CMPE-242
Applied Feedback Control

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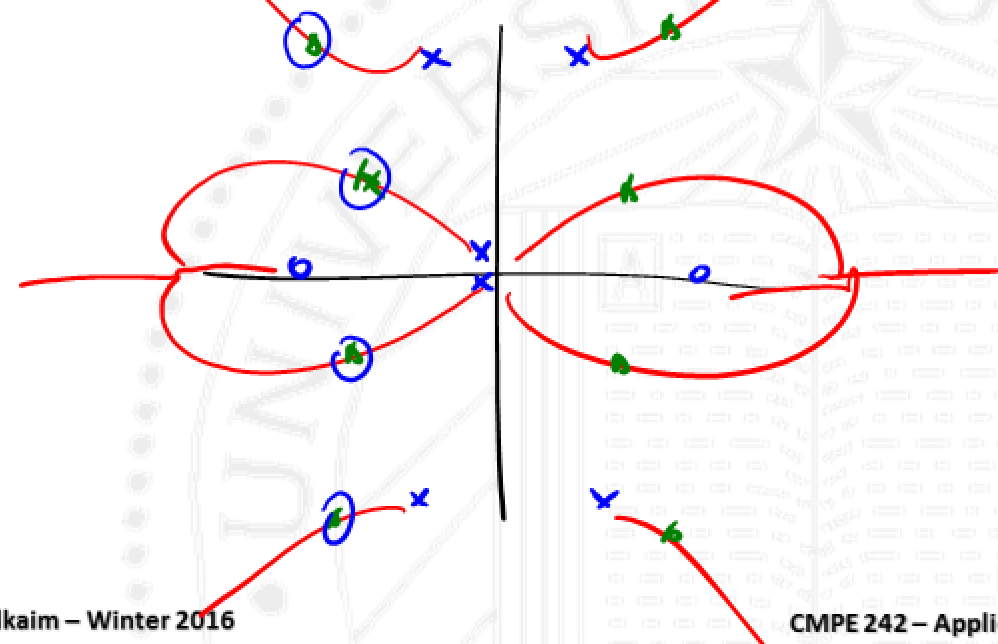
Winter 2016

Symmetric Root Locus (SRL)

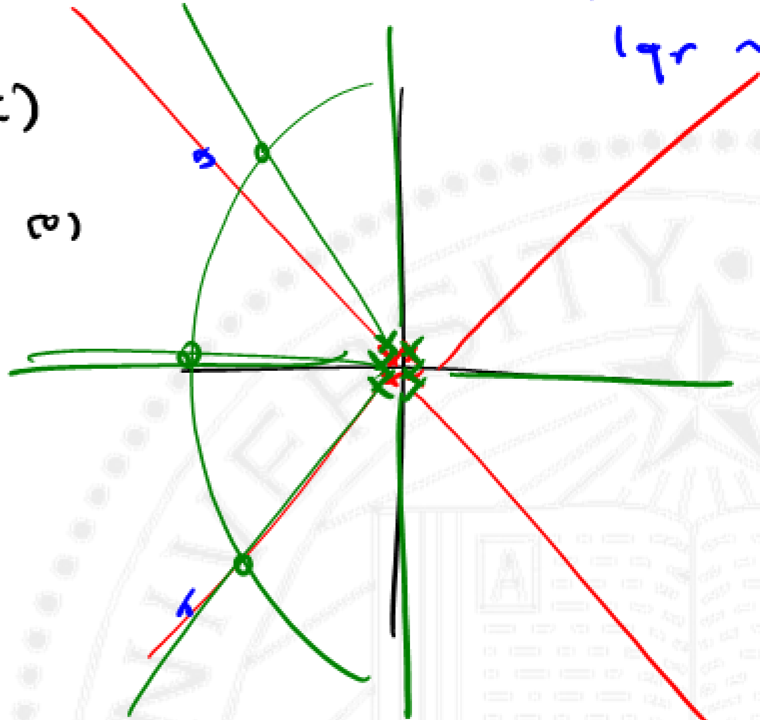
SINGLE INPUT / SINGLE OUTPUT (SISO)

