CMPE-242
Applied Feedback Control

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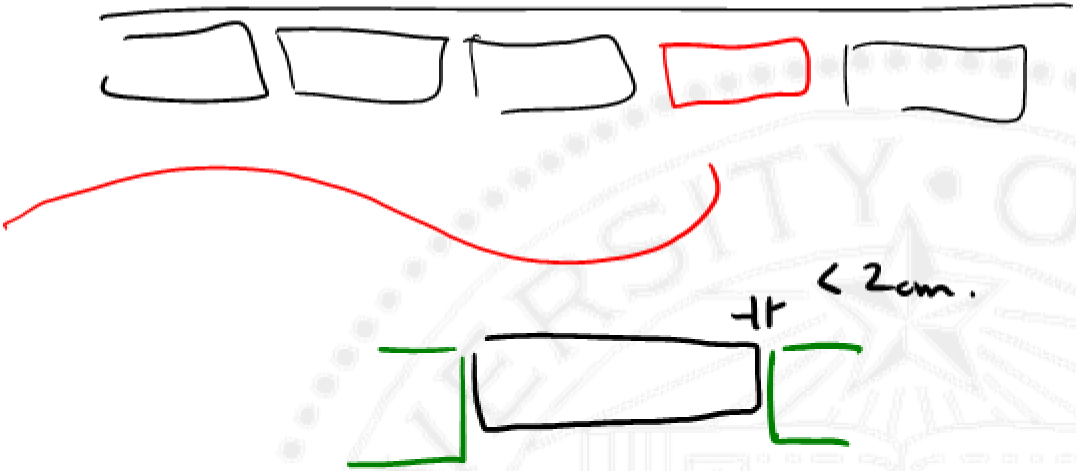
Winter 2016


< 2cm.

$$G(s) = \frac{1}{s^2} \longrightarrow \frac{Y}{U} = \frac{1}{s^2 + 0s + 0}$$

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} \dot{y} \\ y \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Phi = e^{AT} = \overbrace{I + AT}^{} + \frac{A^2T^2}{2!} + \frac{A^3T^3}{3!} + \dots$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ T & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} = e^{AT} = \Phi$$

$$\Gamma = \left(\int_0^T e^{A\eta}\, d\eta\right) B = \int_0^T e^{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\eta} d\eta \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \int_0^T \begin{bmatrix} 1 & 0 \\ \eta & 1 \end{bmatrix} d\eta \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Gamma = \int_0^T \begin{bmatrix} 1 & 0 \\ \eta & 1 \end{bmatrix} d\eta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ \frac{\eta^2}{2} & \eta \end{bmatrix} \Bigg|_0^T \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} T & 0 \\ \frac{T^2}{2} & T \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} T \\ \frac{T^2}{2} \end{bmatrix} = \Gamma}$$

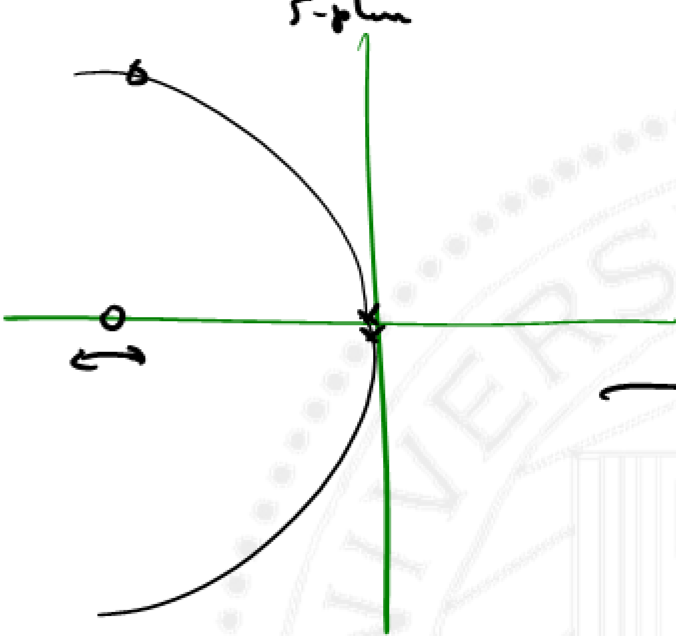
$$T_s = 0.1 \rightarrow \Phi = \begin{bmatrix} 1 & 0 \\ .1 & 1 \end{bmatrix} \quad \Gamma = \begin{bmatrix} .1 \\ .005 \end{bmatrix}$$

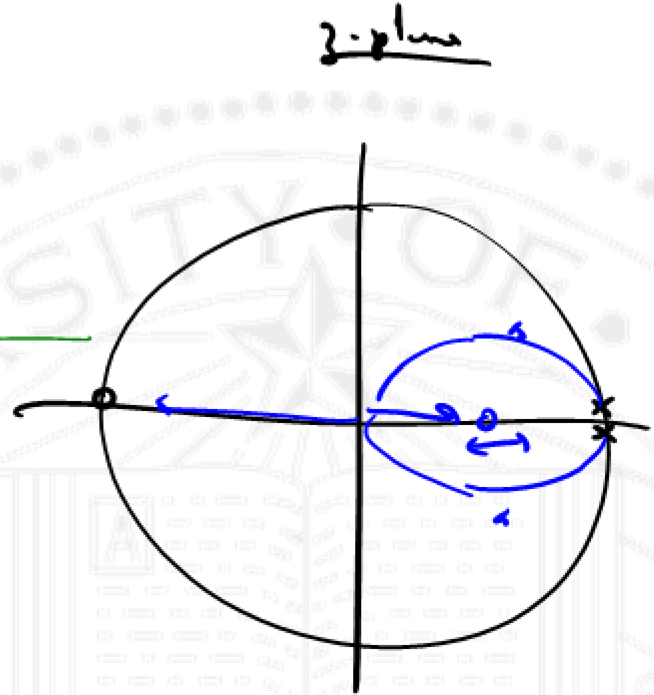
$$x_{k+1} = \begin{bmatrix} 1 & 0 \\ .1 & 1 \end{bmatrix} x_k + \begin{bmatrix} .1 \\ 0.005 \end{bmatrix} u_k$$

