CMPE-242
Applied Feedback Control

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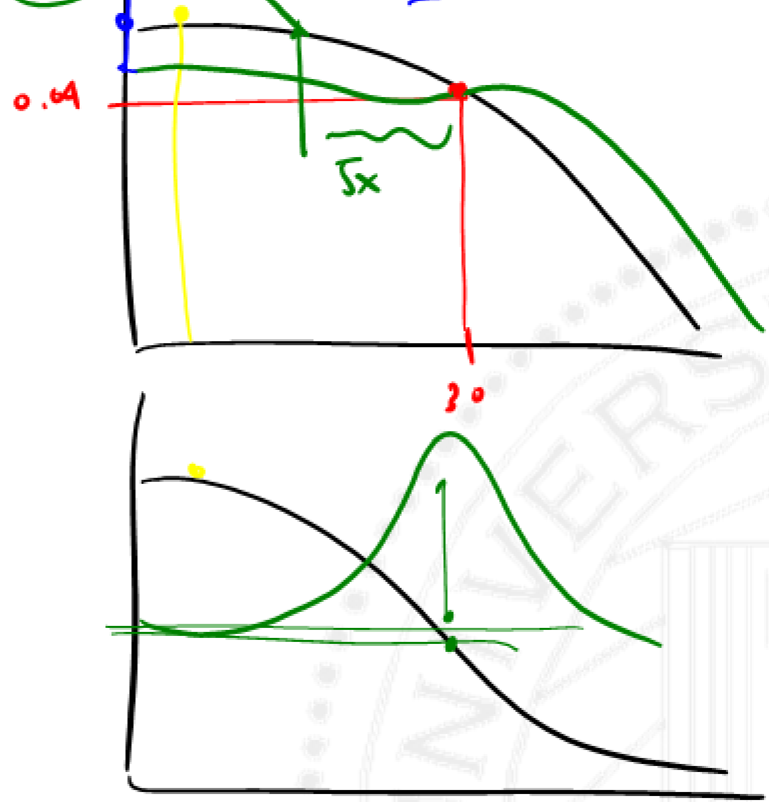
Winter 2016

Announcements

(1) Midterms are graded — /100 $\mu = 39$ $\sigma = 18$ 70 | 11

$e_{ss} < 1\%$



$\boxed{4.75\ \text{Hz}}$ (note units)

↳ 30 rad/sec.

$K_0 = \frac{1}{0.04} = 25$

2x LEAD $\underline{55°}$

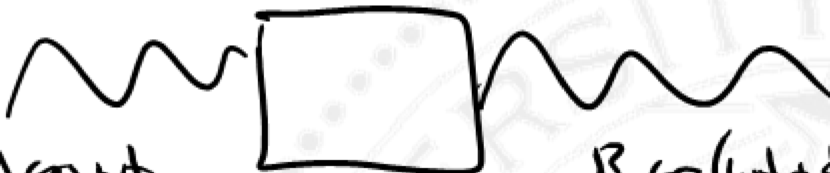
$\sqrt{ab} = 30$

$\frac{b}{a} = 10$

$K_0\left(k\frac{s+a}{s+b}\right)^2 - 250\left(\frac{s+a}{s+b}\right)^2$

$$\frac{E}{R} = \frac{1}{1+GK} \qquad |GK| \qquad \frac{E}{R} < .01$$

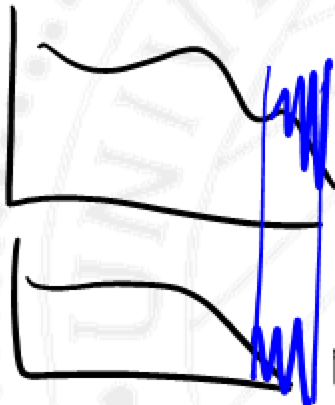
$$|GK| > 100.$$



$A \cos \omega t$ $\qquad$ $B \cos(\omega t + \phi)$

$\left|\frac{B}{A}\right|$

$\angle \phi$

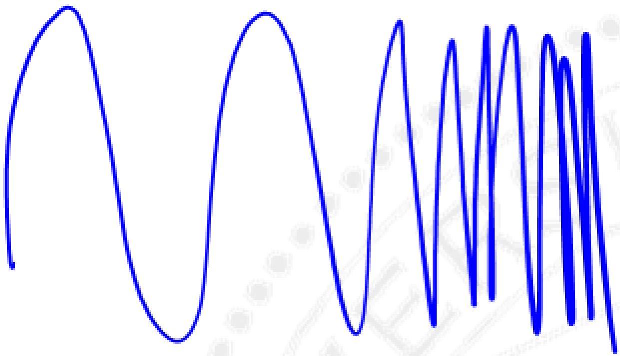


log ω

log ω

"periodogram"

chirp



Control

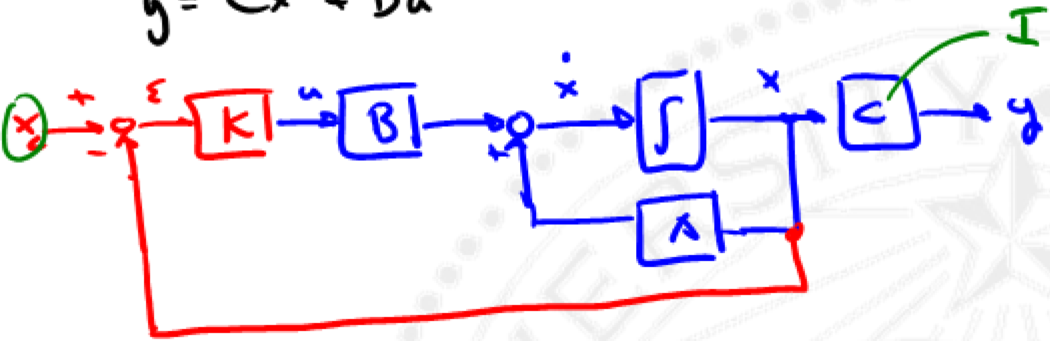
$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