lqr — bryson's rule → $1 + \rho\, G(s)G(-s) = 0.$



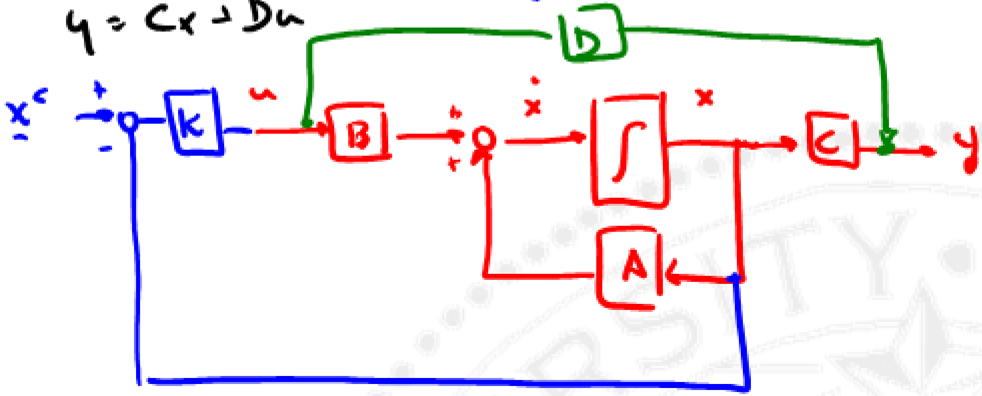
rlocus(-G*G)

optimal damping for lqr ~ $\frac{\sqrt{2}}{2}$.



$\dot{x} = Ax + Bu$ $\qquad u = -K(x - x^c) \longrightarrow u = K(x^c - x)$

$y = Cx + Du$



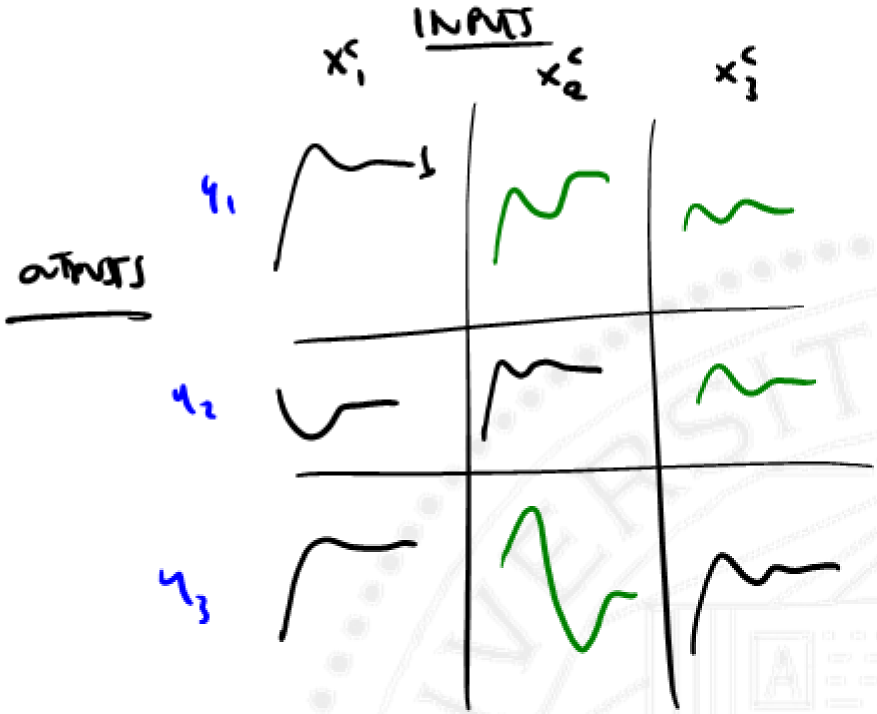
$$\dot{x} = Ax + BK(x^c - x) = (A - BK)x + BK\underline{x}^c$$

$$y = Cx + DK(x^c - x) = (C - DK)x + DK\underline{x}^c \Big] \text{ input}$$

$$\dot{x}_{cl} = [A - BK]x_{cl} + BK\underline{x}_c$$

$$y_{cl} = [C - DK]x_{cl} + DK\underline{x}_c$$

ss(A-BK, BK, ...)



step (resp)

n×1

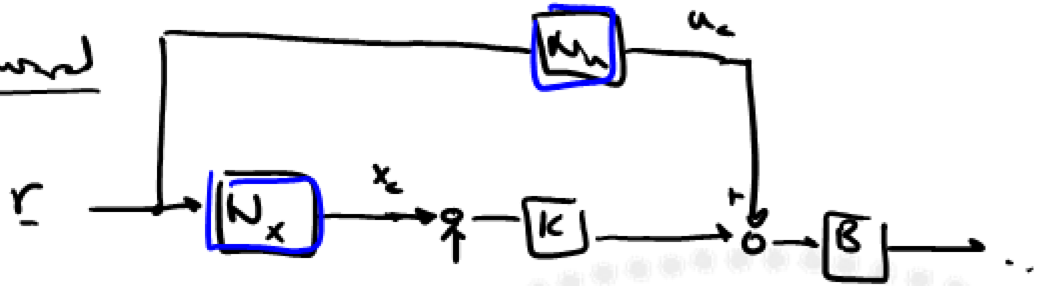
r×1

$$\begin{bmatrix}\dot{x}\\ \hline y\end{bmatrix} = \left[\begin{array}{c|c} \overset{n\times n}{A} & \overset{n\times m}{B} \\ \hline \underset{r\times n}{C} & \underset{r\times m}{D}\end{array}\right]\begin{bmatrix} x \\ \hline u\end{bmatrix}$$

