CMPE-242
Applied Feedback Control

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Winter 2016

Converting things to discrete w/ different δT's.

Stability should be preserved.

$D_1(s)$

$\downarrow$ ~ 9-10ω_B

$D_1(z)\big|_{\text{Tustin}}$

$\downarrow$

$G_{AFT}(s)$

$\downarrow$

$G_{AFT}(z)$ ← c2d 'zoh'

$$\frac{Y}{R}(z) = \frac{D_1(z)\,G_{AFT}(z)}{1 + D_1(z)\,G_{AFT}(z)}$$

STATE SPACE

CONTROL → LQR

ESTIMATOR — LQE/KALMAN FILTER

STATE COMMAND

BIAS ESTIMATION

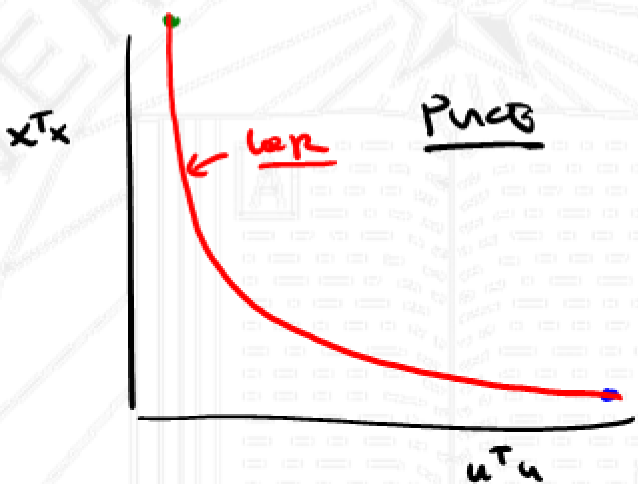
ABUSE LQR TO DO FUN THINGS

regulator

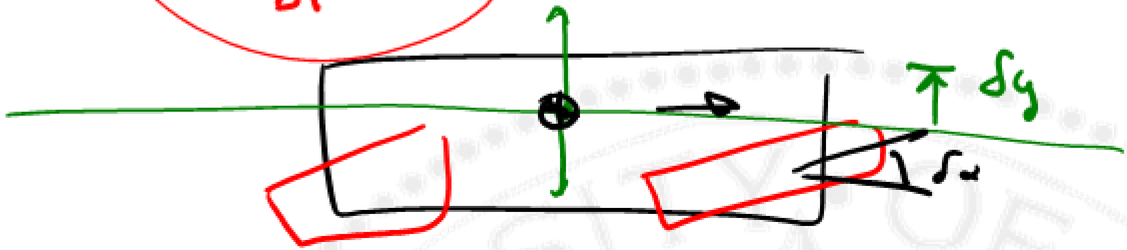
LQR: $$\min_{u} \int_0^{\infty} \left(x^T Q x + u^T R u\right) dt$$

subj: $\underline{\dot{x}} = A\underline{x} + B\underline{u}$

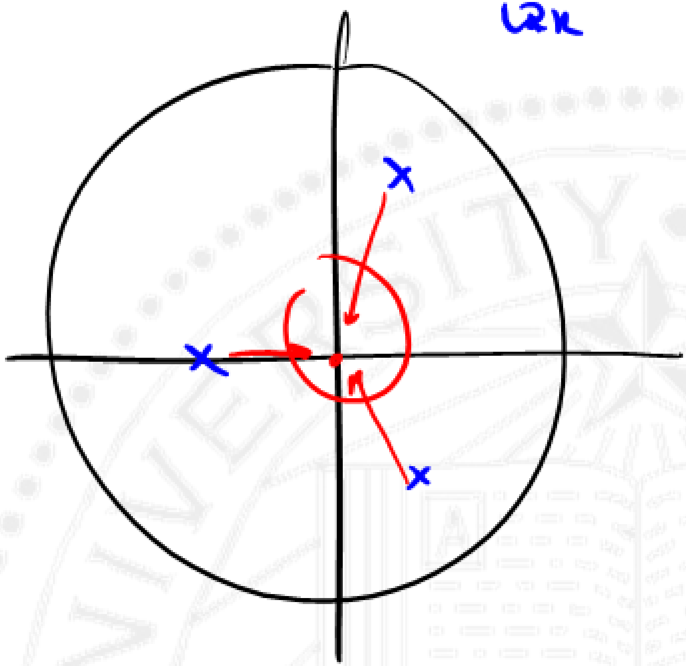
Solution: $u = -\underline{K}x$ ← optimum solution



$$\dot{\delta y} \approx \frac{\delta y_k - \delta y_{k-1}}{\Delta T}$$



$$x = \begin{bmatrix} \cancel{\dot{\delta y}} \\ \delta y \\ \delta \alpha \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \ddot{\delta y} \\ \dot{\delta y} \\ \dot{\delta \alpha} \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