$$s_{des} = -2.8 \pm 2.8j$$

$$z_{des} = e^{s_{des} T} = .72 \pm .21j$$

$K = \text{place}(\Phi, \Gamma, z_{des}) \longrightarrow K = [9.9 \quad 12]$





$$lqr = \int_0^\infty y^T Q_y y + u^T R u = \int_0^\infty x^T C^T Q_y C x + u^T R u$$

$$\left(\frac{1}{y_{max}^2}\right) \quad \rho 1 \qquad \begin{bmatrix} 0 & 0 \\ 0 & 1/y_{max}^2 \end{bmatrix} \qquad [1]$$

$$dlqr(\Phi, \Gamma, Q, R)$$

$$Q = \rho\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad R = [1]$$

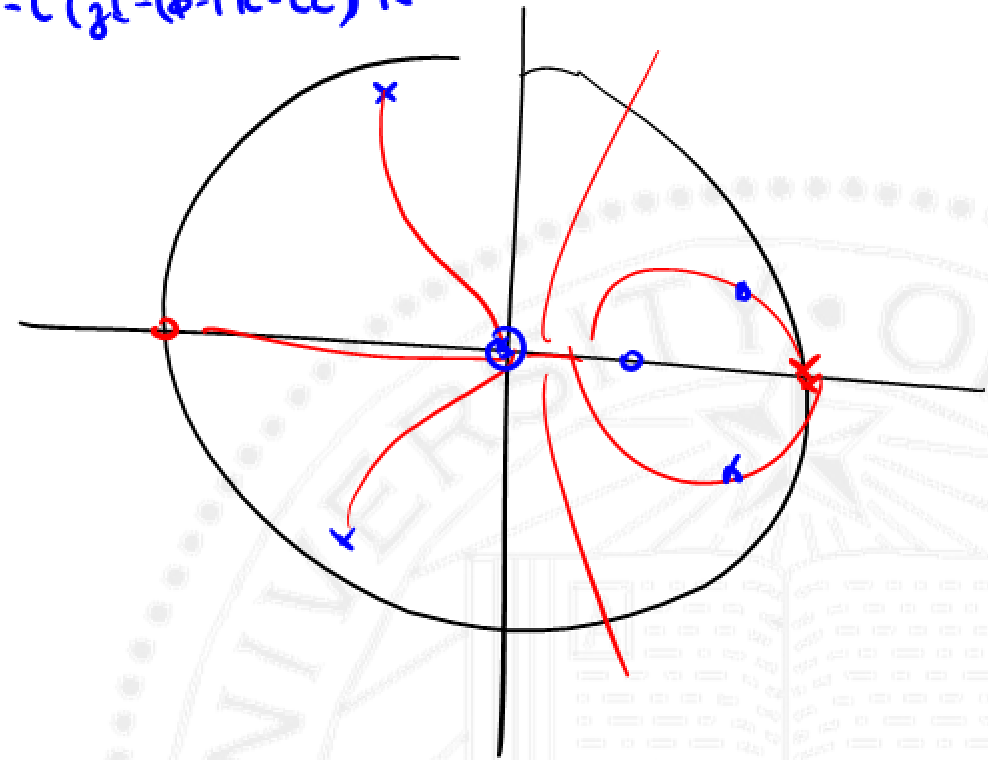
$$\rho = 256 \sim \frac{1}{y_{max}^2} = \frac{1}{(1/16)^2} \rightarrow K = [9.7 \;\; 12]$$

$$lqr(sys, Q, R)$$

$z_D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ "deadband" – error goes to zero in 2 steps

$$L = \begin{bmatrix} 1.5 \\ 1.0 \end{bmatrix}$$

$$\frac{u}{y} = K(s) = -L\left(zI - (\Phi - \Gamma K - LC)\right)^{-1} K$$

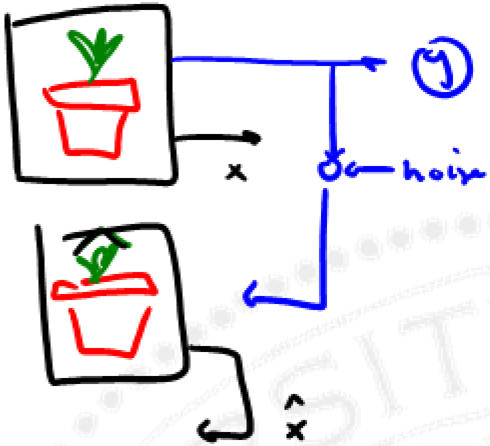


- Reduced Order Estimator

- Ways in which to Abuse LQR.