commanded state

$$u = -K(x - x_c)$$

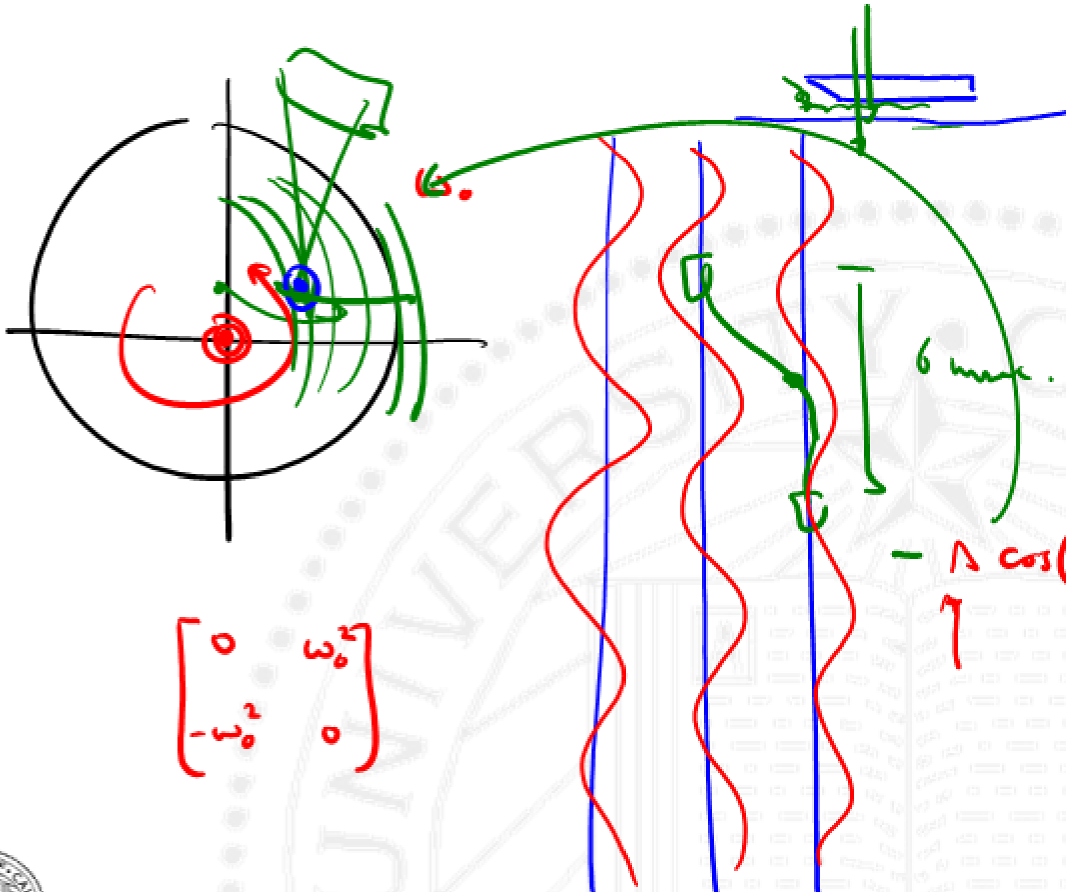
Must measure full state



Form $\mathbb{C} = [B \; AB \; A^2B \; \dots \; A^{n-1}B]$ $\text{rank}(\mathbb{C}) = n$

$\text{rcond}(\mathbb{C}) = 1 \rightarrow 0$.

put poles of $A-BK$ anywhere limit.



$$-A\cos(\omega_0 t+\phi)$$

$$\begin{bmatrix} 0 & \omega_0^2 \\ -\omega_0^2 & 0 \end{bmatrix}$$

Kalman Filter

$$\dot{x} = Ax + Bu + \Gamma w$$ process noise

$$y = Cx + v$$ measurement noise

$w \sim N(0, R_w)$

$v \sim N(0, R_v)$

GAUSSIAN DISTRIBUTION

BLUE — Best Linear Unbiased Estimator

small L ← trust model, don't trust measurements

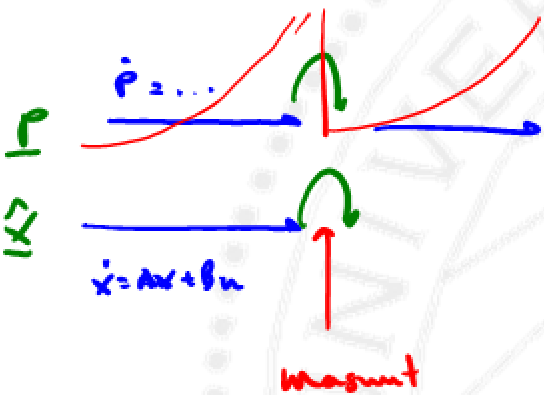
big L ← trust measurements don't trust model

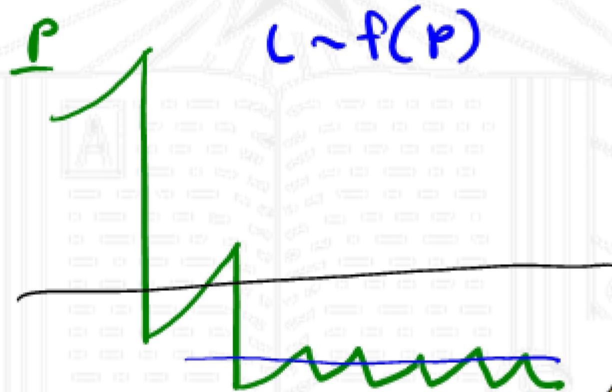
Kalman Filter — lqe ← matlab   linear quadratic estimator

P ← covariance matrix $E[\tilde{x}^T \tilde{x}]$

True Kalman Filter → continuously variable linear estimator.