n×1

m×1

Feedforward



$$\cancel{\dot{x}_{ss}}^{\,0} = Ax_{ss} + Bu_{ss}$$

$$y_{ss} = Cx_{ss} + Du_{ss}$$

$$x_{ss} \triangleq N_x y_{ss}$$

$$u_{ss} \triangleq N_u y_{ss}$$

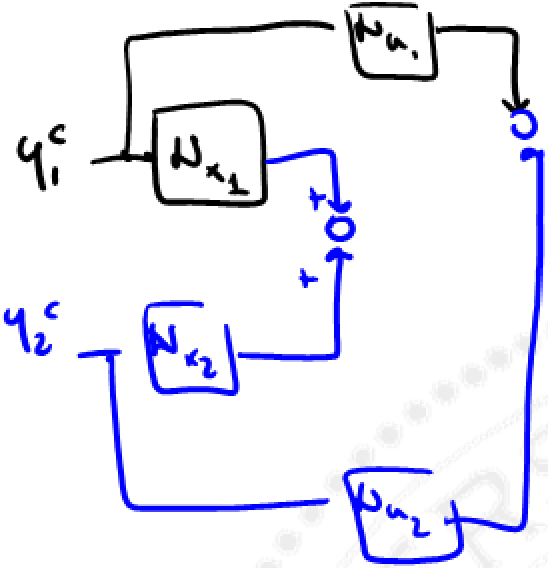
$$\overset{n\times 1}{0} = AN_x y_{ss} + BN_u y_{ss}$$

$$y_{ss} = CN_x y_{ss} + DN_u y_{ss}$$

$$\longrightarrow \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} AN_x + BN_u \\ CN_x + DN_u \end{bmatrix} y_{ss}\right]$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} N_x \\ \cdots \\ N_u \end{bmatrix} \longrightarrow \begin{bmatrix} N_x \\ \cdots \\ N_u \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

FPE 7-16



$y_c^1 - [\quad\quad]^{c_1}$

$y_c^2 \; [\quad\quad]^{c_2}$

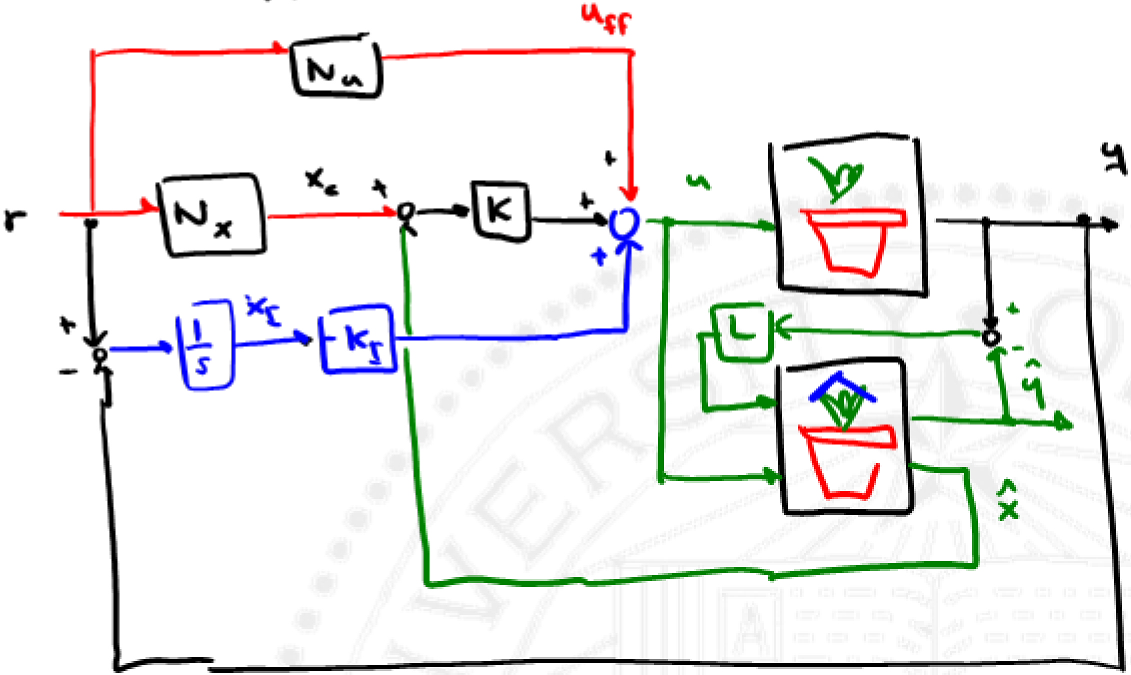
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = M^{-1}$$