  - Pricks

  - Implicit Model Following.



$$\begin{bmatrix} y \\ v \end{bmatrix} \qquad \begin{bmatrix} \hat{y} \\ \hat{v} \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} \cancel{\hat{y}}\ y \\ \hat{v} \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x_A \\ \hline x_B \end{bmatrix}$$

$x_A \leftarrow$ all that I measure

$x_B \leftarrow$ everything else.

$$\begin{bmatrix} \dot{x}_a \\ \hline \dot{x}_b \end{bmatrix} = \left[\begin{array}{c|c} A_{aa} & A_{ab} \\ \hline A_{ba} & A_{bb} \end{array}\right] \begin{bmatrix} x_a \\ \hline x_b \end{bmatrix} + \begin{bmatrix} B_a \\ \hline B_b \end{bmatrix} \underline{u}$$

$$y = \left[\begin{array}{c|c} I & 0 \end{array}\right] \begin{bmatrix} x_a \\ \hline x_b \end{bmatrix}$$

$$I \triangleq \text{identity} \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$$

$$\dot{x}_B = A_{ba} \underbrace{x_a}_{y} + A_{bb} x_b + B_b \underline{u}$$

$$\dot{\underline{x}}_B = A_{BB} \underline{x}_B + \left[\begin{array}{c:c} A_{ba} & B_b \end{array}\right] \begin{bmatrix} y \\ \underline{u} \end{bmatrix}$$

$$\dot{\hat{x}}_b = A_{bb}\hat{x}_b + [A_{ba}\underset{y}{\underset{\uparrow}{x_a}} + B_b u] + L_r A_{ab}(x_b - \hat{x}_b)$$

$$\tilde{x}_b \triangleq x_b - \hat{x}_b$$

$$\dot{\tilde{x}}_b = -A_{bb}\hat{x}_b - [\cancel{A_{ba}x_a + B_b u}] - L_r A_{ab}\tilde{x}_b$$

$$+ A_{bb}x_b + [\cancel{A_{ba}x_a + B_b u}]$$

$$L_r^T = \text{place}(A_{bb}^T, A_{ab}^T, P_{des})$$

$$\dot{\tilde{x}}_b = A_{bb}\tilde{x}_b - L_r A_{ab}\tilde{x}_b$$

$$\dot{\tilde{x}}_b = \underline{[A_{bb} - L_r A_{ab}]}\,\tilde{x}_b.$$

$$\dot{\hat{x}}_b = A_{bb}\hat{x}_b + \begin{bmatrix} A_{ba} & B_b \end{bmatrix}\begin{bmatrix} y \\ u \end{bmatrix} + L_r\left[A_{ab}x_b - A_{ab}\hat{x}_b\right]$$

$$\dot{x}_a = A_{aa}x_a + \underline{A_{ab}x_b} + B_a u$$

$$(\dot{y} - A_{aa}y - B_a u) = A_{ab}x_b$$

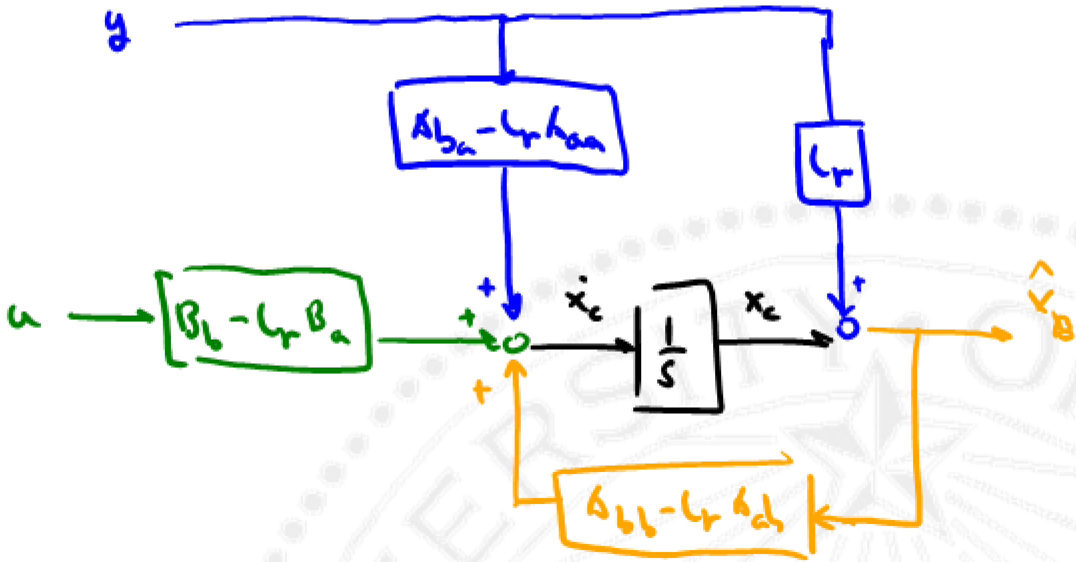
$$\dot{\hat{x}}_b = \left[A_{bb} - L_r A_{ab}\right]\hat{x}_b + \left[A_{ba} - L_r A_{aa}\right]y + \left[B_b - L_r B_a\right]u - L_r\dot{y}$$

$$x_c \triangleq \hat{x}_b - L_r y$$

$$\dot{x}_c = \dot{\hat{x}}_b - L_r\dot{y}$$

$$\dot{\underline{x}}_c = [A_{bb} - L_r A_{ab}]\hat{x}_b + [A_{ba} - L_r A_{aa}]y + [B_b - L_r B_a]u$$

$$\hat{x}_b = \underline{x}_c + L_r y$$



$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \dot{x}_A \\ \dot{x}_B \end{bmatrix} = \left[\begin{array}{c|c} A_{aa} & A_{ab} \\ \hline A_{ba} & A_{bb} \end{array}\right] \begin{bmatrix} x_A \\ x_B \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u$$

$$u = -\left[K_A \,\vdots\, K_B \right] \begin{bmatrix} x_A \\ x_B \end{bmatrix}$$