Does not need measurements at fixed time intervals





FVT :

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

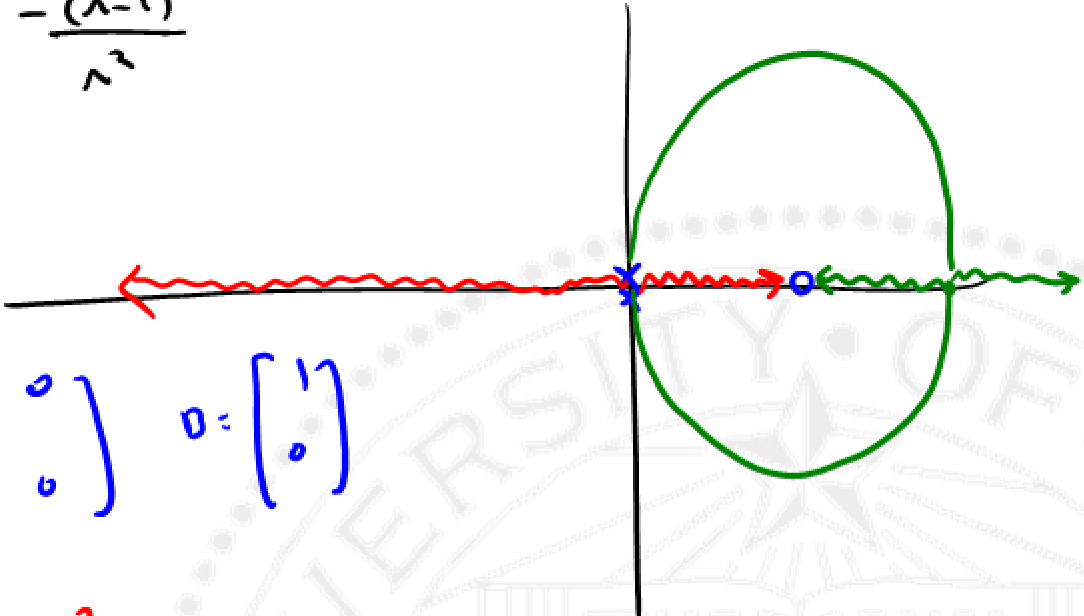
$\Rightarrow$ FVT $\rightarrow$ $\dot{x} = \phi$.

