CMPE-242
Applied Feedback Control

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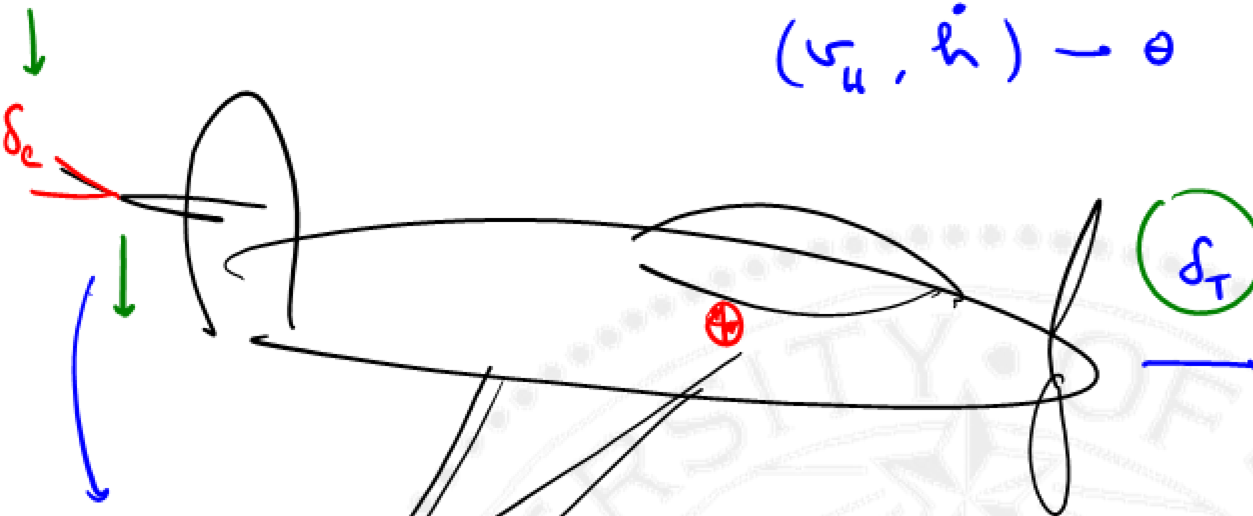
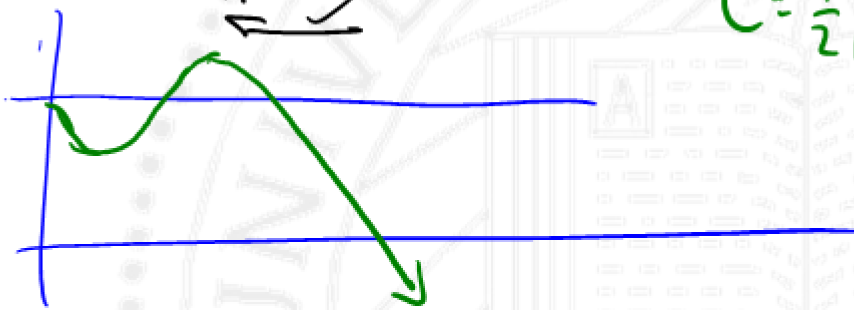
$(v_u, \dot{h}) \rightarrow \theta$



$L = \frac{1}{2}\rho V^2 C_L \alpha$



$$\left[B \right] \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

$$\boxed{C_{r, B_{\delta_e}}}$$

$$\boxed{C_{h, B_{\delta_T}}}$$

State Space Control

→ regulator design $(r = 0)$

$\dot{x} = Ax + Bu \qquad u = -Kx$

$y = Cx + Du$

"Full State Feedback"

Choose K to put poles in desirable locations

"acker" / "place"

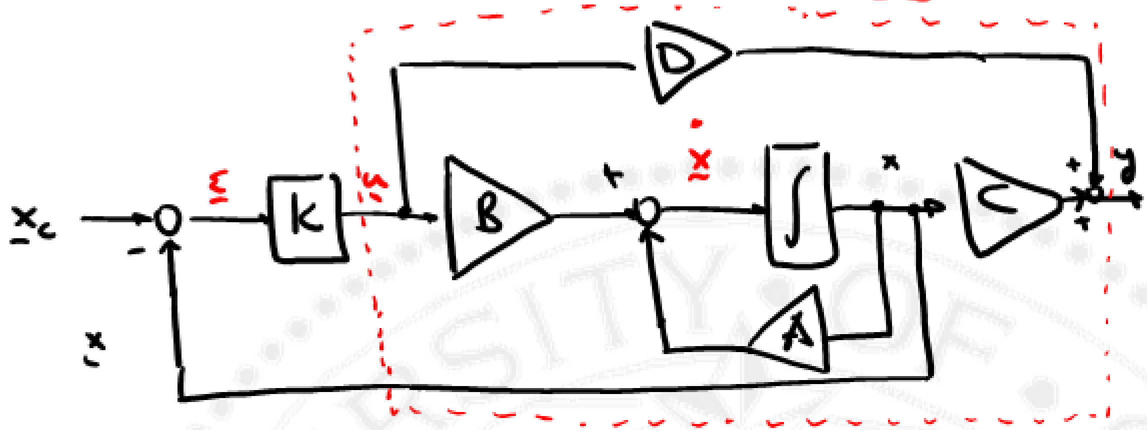
- Check rank of $\mathcal{C} \triangleq [B \;\; AB \;\; A^2B \cdots A^{n-1}B]$

  cond $(\mathcal{C})$

- Must measure ALL states.