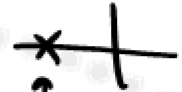
$$u = -K_A y - K_B \hat{x}_B$$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$A = \left[\begin{array}{c|c} 0 & 1 \\ \hline -2 & 0 \end{array}\right] \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$



$A_{aa} = 0$

$A_{ab} = 1$

$A_{ba} = -2$

$A_{bb} = 0$

$x_1 = y \leftarrow$ measurement

x_2 — estimate

$$\dot{\tilde{x}}_2 = (0 - L_r)\,\hat{x}_2$$

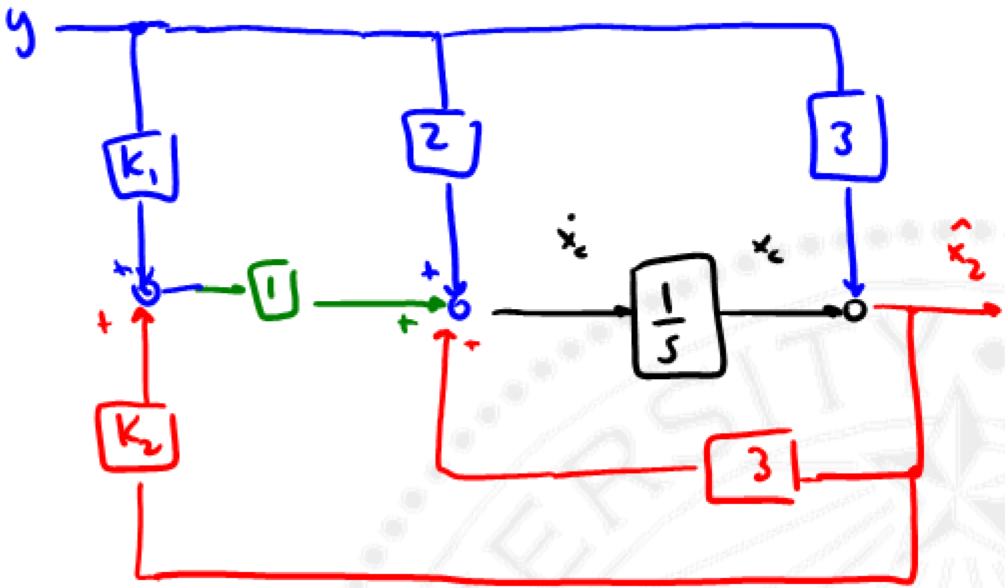
$$\tilde{x}_2(t) = e^{-L_r t}\,\tilde{x}_2(0).$$