$$\overset{n\times 1}{\phi} = Ax_{ss} + Bu_{ss} \qquad x_{ss} = -A^{-1}Bu_{ss}$$

$$y_{ss} = Cx_{ss} + Du_{ss} = C[-A^{-1}B]u_{ss} + Du_{ss}$$

$$= \underbrace{[-CA^{-1}B + D]}_{\text{DC gain matrix.}} u_{ss}$$

$$G(s) = \frac{-(s-1)}{s^2}$$



$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

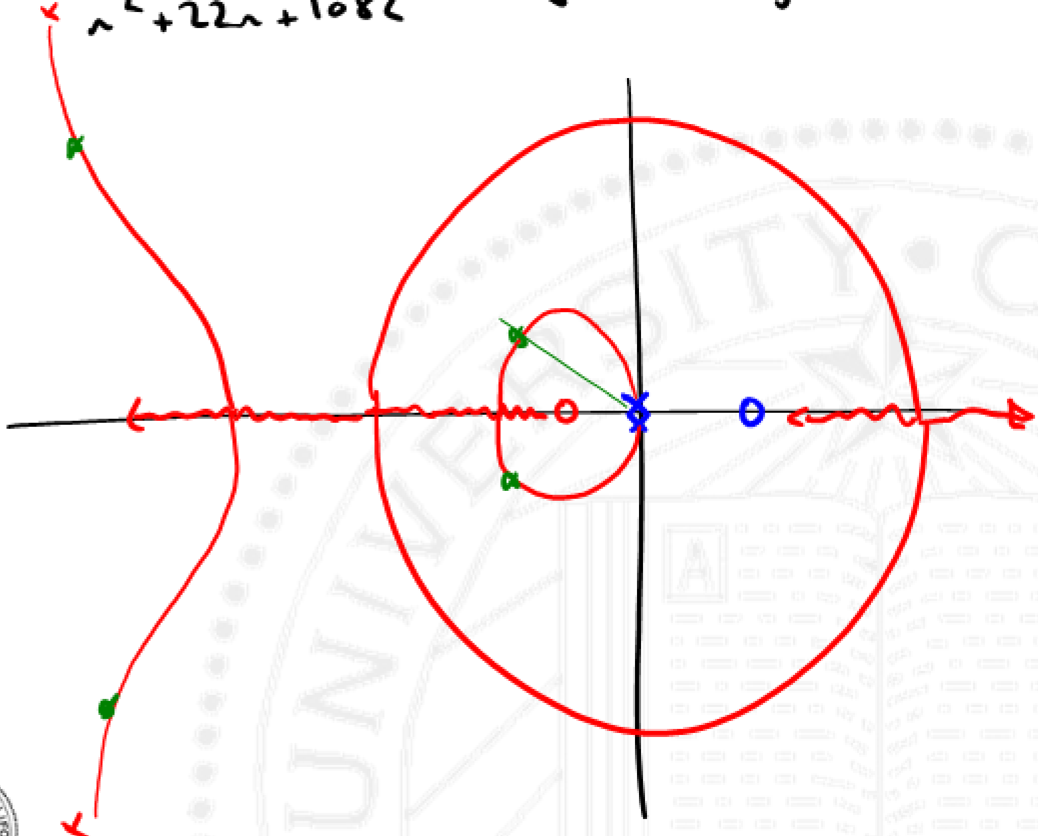
$\mathcal{C}$ – unchanged – full rank

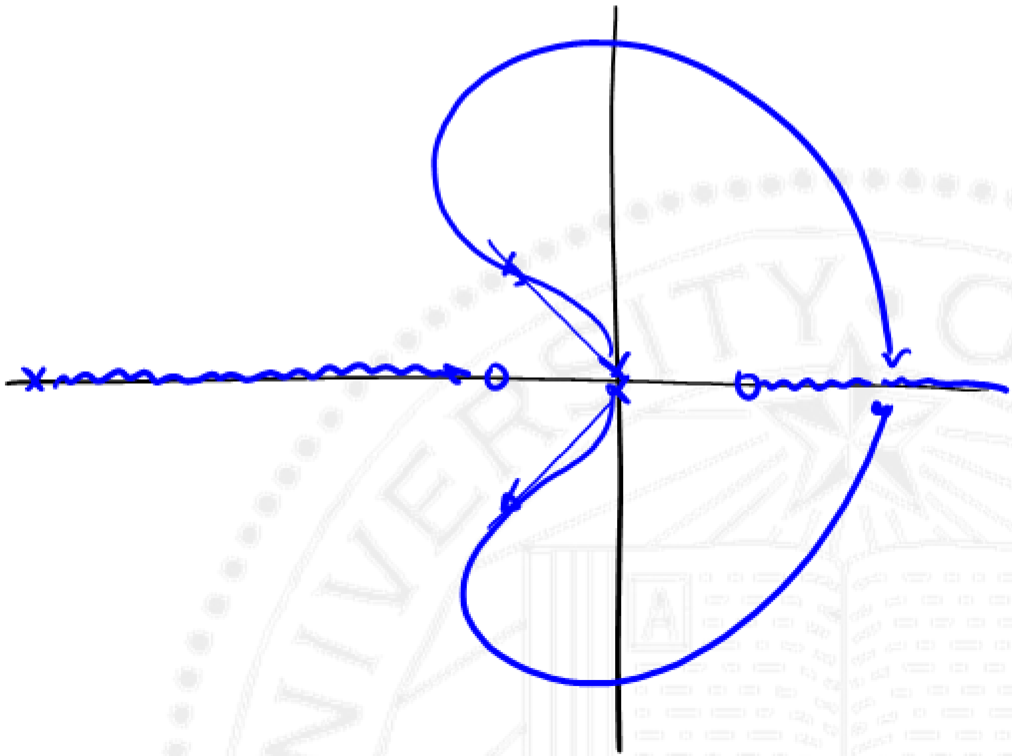
$$\mathcal{O} = \begin{bmatrix} C \\ AC \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{full rank}$$

$$P_{des} = -1 \pm j$$

$$P_{est} = -10 \pm 10j$$

$$K(s) = \frac{840\,(s+0.476)}{s^2+22s+1082} \leftarrow -11 \pm 31j$$

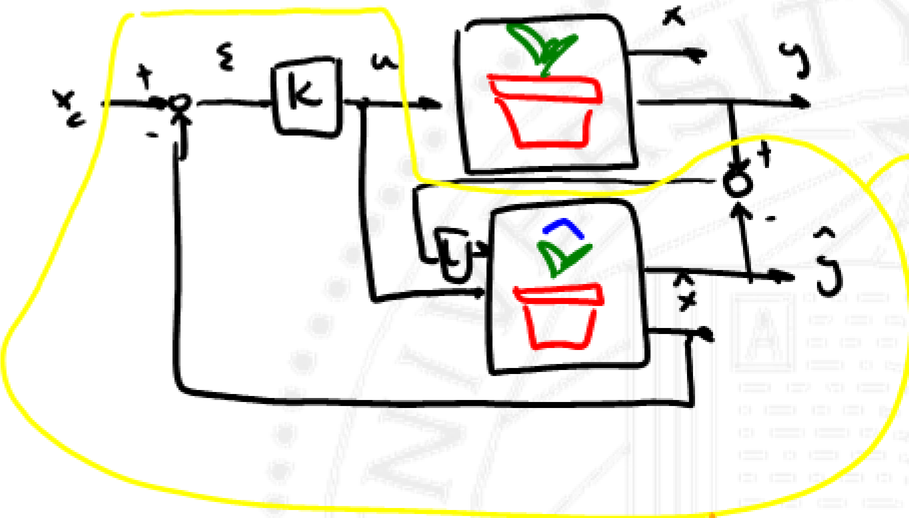




$K = \text{place}(A, B, P_{contrl})$ "$A - BK$"

$u = -K(\hat{x} - x_c)$

$L^T = \text{place}(A^T, C^T, P_{est})$ "$A - LC$"



$K(s)$

$$\dot{\hat{x}} = (A - LC)\hat{x} + [B \; L]\begin{bmatrix} u \\ y \end{bmatrix}$$