LQR

Poles
5-10

Kalman Filter

$w \sim N(0, R_w)$

$v \sim N(0, R_v)$

process matrix

$$\dot{x} = Ax + Bu + \Gamma w$$

process noise

$$y = Cx + v$$

measurement noise

$R_w \gg R_v$ – believe measurements

$L \leftarrow$ big

$R_w \ll R_v$ – believe model

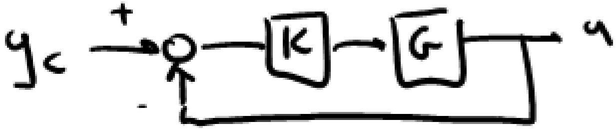
$L \leftarrow$ small.

$$\dot{\hat{x}} = A\hat{x} + Bu - L(y - \hat{y})$$

Kalman Filter design "L"

$L = \text{lqe}(A, \Gamma, C, R_w, R_v)$ – Kalman Filter (steady state)

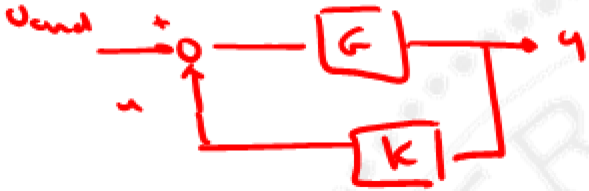
$$L^T = \text{lqr}(A^T, C^T, \underbrace{\Gamma R_w \Gamma^T}_{Q}, \underbrace{R_v}_{R})$$



output follows input

"AIRBUS"

Full AUTOPILOT



Augment stability of the plant

"747"

STABILITY AUGMENTATION

DIGITAL

$$\dot{x} = Ax + Bu \qquad \longrightarrow \qquad x_{k+1} = \Phi x_k + \Gamma u_k$$

$$y = Cx + Du \qquad \qquad y_k = Hx_k + Du_k$$

$\Phi = e^{A\Delta T}$ ← matrix exponential: $e^{At} \triangleq I + At + \frac{A^2t^2}{2!} + \dots$

$\Gamma = \left[\int_0^{\Delta T} e^{A\tau} d\tau\right] B \qquad H = C \qquad D = D$.

Discrete equivalents to $\dot{x} = Ax + Bu$, $y = Cx + Du$

c2d → 'zoh'

$$\text{lqr} \quad - \quad J = \int_0^{\infty} x^T Q x + u^T R u \, dt$$

$$\text{dlqr} \quad - \quad J = \sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k$$

lqr overloaded

$K = \text{lqr}(A, B, Q, R)$ $\qquad$ $K = \text{lqr}(\Phi, \Gamma, Q, R)$

[illegible]

$$u_k = -K(\hat{x}_k - \underline{x}_e)$$

$$x_{k+1} = \Phi x_k + \Gamma u_k$$

$$y_k = H x_k + D u_k$$

$$u_k = -K(\hat{x}_k - \underline{x}_k^c)$$

$$\hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k) = [\Phi - LH]\hat{x}_k + [\Gamma \;\; L]\begin{bmatrix} u_k \\ y_k \end{bmatrix}$$

$$\hat{y}_k = H x_k + D u_k$$

initialize $\hat{x}_k, u_k$

ΔT

measure y_k

$$\hat{x}_{k+1} = [\Phi - LH]\hat{x}_k + \Gamma u_k + L y_k$$

$$u_{k+1} = -K(\hat{x}_{k+1} - x_{k+1}^c)$$

$$\hat{x}_k \leftarrow \hat{x}_{k+1}$$

$$u_k \leftarrow u_{k+1}$$

Problems w/ Kalman

← Time varying are numerically unstable.

Square root form of K.F.