$L_r = 3$

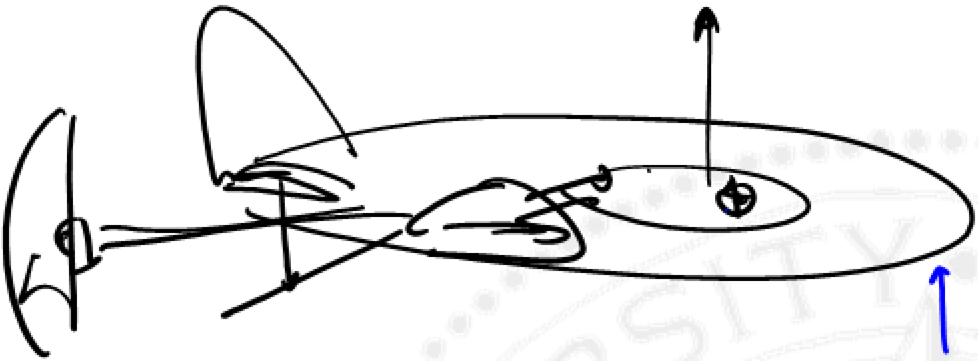
$$\dot{x}_c = -3\hat{x}_2 + 2y + u$$

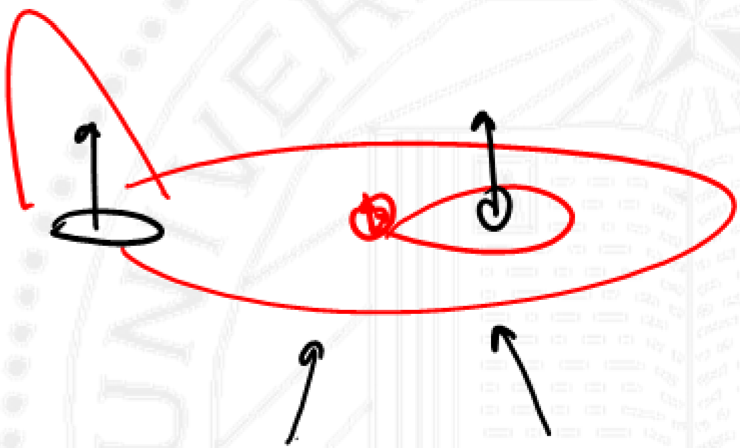
$$\hat{x}_b = x_c + L_r y$$

$$u = -K_1 y - K_2 \hat{x}_2$$

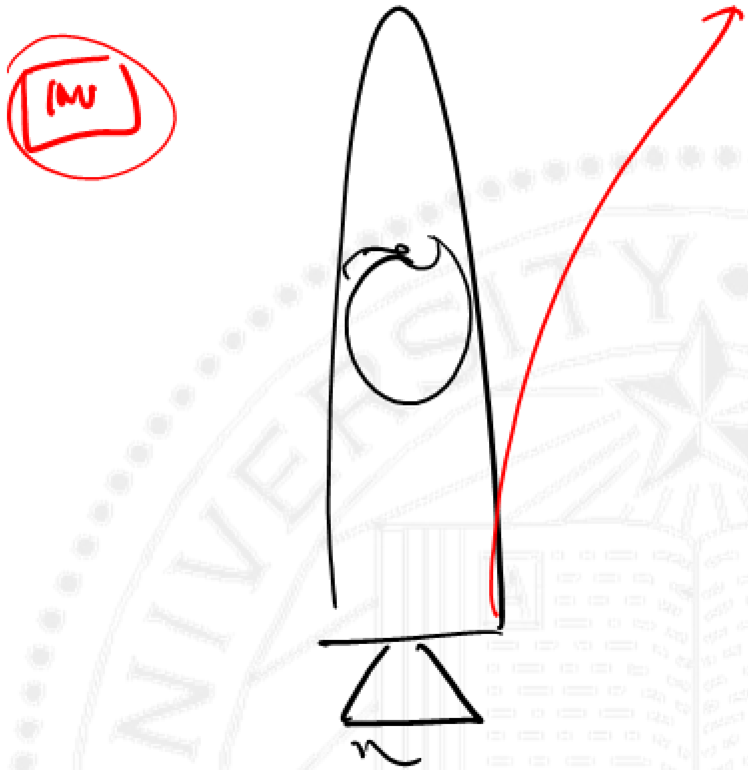


y
K1
2
3
1
1/s
K2
3



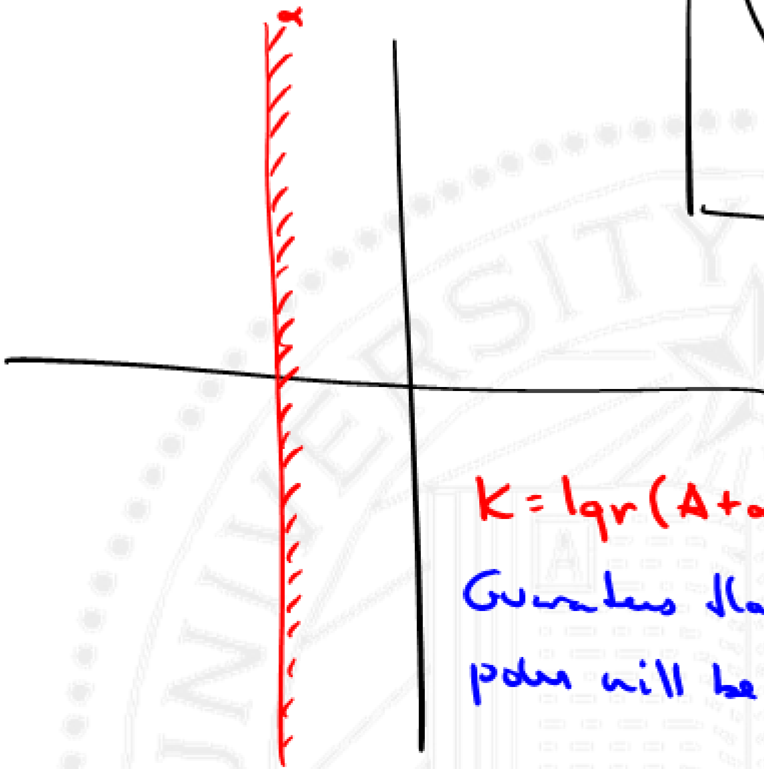


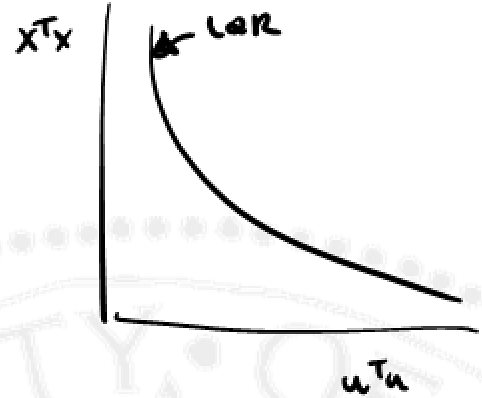
MILITARY
FIGHTER


IMU

Abuse of LQR

"pinder"