$$u = -K\hat{x}$$

$$\frac{U}{Y}(s) = -K[sI - (A - BK - LC)]^{-1} L \leftarrow \text{compensator}$$

Ways to choose L

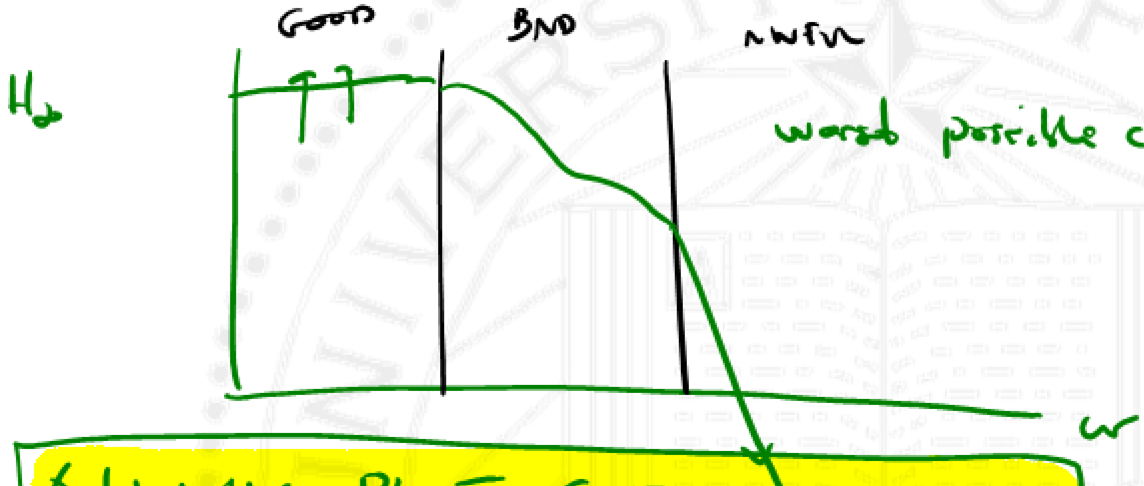
$$L^T = \text{place}(A^T, C^T, P_{obs}) \qquad P_{obs} \approx 5\text{-}10 \times P_{control}$$

Kalman Filter ~ R_w, R_v

Better ways to choose Pole locations

H_2 — LQR — min 2 norm of $x^T x$ w.r.t. u

H_∞ — " ∞ norm of x^T w.r.t.



ALWAYS PLOT CONTROL EFFORT

$\dot{x} = Ax + Bu \qquad u = -K(x - x_c) = K(x_c - x)$

$$\dot{x} = (A - BK)x + BK\underline{x}_c$$

$$y = (C - DK)x + DK\underline{x}_c$$

$$\begin{bmatrix} y \\ \hdashline u \end{bmatrix} = \begin{bmatrix} C - DK \\ \hdashline -K \end{bmatrix} \underline{x} + \begin{bmatrix} DK \\ \hdashline K \end{bmatrix} \underline{x}_c$$ ← extend control effort.

LQR control – Linear Quadratic Regulator

$$\min_K J = \int_0^\infty (x^TQx + u^TRu)\,dt$$

s.t.: $\dot{x} = Ax + Bu$
$y = Cx + Du$

Algebraic Riccati Equation:

$$\left(A^TP + P^TA - P^TBR^{-1}B^TP + Q = \phi\right) \leftarrow \text{soln for } P$$

$$u = -\underbrace{R^{-1}BP}_{K}x$$

$$\boxed{u = -Kx}$$

$K = \mathrm{lqr}(A, B, Q, R)$

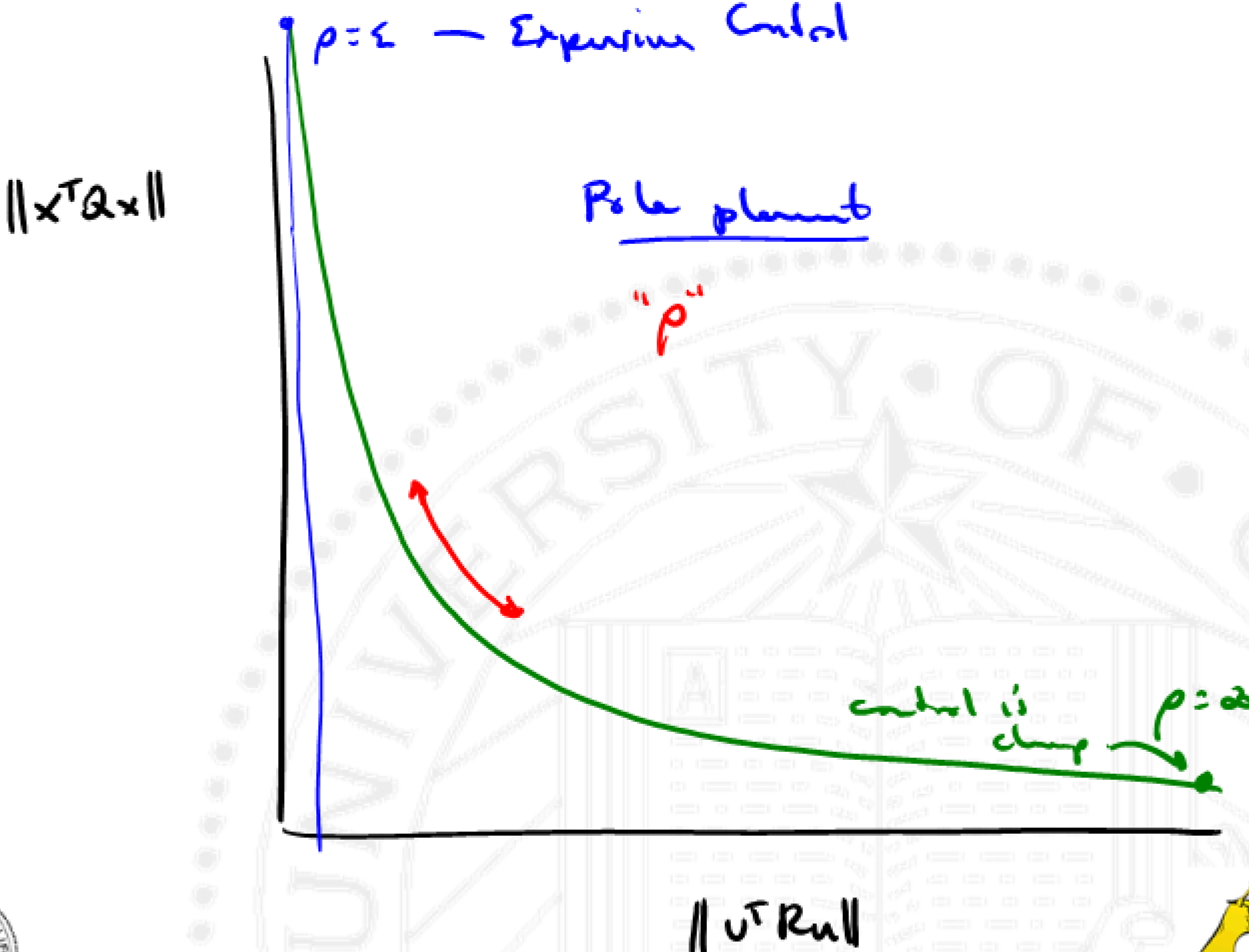
tuning knobs ...