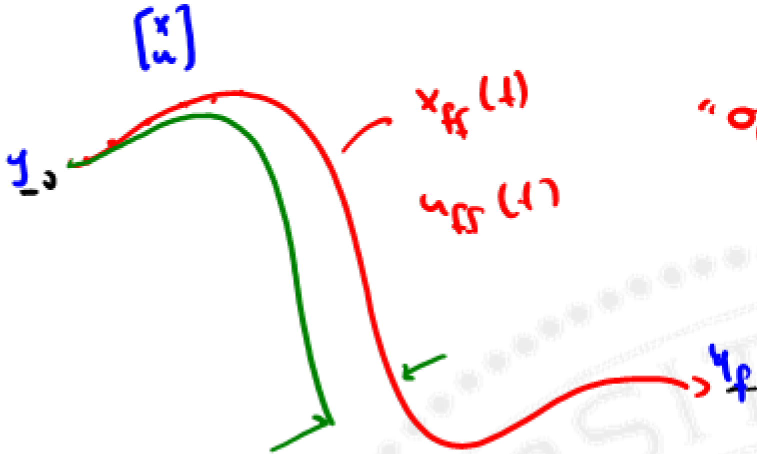
Tychonov Regularization

$$M^{-1} \simeq \underline{(M^T M + \mu I)^{-1} M^T}$$

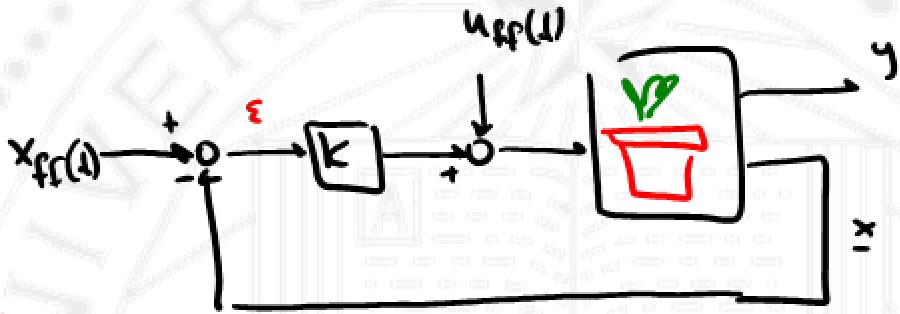
small $\mu > 0$

"Full Monde"





"optimal trajectory"



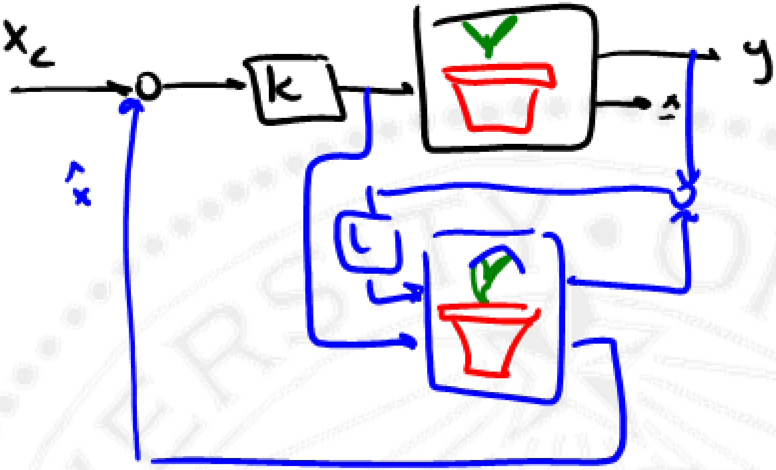
feedforward +
feedback.