$K = \mathrm{lqr}(A + \alpha I, B, Q, R)$

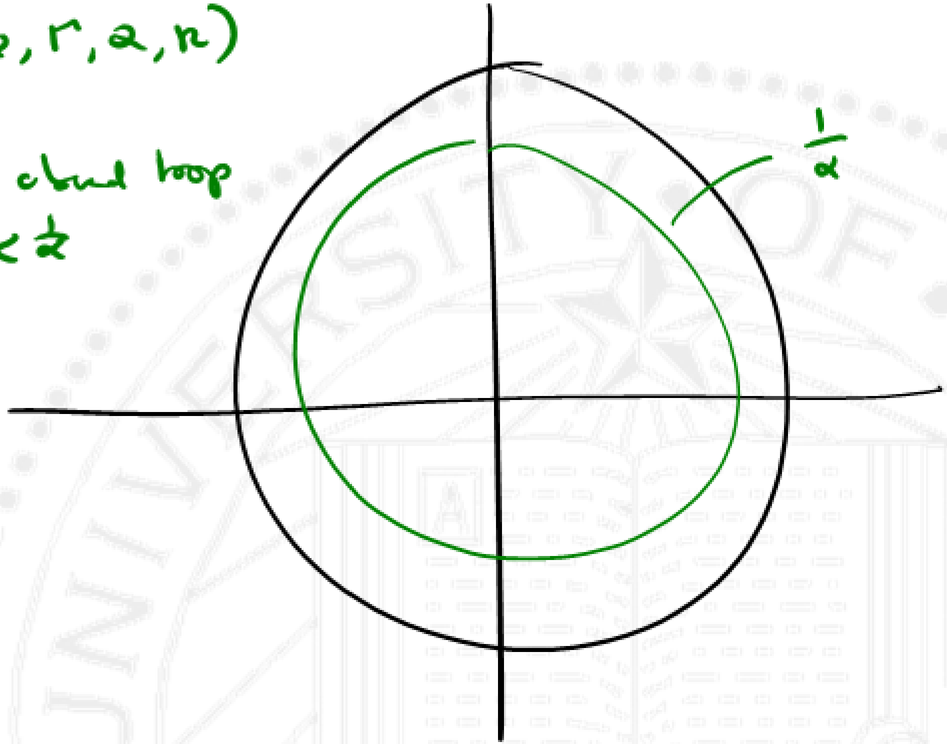
Guarantees that all closed loop poles will be to LEFT of α.

$$x_{k+1} = \Phi x_k + \Gamma u_k$$

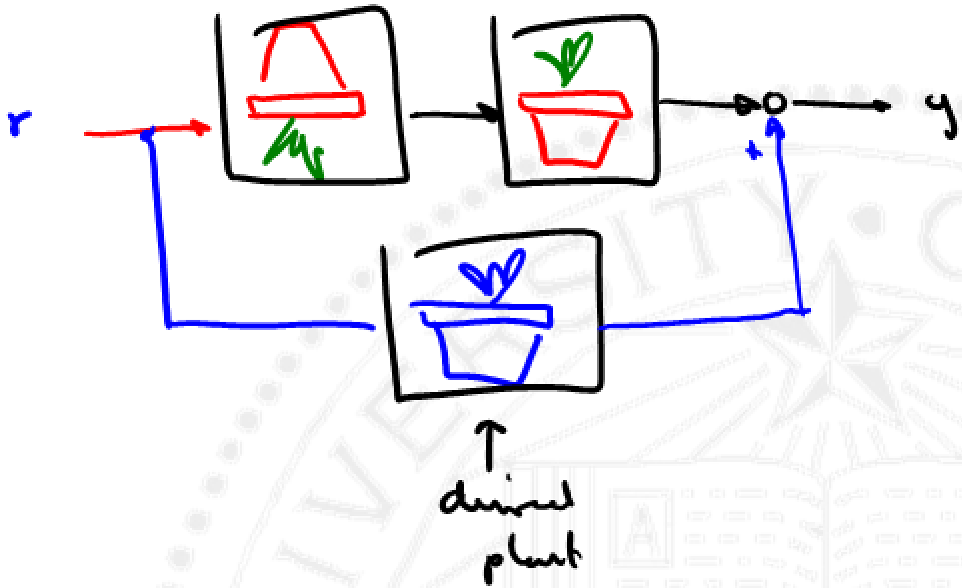
"pincher"

$$K = \text{dlqr}(\alpha\Phi, \Gamma, Q, R)$$

Guarantee <u>all</u> closed loop poles are $|\ | < \frac{1}{\alpha}$



Implicit Model Following



$$J = \int_0^{\infty} \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$u = -Kx$$

$$\dot{z} = A_m z$$ model

$$z \approx y.$$

$$\dot{y} = C\dot{x} = C(Ax + Bu)$$

$$J = \int_0^\infty \{(\dot{y} - \dot{z})^T Q (\dot{y} - \dot{z}) + u^T R u\} dt$$

$$\dot{z} = A_m z \approx A_m y = A_m C x$$

$$\dot{y} = CAx + CBu$$

$$\dot{z} = A_m z \approx A_m y = A_m C x$$

$$(\dot{y} - \dot{z}) = [CA - A_m C]\underline{x} + CB\underline{u}$$

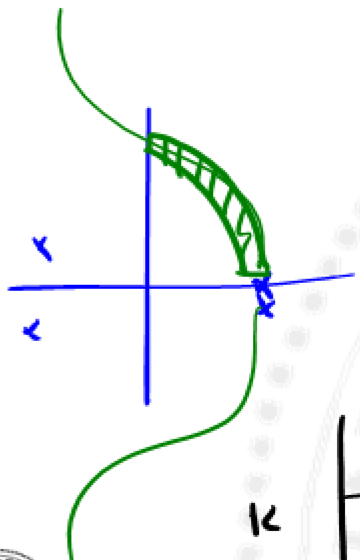
$$J = \int_0^\infty (\dot{y}-\dot{z})^T Q (\dot{y}-\dot{z}) + u^T R u$$

$$J = \int_0^\infty \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} \underbrace{(CA - A_m C)^T Q [CA - A_m C]}_{\bar{Q}} & \overbrace{[CA - A_m C]^T Q [CB]}^{N} \\ \underbrace{[CB]^T Q [CA - A_m C]}_{N^T} & \underbrace{R + [CB]^T Q [CB]}_{\bar{R}} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} d\tau$$