$\rho \to \varepsilon$: huge control effort

$Q, R \leftarrow$ Bryson's Rule

$$Q = \rho \begin{bmatrix} \frac{1}{x_{1,max}^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{x_{n,max}^2} \end{bmatrix} \qquad R = \begin{bmatrix} \frac{1}{u_{1,max}^2} & & \\ & \ddots & \\ & & \frac{1}{u_{m,max}^2} \end{bmatrix}$$

$$J = \int_0^\infty \Big\{ \underbrace{\rho \Big[\Big(\frac{x_1}{x_{1,max}}\Big)^2 + \Big(\frac{x_2}{x_{2,max}}\Big)^2 + \cdots + \Big(\frac{x_n}{x_{n,max}}\Big)^2 \Big]}_{\text{tune } \rho} + \Big[\Big(\frac{u_1}{u_{1,max}}\Big)^2 + \cdots + \Big(\frac{u_m}{u_{m,max}}\Big)^2 \Big] \Big\} dt$$


ρ=ε — Expensive Control
||x^T Q x||
Pole placement
"ρ"
control is cheap
ρ=∞
||u^T R u||

LQR — full state feedback

$\mathbb{C}$ full rank

design knob

Bryson's Rule $\rightarrow$ Q, R tune w/ ρ.

$$\boxed{PM \gtrsim 50^\circ \qquad \tfrac{1}{2} < GM < 2}$$

$$J = \int_0^\infty \left(\rho\, y^T Q_y y + u^T R u\right) dt \qquad y = Cx, \quad y^T = x^T C^T$$

$$J = \int_0^\infty \rho\, x^T \underbrace{C^T Q_y C}_{Q} x$$

$$Q_y = \begin{bmatrix} 1/y_{1,max}^2 & & \\ & \ddots & \\ & & 1/y_{r,max}^2 \end{bmatrix}$$

"Optimal Control" — LQ"R" regulator

"E" estimator/Kalman

"G" gaussian (Both)

$K = \text{lqr}(A, B, Q, R)$

$L = \text{lqe}(A, C, R_w, R_v)$

$K = \text{lqy}(A, B, Q_y, R)$ ← check

LQR performance

$PM \geq 50^\circ$

$GM \; -\frac{1}{2} \rightarrow (2, \infty)$

John Doyle — Performance Guarantees of LQG control

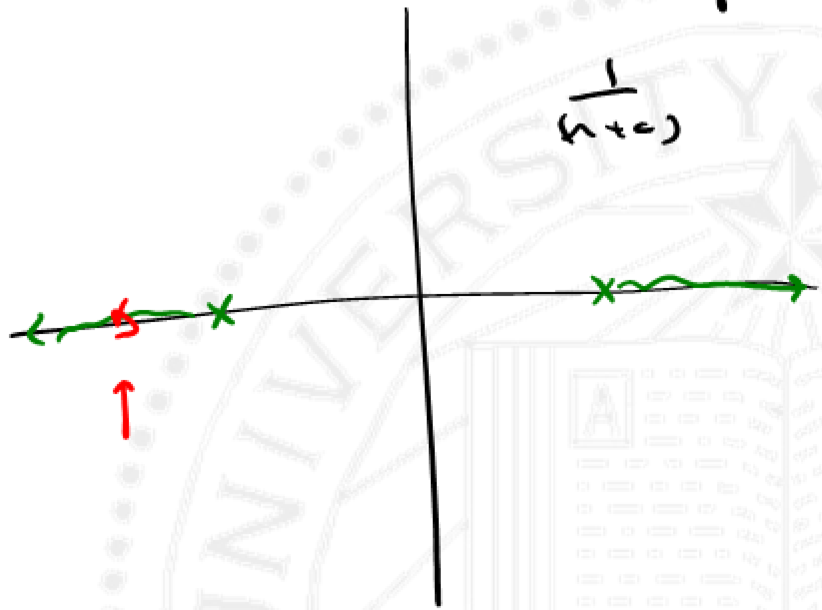
Abstract: There are none.