$y = Cx + v$

$\uparrow$

noise

$\dot{\hat{x}} = ($



$$\dot{x} = Ax + Bu \qquad \dot{\hat{x}} = Ax + Bu \qquad u = -K(\hat{x} - \rho)$$

$$y = Cx + v \qquad \hat{y} = Cx$$

$$\dot{x} = Ax - BK\hat{x}$$

$$y = Cx + v$$

$$\dot{\hat{x}} = (A - BK)\hat{x} + LCx - LC\hat{x} + Lv$$

$$= (A - BK - LC)\hat{x} + LCx + Lv$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A & -BK \\ \hline LC & A - BK - LC \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \boxed{\begin{bmatrix} 0 \\ L \end{bmatrix} v}$$

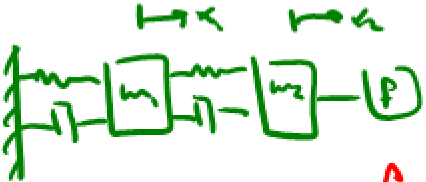
poles of controller @ $(A - BK)$

estimator @ $(A - LC)$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{bmatrix} N_x \\ \vdots \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$



$$m_1 = m_2 = k_1 = k_2 = b_1 = b_2 = 1$$

$$\begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} f$$

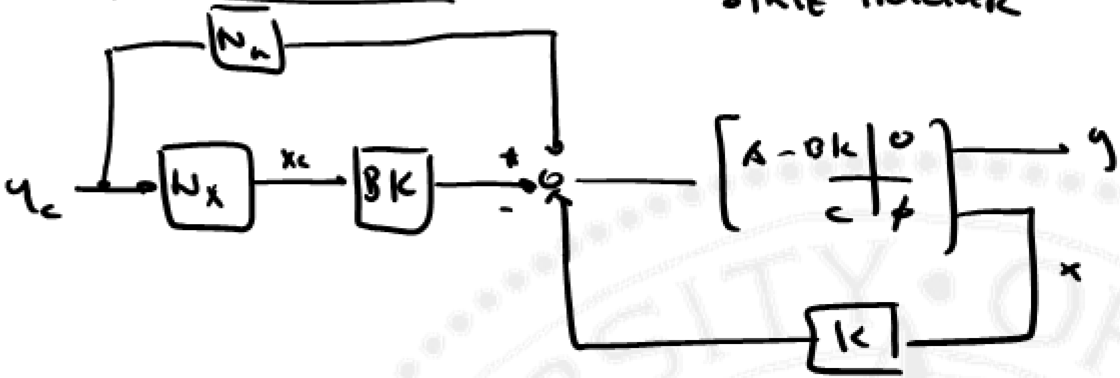
$$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \underline{x}$$

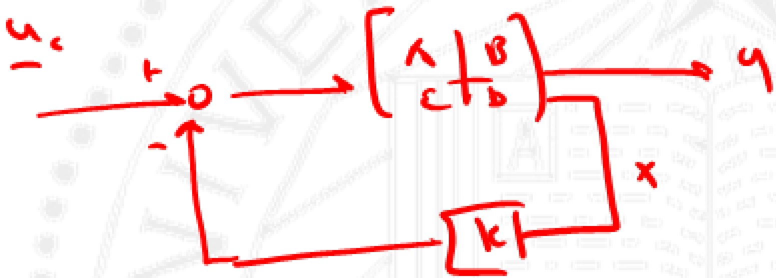
$$\begin{bmatrix} N_x \\ \vdots \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{array}{c} \left.\begin{matrix} 0 \\ 1/2 \\ 0 \\ 1 \end{matrix}\right\} N_x \\ \hline \left. 1/2 \right\} N_u \end{array}$$

command x_2

STRUCTURES

"STATE TRACKER"

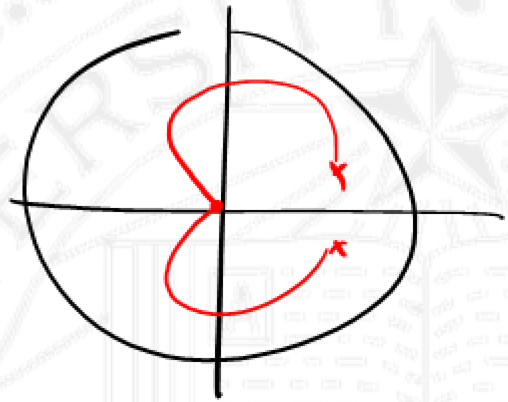




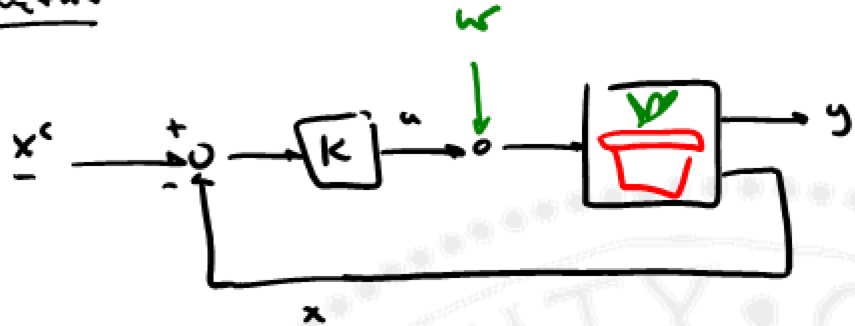
"CONTROL AUGMENTATION"

