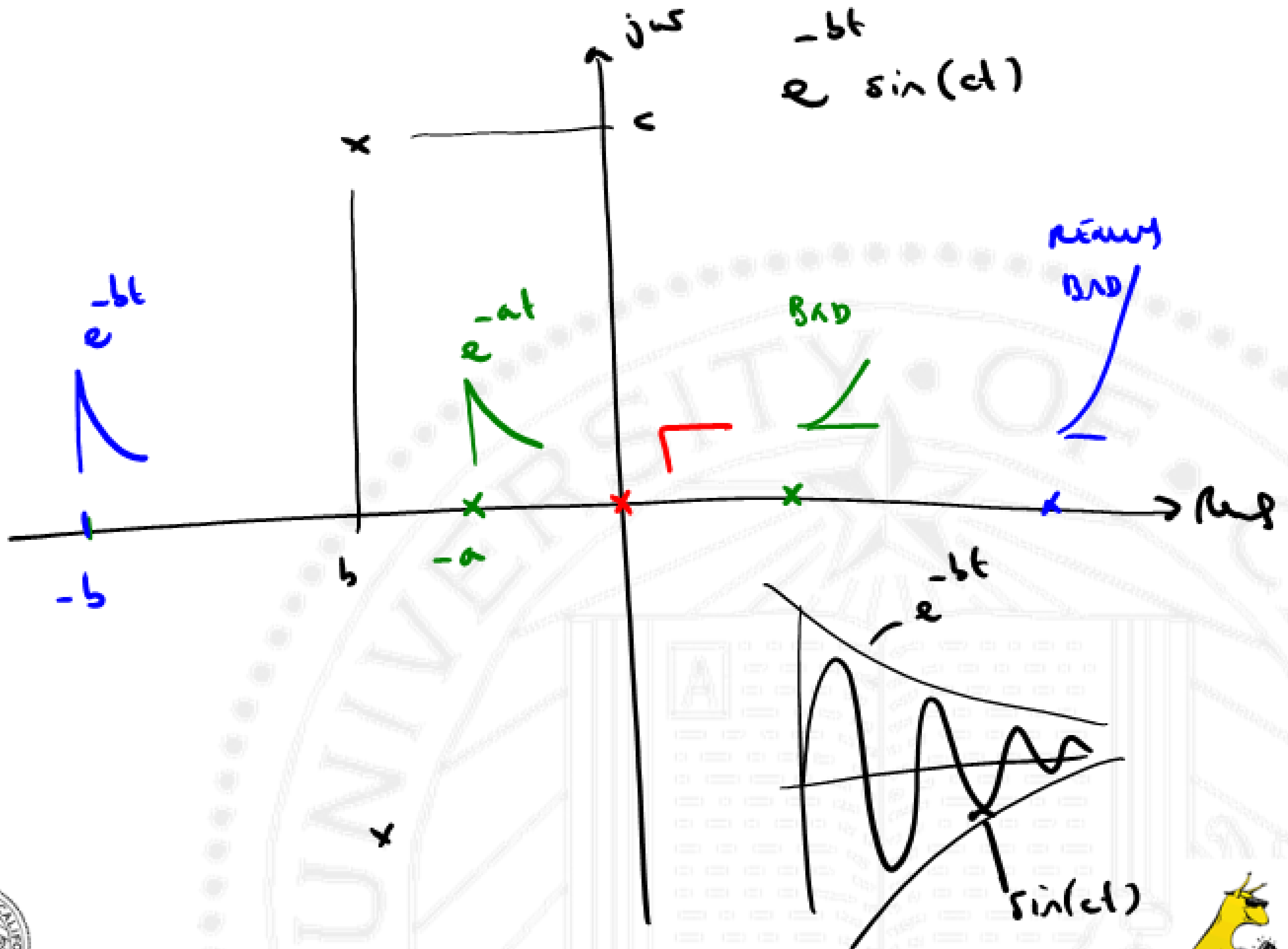
← poles – type of response "ingredients"

$Y(s) = H(s)U(s)$ ∧ $U(s) = \delta(t) = 1$

$$H(s) = \frac{\text{num}}{(s+a)\left[(s+b)^2 + c^2\right]} = \frac{(\quad)}{(s+a)} + \frac{(\quad)}{(s+b)^2 + c^2}$$

modal responses.

$D(s)$ — $\Delta(s)$ – characteristic equation


jω
e^-bt sin(ct)
c
e^-bt
e^-at
BAD
REALLY BAD
Re
-b
b
-a
e^-bt
sin(ct)