Information form of KF

$S = P^{-1}$

$P = E(\hat{x}^T \hat{x})$

Process noise } ergodic
measurement noise

↓

STAT'S DON'T CHANGE

"white"

$(0, \sigma^2)$

E.K.F

↑

$\dot{x} = f(x, u)$ at each $\hat{x}_k$

$A = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k,u_k}}$ $B = \frac{\partial f}{\partial u}\Big|_{\hat{x}_{k,u_k}}$

Continuous Time

$$\phi = e^{A\Delta T}$$

$$\Gamma = \int_0^{\Delta T} e^{A\tau}\, d\tau \cdot B$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$u = -Kx \;/\; -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x} + Du$$

$K = \mathrm{place}(A, B, P_{con})$

$L^T = \mathrm{place}(A^T, C^T, P_{obs})$

$K = \mathrm{lqr}(A, B, Q, R)$

$L = \mathrm{lqe}(A, B, \Gamma, R_w, R_v)$

Digital

$$x_{k+1} = \phi x_k + \Gamma u_k$$

$$y_k = Hx_k + Du_k$$

$$u_k = -Kx_k \;/\; -K\hat{x}_k$$

$$\hat{x}_{k+1} = \Phi\hat{x}_k + \Gamma u_k + L(y_k - \hat{y}_k)$$

$$\hat{y}_k = Hx_k + Du_k$$

$K = \mathrm{place}(\Phi, \Gamma, P_{con})$

$L^T = \mathrm{place}(\Phi^T, H^T, P_{obs})$

$K = \mathrm{dlqr}(\Phi, \Gamma, Q, R)$

$L = \mathrm{dlqe}(\Phi, H, \Gamma, R_w, R_v)$

$\Phi = e^{AT_s}$ ← matrix exponential (expm)

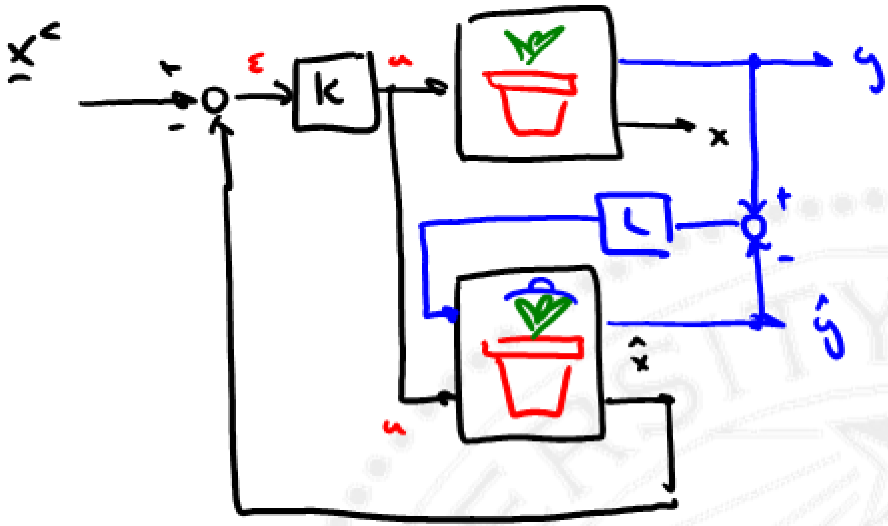
$e^{At} \triangleq I + At + \frac{A^2t^2}{2!} + \dots +$