digital

$z_D : \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow$ deadbeat



Bias Estimation



$$\frac{1}{s^2} \rightarrow A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} y \\ v \end{bmatrix} = \underline{x}$$

$m = 1$

$$\dot{y} = v$$
$$\dot{v} = u + w$$
$$\dot{w} = \phi$$

$$\begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 0 \end{array}\right] \begin{bmatrix} y \\ v \\ \hline w \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\boxed{u_{cmd} = u - \hat{w}}$$

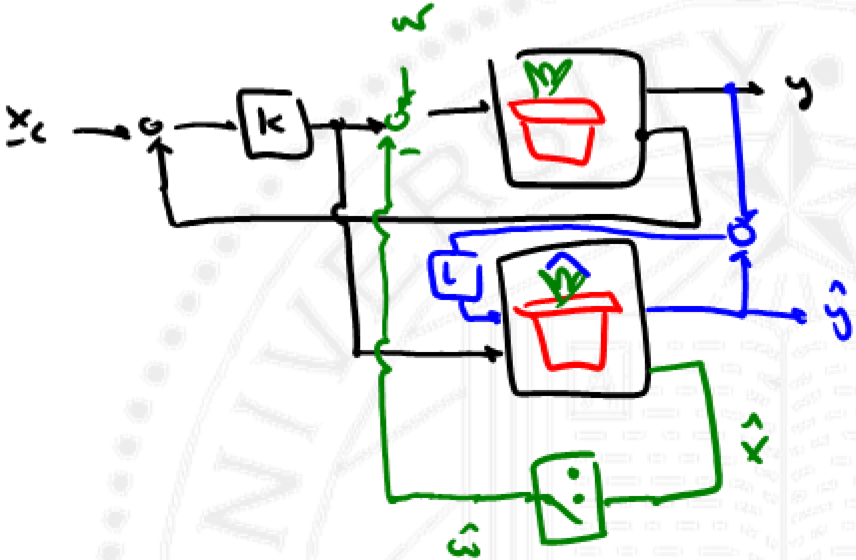
$$\mathcal{C} = [B \;\; AB \;\; A^2B] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \underline{\text{Rank}(2)}.$$

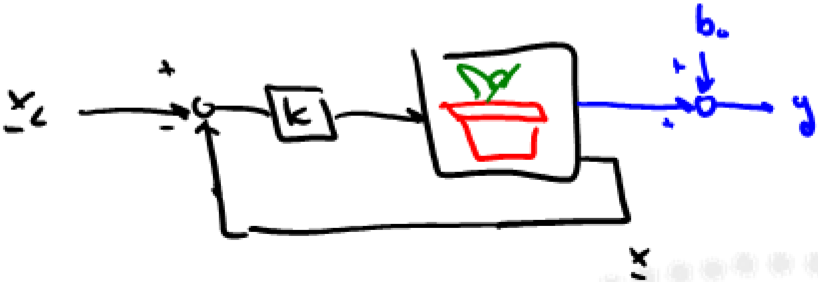
$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Rank}(3).$$