$$u = -Kx.$$

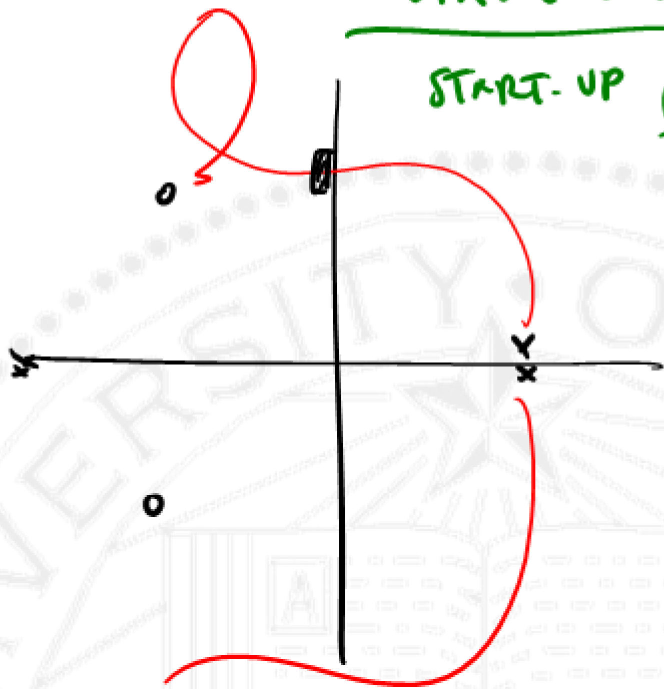
WARNINGS

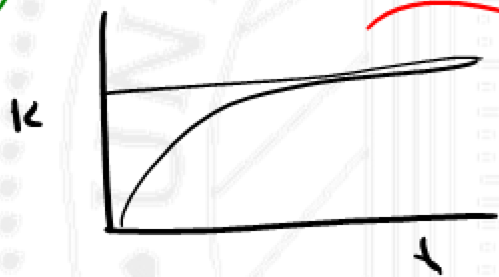
UNSTABLE CONTROLLER



UNSTABLE SYSTEM

START-UP PROBLEM





Non-linear systems

$$\dot{x} = Ax + Bu \quad — \quad \dot{x} = f(x,u)$$

$$\dot{\underline{x}} = \underbrace{\frac{\partial f}{\partial x}\Big|_{x_0,u_0}}_{A}(x-x_0) + \underbrace{\frac{\partial f}{\partial u}\Big|_{x_0,u_0}}_{B}(u-u_0)$$

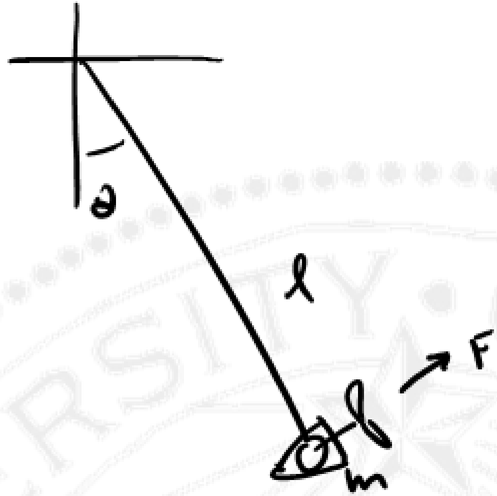
$$u = -f_{NL}(x,u) + K\underline{x}$$

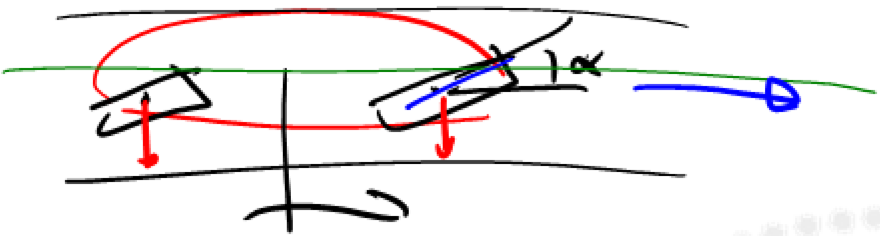
$$m\ell^2\ddot{\theta} + mg\ell\sin\theta = F.$$