$\Gamma = \int_0^{\Delta T} e^{A\tau} d\tau \cdot B$ — zoh equivalent for input

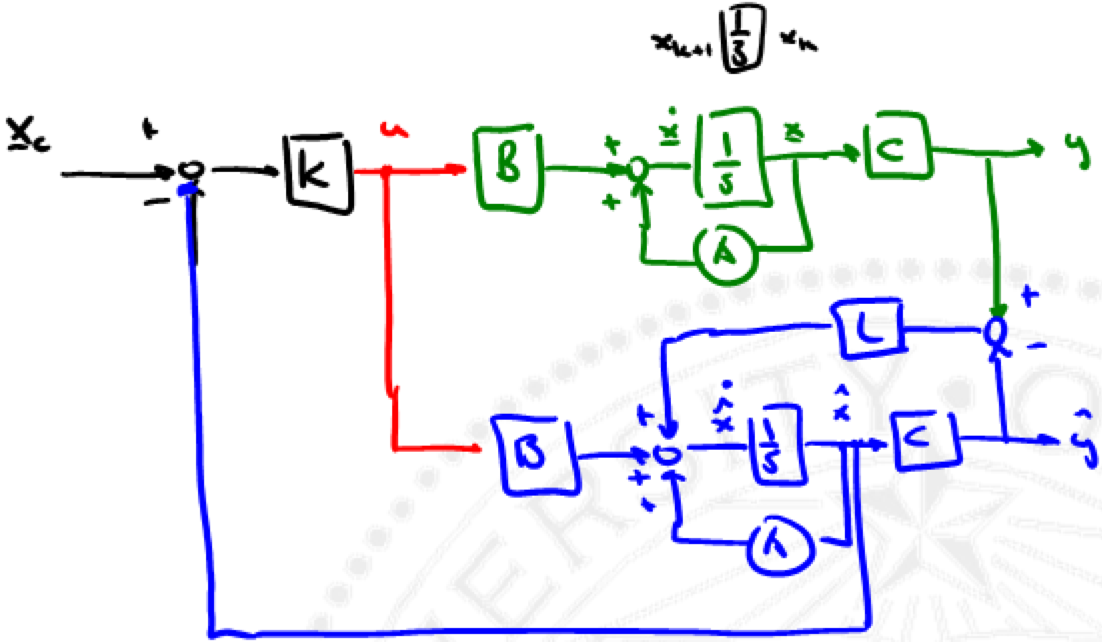
$\dot{x} = Ax \rightarrow x(t) = e^{At} x(0)$

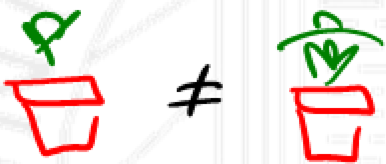
c2d 'zoh'

$R_w \neq R_{w_k}$ ← disrw( )

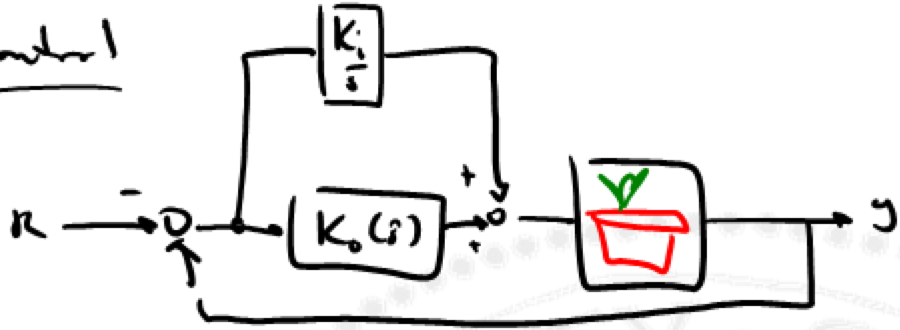


x
r
ε
K
u
y
x
L
u

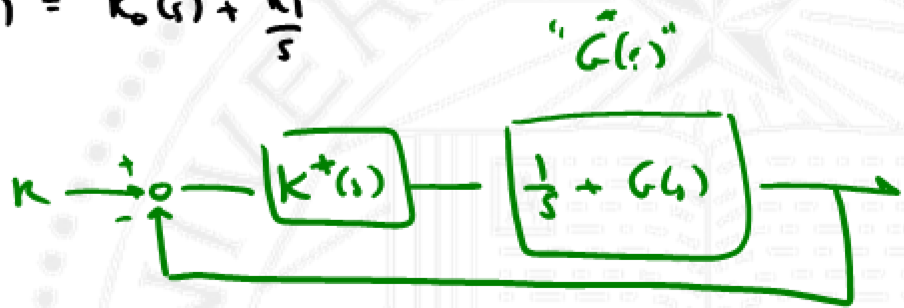




Integral Control



$$K^*(s) = K_0(s) + \frac{K_i}{s}$$



$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$e = r - y = r - Cx$$

$$x_I = \int e\,dt \quad \therefore \quad \dot{x}_I = e = r - Cx$$

$$\dot{\begin{bmatrix} x \\ \hline x_I \end{bmatrix}} = \underbrace{\left[\begin{array}{c|c} A & 0 \\ \hline -C & 0 \end{array}\right]}_{A^{aug}} \begin{bmatrix} x \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ \hline 0 \end{bmatrix}}_{B^{aug}} u + \begin{bmatrix} 0 \\ \cdots \\ I \end{bmatrix} r$$