Can estimate $\hat{x}$ from $\underline{y}$

$u = K(x_c - x) - \hat{w}$

null out any nuisance signal

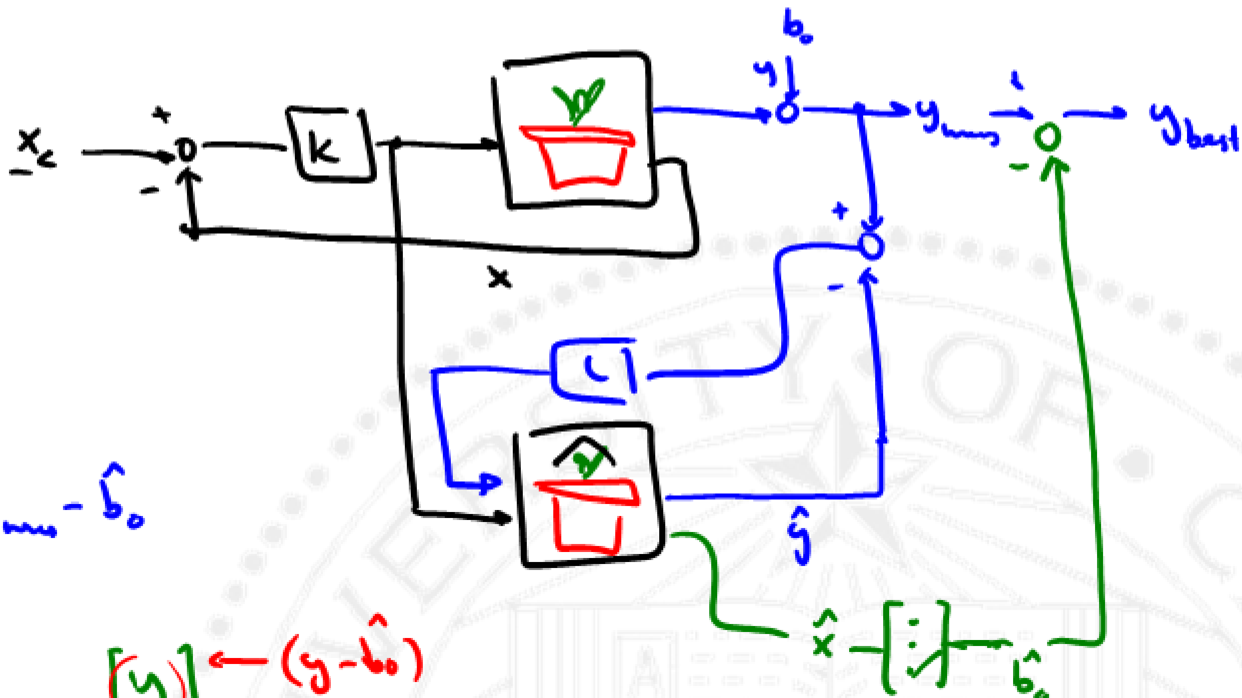




$$\dot{b}_0 = \phi$$

$$\begin{bmatrix}\dot{x}\\ \cdots \\ \dot{b}_0\end{bmatrix} = \begin{bmatrix}A & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ b_0\end{bmatrix} + \begin{bmatrix}B\\ \hline 0\end{bmatrix}u$$

$$c = \left[C \;\vdots\; 1\right]\begin{bmatrix}x\\ \hline b_0\end{bmatrix}$$



$y = y_{meas} - \hat{b}_0$

$$\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} \leftarrow (y - \hat{b}_0)$$

$$\omega_0 = \frac{1}{T} \qquad \ddot{\omega} + \omega_0^2\,\omega = \phi$$

$$\dot{x}_w = \underbrace{\begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix}}_{A_w} x_w \qquad x_w = \begin{bmatrix} \omega \\ \dot{\omega} \end{bmatrix}$$

$$\dot{\begin{bmatrix} x \\ \hline x_w \end{bmatrix}} = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & A_w \end{array}\right]$$

$$u = -K(x - x^c) - \hat{\omega}$$

Go Digital

$$\dot{x} = Ax + Bu \longrightarrow \mathcal{L}\{\} \longrightarrow sX = AX + BU$$

$$X = \underbrace{(sI - A)^{-1}B}_{G_{cl}} U$$

$$x_{k+1} = \Phi x_k + \Gamma u_k \longrightarrow Z\{\} \longrightarrow zX = \Phi X + \Gamma U$$

$$X = \underbrace{(zI - \Phi)^{-1}\Gamma}_{G_{cl}} U$$

Scalar Case

$\dot{x} = ax + bu \qquad y = 1x$

$\frac{x}{u} = \frac{b}{s-a} \qquad h(t) = be^{at}$ 1st. order system

$\boxed{\text{I.C. response}} \qquad x(t) = e^{a(t-t_0)} x(t_0)$

forced: $\int_0^t h(t-\tau)u(\tau)\,d\tau = \int_0^t e^{a(t-\tau)} u(\tau)\,d\tau$

$e^{at} \simeq 1 + at + \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} + \dots$

$e^{At} \simeq I + At + \frac{A^2t^2}{2!} + \dots$

$t_0 = kT$

$t_0 + T = kT + T = (k+1)T$ } assume a single time step

$$x(kT+T) = e^{AT} x(kT) + \int_{kT}^{kT+T} e^{A[(k+1)T-\tau]} B u(\tau)\, d\tau$$

$$x_{k+1} = \underbrace{e^{AT}}_{\Phi} x_k + \underbrace{\left[\int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}\, d\tau\right] B}_{\Gamma} u_k$$