SISO :

$$J = \int_0^\infty (\rho y^2 + u^2)\, dt$$

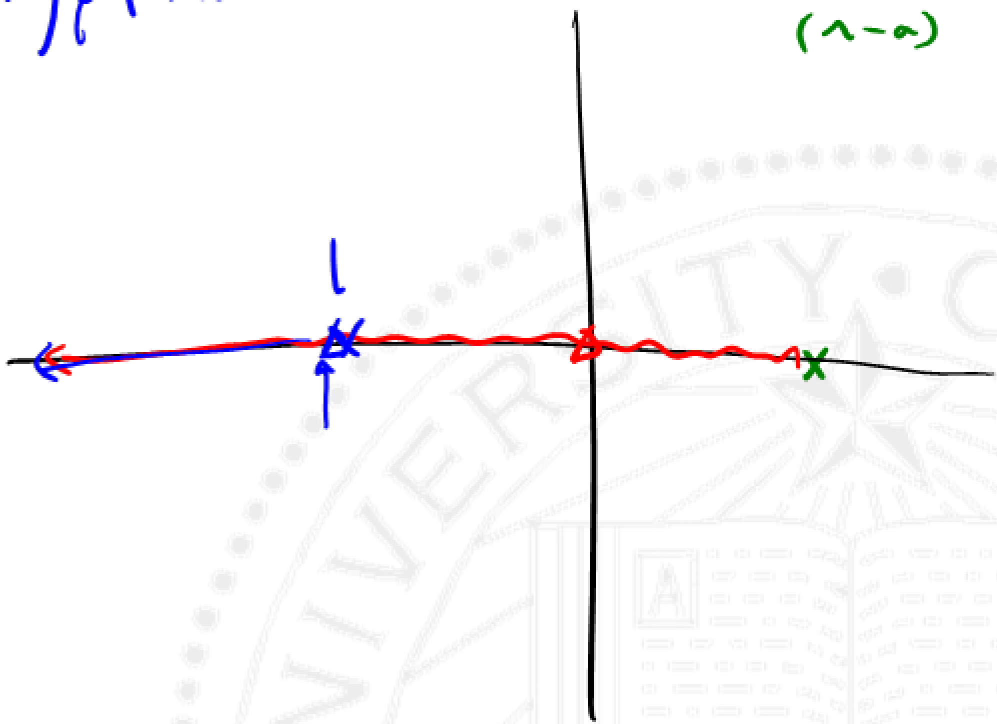
Soln is symmetric root locus.