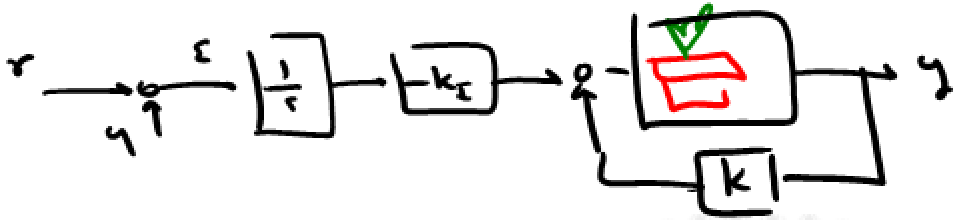
$$u = -\begin{bmatrix} K & K_I \end{bmatrix} \begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix}$$

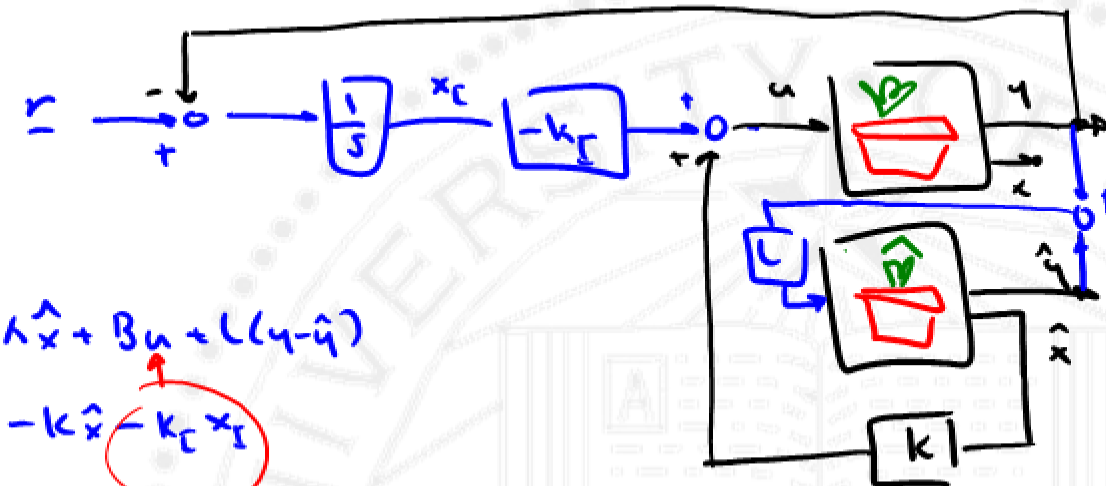
$$\operatorname{eig}\left(A^{aug} - B^{aug} \begin{bmatrix} K & K_I \end{bmatrix}\right)$$

$\underline{x} \in \mathbb{R}^5 \qquad u \in \mathbb{R}^2$

- Drive as many states to a reference as I have actuators/inputs

- Drive number of linear combination of states to reference as I have actuators/inputs





$$\dot{\hat{x}} = A\hat{x} + Bu + L(y-\hat{y})$$

$$u = -K\hat{x} - K_I x_I$$

$$\frac{d}{dt}\underbrace{\begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}}_{A_a} \underbrace{\begin{bmatrix} x \\ \cdots \\ x_I \end{bmatrix}}_{x_a} + \underbrace{\begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}}_{B_a} \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{u_a}$$