$\eta = (k+1)T - \tau$

$d\eta = -d\tau$

$\tau = kT \to \eta = T$

$\tau = (k+1)T \to \eta = 0$

$$\left[\int_T^0 e^{A\eta}(-d\eta)\right] = \int_0^T e^{A\eta}\, d\eta$$

$$x_{k+1} = \underbrace{e^{AT}}_{\Phi} x_k + \underbrace{\left[\int_0^T e^{A\eta} d\eta\right] B}_{\Gamma} u_k$$

$\Phi \leftarrow \mathrm{expm}(A*T)$

c2d(sys, Ts, 'zoh')

$u = -Kx$ or $-K\hat{x}$

$\dot{x} = (A - BK)x$

$sIx = (A - BK)x$

$x = \underbrace{[sI - (A - BK)]}_{\Delta_{cl} = \phi}^{-1}$

T

$\uparrow \lambda$

$u_k = -Kx_k$ or $-K\hat{x}_k$

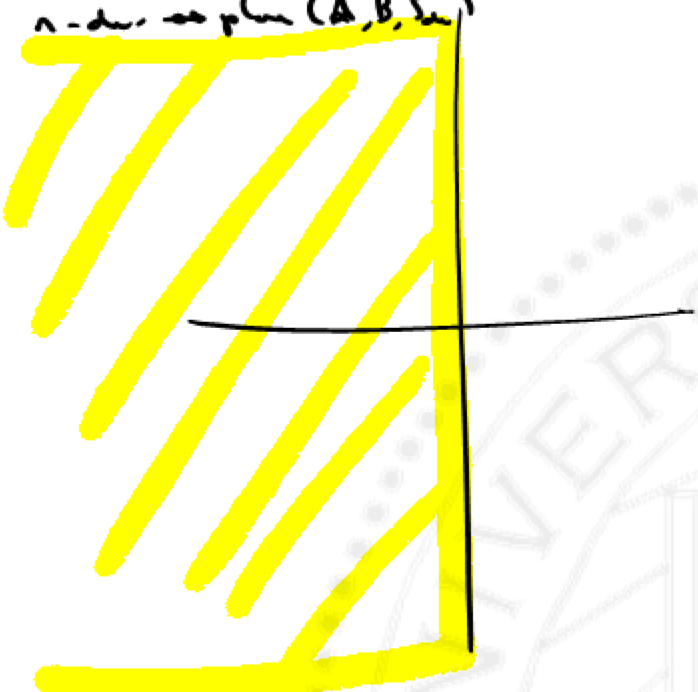
$x_{k+1} = (\Phi - \Gamma K)x_k$

$zIX = (\Phi - \Gamma K)X$

$X = \underbrace{[zI - (\Phi - \Gamma K)]}_{\Delta_{cl}(z) = \phi}^{-1}$

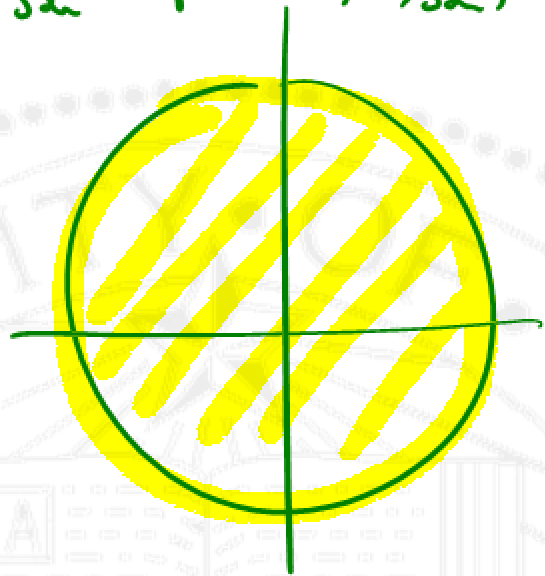
s - plane

n-dim → plane (A, B, s_c)



z - plane

z_d → plane (Φ, Γ, z_d)



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w

$$J = \int_0^\infty (x^T Q x + u^T R u)\,dt \longrightarrow \text{lqr}(A, B, Q, R)$$

$\neq$

$$J = \sum_0^\infty x_k^T Q x_k + u_k^T R u_k \longrightarrow \text{dlqr}(\Phi, \Gamma, Q, R)$$

$$\dot{x} = Ax + Bu$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$x_{k+1} = \Phi x_k + \Gamma u_k$$

$$\hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k)$$