$$\ddot{\theta} + \frac{g}{\ell}\sin\theta = F$$

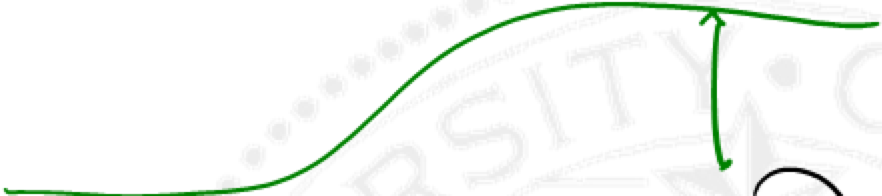
$$u = F - \frac{g}{\ell}\sin\theta$$

$$\ddot{\theta} = u.$$





E.O.M ~ unweight

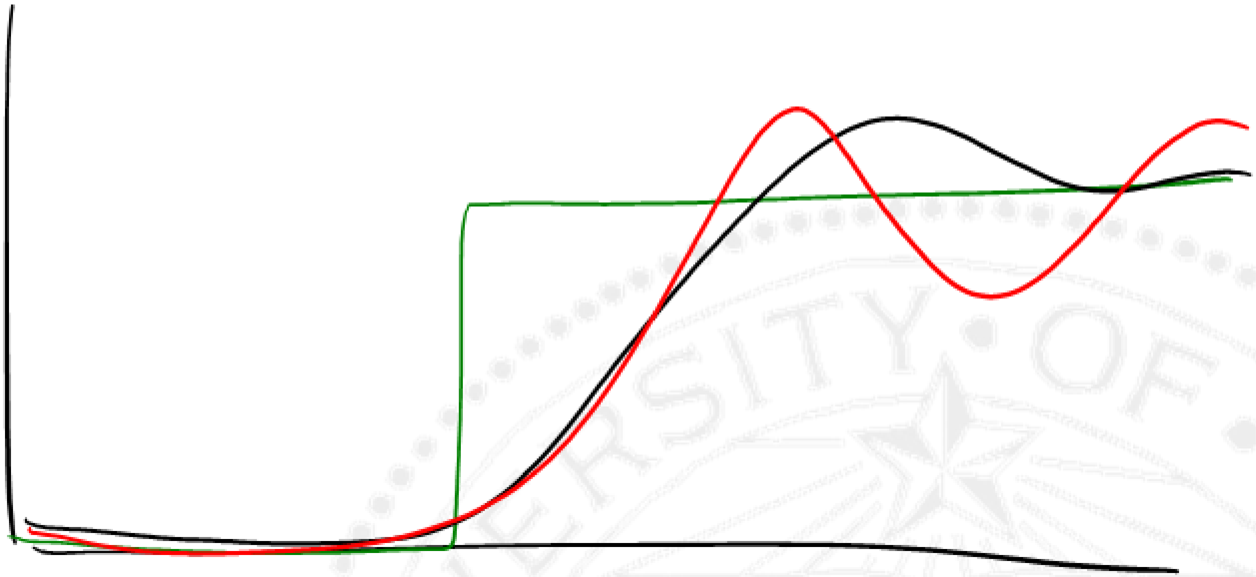


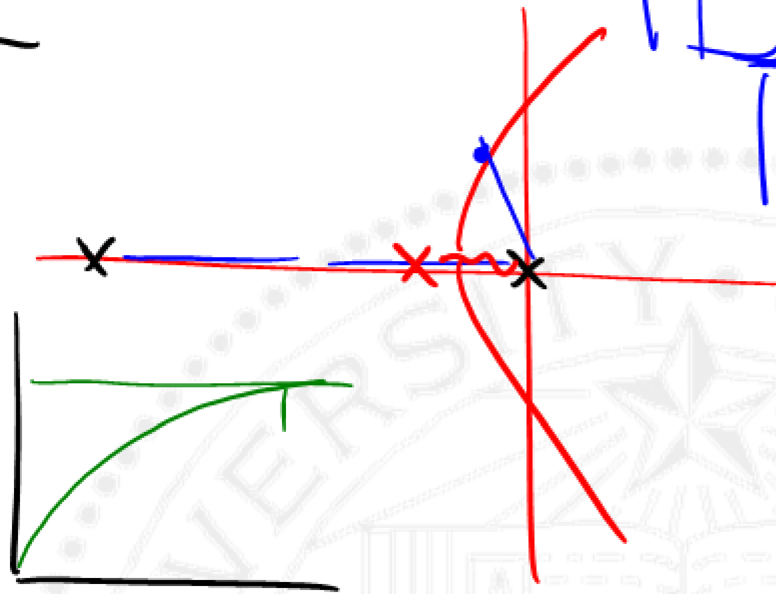
δy	$\delta \dot{y}$	$\delta \alpha$
δu	$\delta \dot{u}$	$\delta \omega$
θ	$\dot{\theta}$	$\delta \alpha_{\theta}$
ϕ	$\dot{\phi}$	$\delta \alpha_{\phi}$

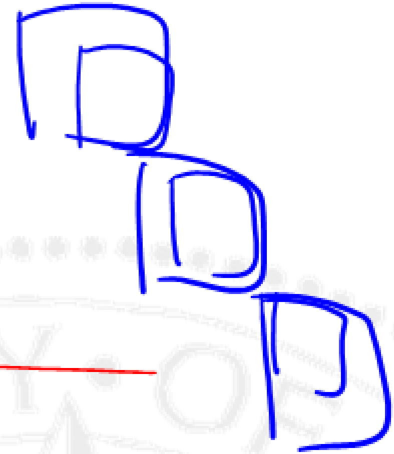
Altitude control

pitch control

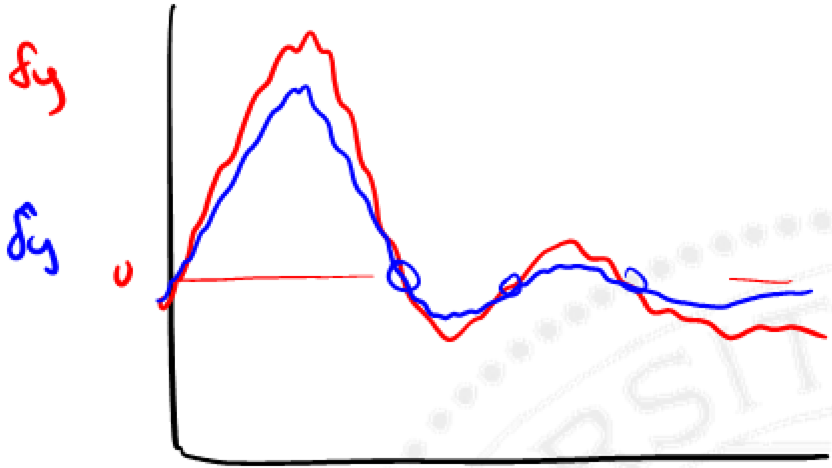
roll control







$$\frac{\alpha_{true}}{\alpha_{cmd}}$$



$$u \rightarrow \delta_a \rightarrow (\dot{\delta}_\omega)$$