$$\dot{\underline{x}}_a = A_a \underline{x}_a + B_a \underline{u}_a$$

$$y_a = C_a x_a$$

$$\dot{\hat{x}}_a = A_a \hat{x}_a + B_a \underline{u}_a + L(y_a - \hat{y}_a)$$

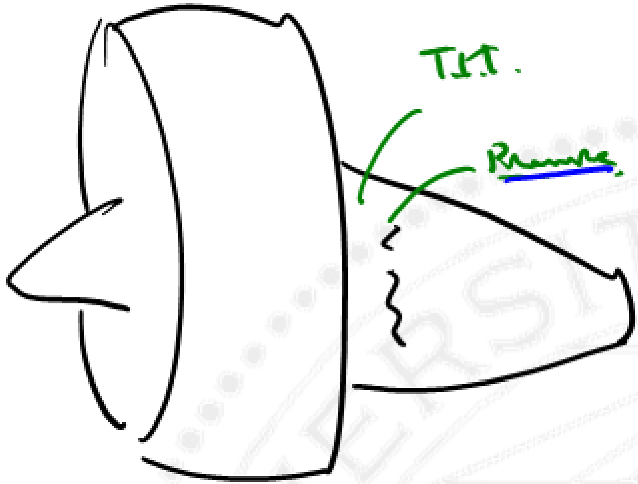
$$y_a = C_a x_a$$

$$u_a = -K x_a = -K\hat{x} - K_I \hat{x}_I .$$

reduced order estimator

x_I

REDUCED ORDER ESTIMATOR



$$u = -K\hat{x}$$

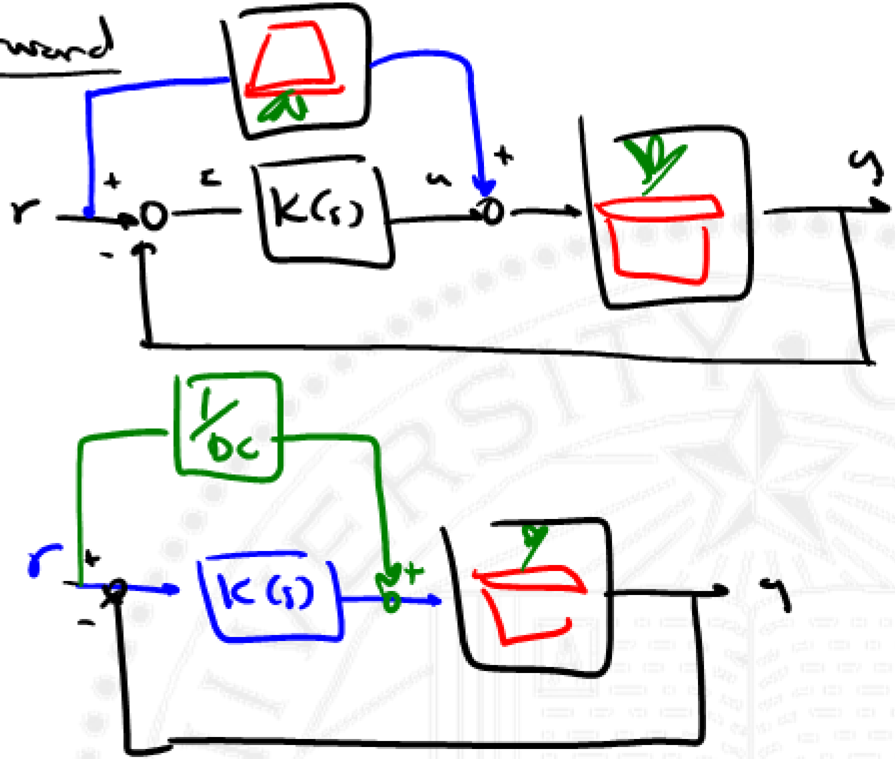
$\hat{x} = \begin{bmatrix} \\ 0 \\ 0 \\ \\ 0 \end{bmatrix} \begin{matrix} \\ \leftarrow y_1 \\ \leftarrow y_2 \\ \\ \leftarrow y_3 \end{matrix}$

$u = -K\hat{x}$

$\hat{x} = \begin{bmatrix} () \\ \\ \end{bmatrix}$

$x_m \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Feedforward



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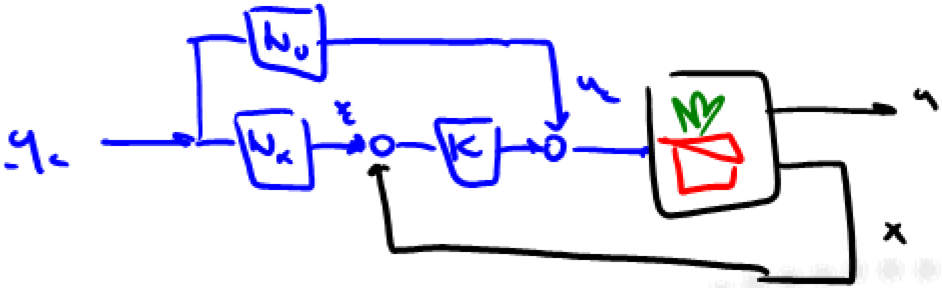

N_x converts $y_c \rightarrow \underline{x}_c$

FPE 7.10

N_u converts $y_c \rightarrow y_{ss}$