$$1 + \rho G(s) G(-s) = 0$$

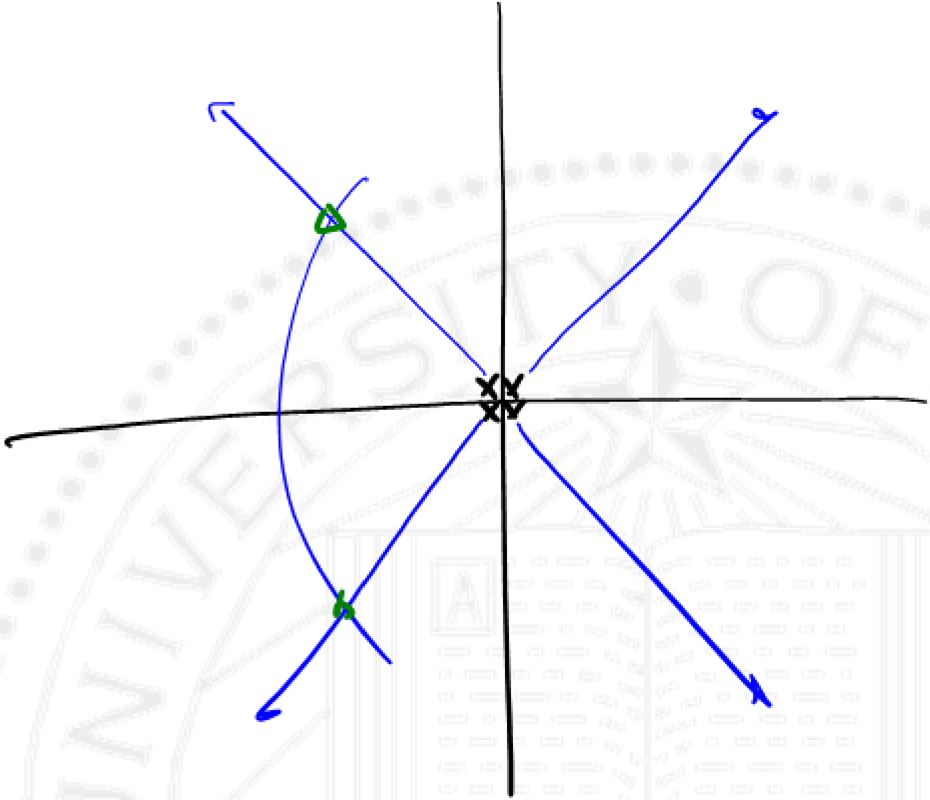


$$J = \int \rho y^2 + u^2$$

$$\frac{1}{(s-a)}$$

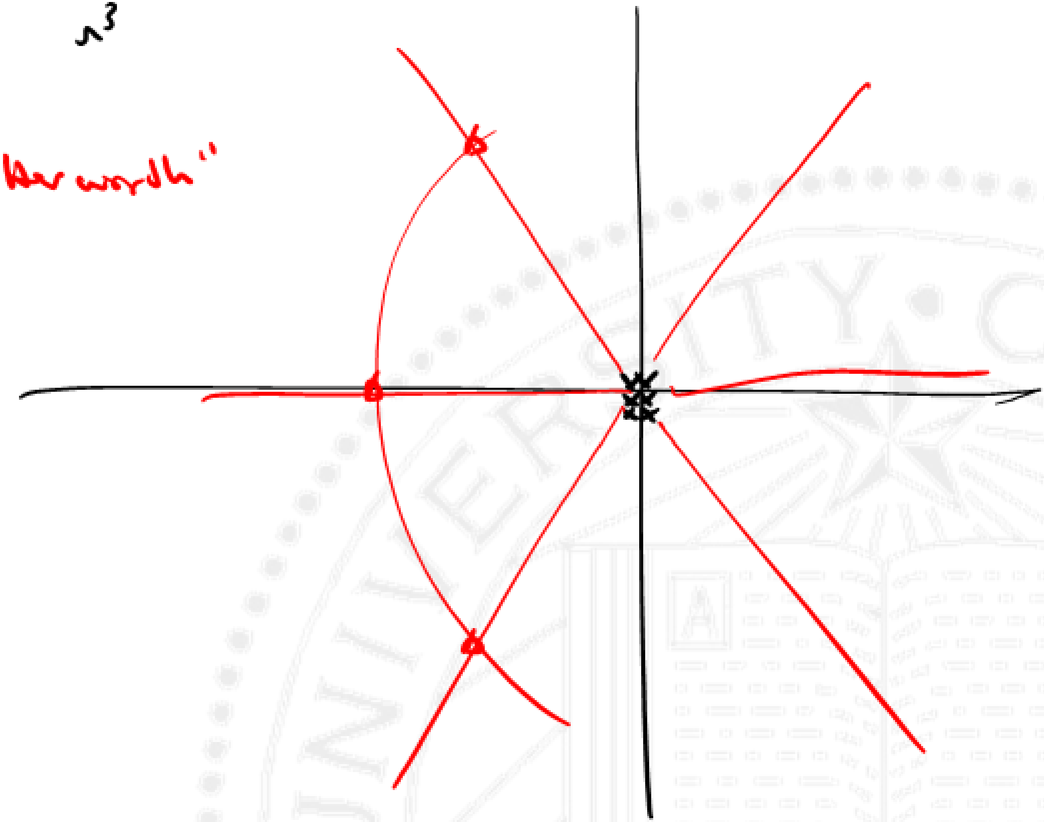


$G(s) = \frac{1}{s^2}$

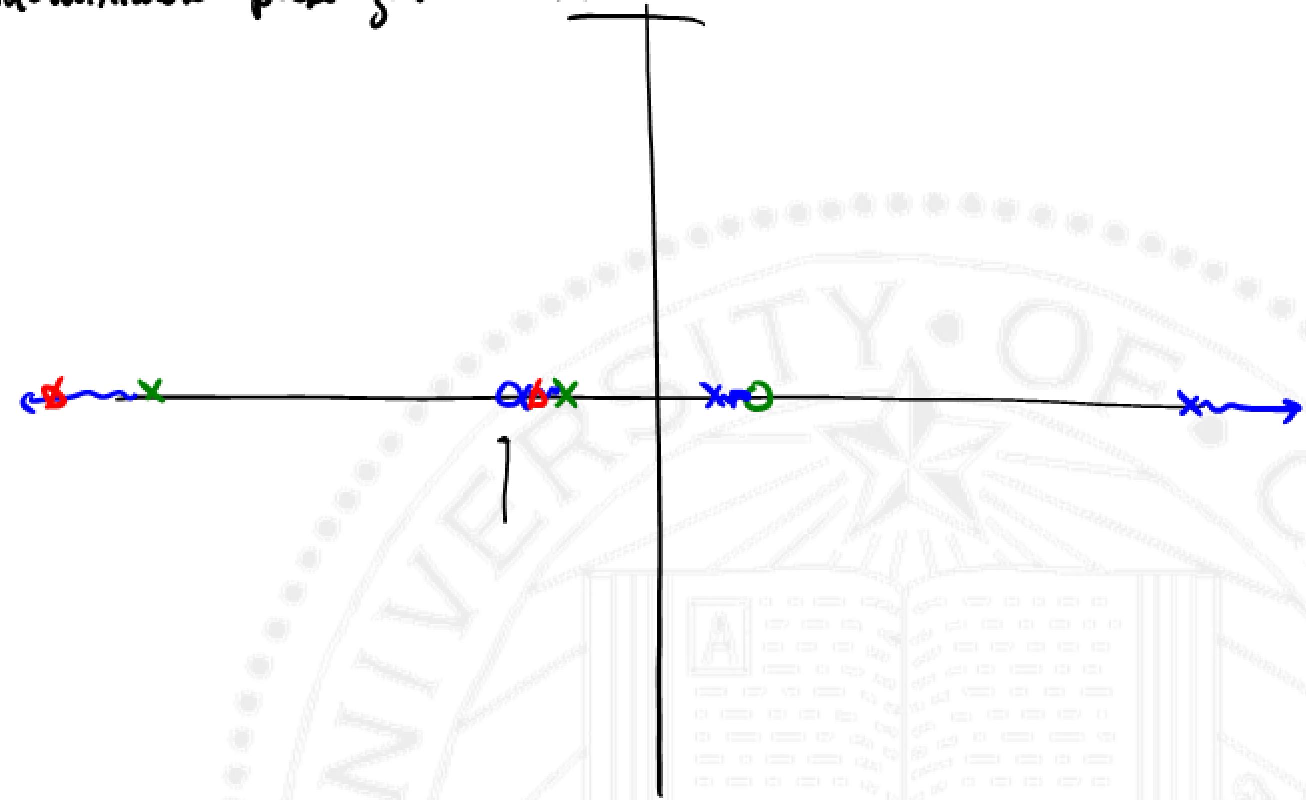


$$G(s) = \frac{1}{s^3}$$

"Butterworth"



non-minimum phase zero "in RHP"



$$G(s) = \frac{1}{s^2} \qquad A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$\rho y^T Q_y y = \rho x^T \begin{bmatrix} 0 \\ 1 \end{bmatrix} q_y \begin{bmatrix} 0 & 1 \end{bmatrix} x + u^T \underset{\overset{\uparrow}{1}}{R_u} u$$

$$\underbrace{\frac{\rho}{q_{max}^2} x^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x + u^T u}_{J}$$

$$K = \mathrm{lqr}(A, B, Q, 1) \quad — \quad \mathrm{eig}(A - BK)$$

ρ	K	$eig(A-BK)$
1	$\sqrt{2}$ 1	$-0.7 \pm .7j$
4	2 2	$-1 \pm j$
100	4.7 10	$-22 \pm 22j$