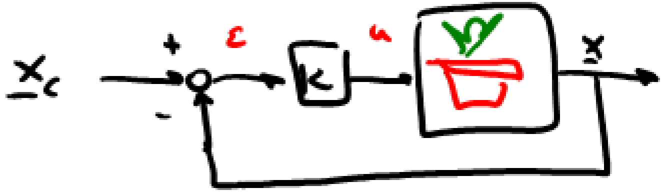
$\dot{x} = Ax + Bu \quad u = -K(x - x_c) \quad u = K(x_c - x)$

$y = Cx + Du$

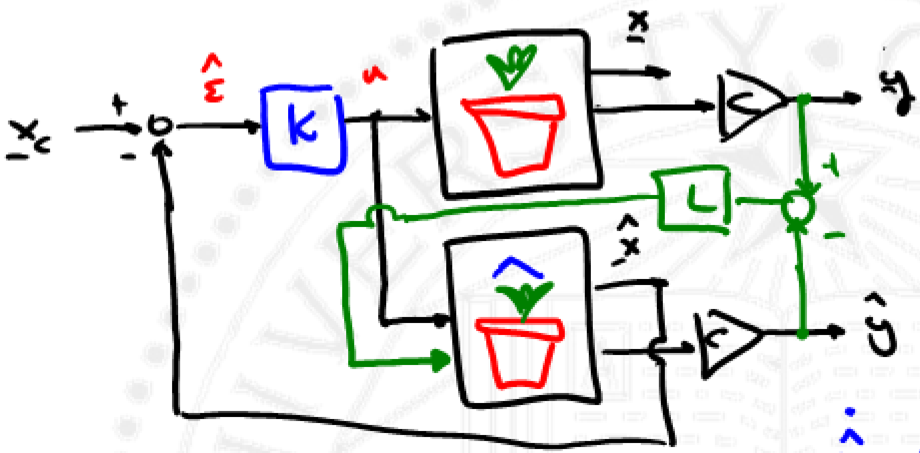


if I have $\underline{x}$, and C is full rank, I have very good control.

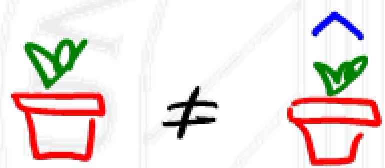
what if I don't have $\underline{x}$?



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$u = -K(x - x_c)$$



$$u = -K(\hat{x} - x_c)$$
$$\dot{\hat{x}} = A\hat{x} + Bu$$
$$\hat{y} = C\hat{x} + Du$$



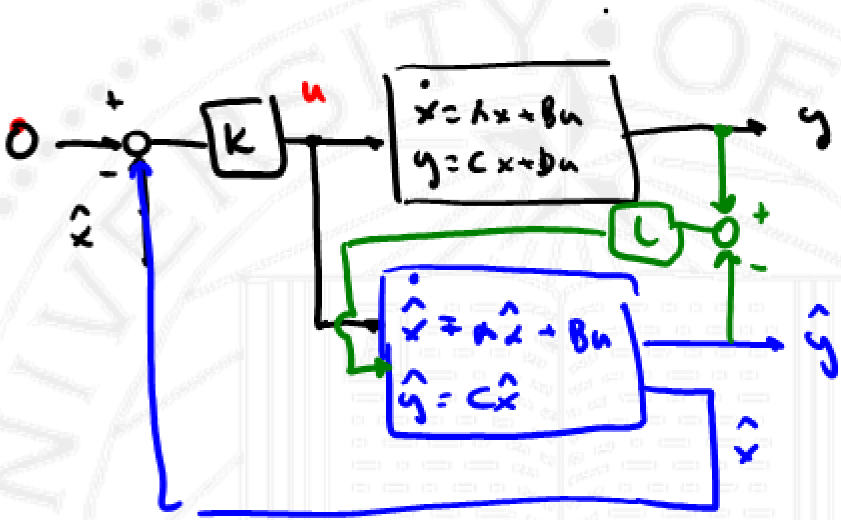
$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L\underbrace{(y-\hat{y})}$$

$$\hat{y} = C\hat{x}$$



$$\dot{x} = Ax + Bu \qquad u = -K\hat{x}$$

control gain

$$y = Cx$$

$$\tilde{x} \triangleq x - \hat{x}$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

estimator gain

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y-\hat{y})$$

$$\dot{\hat{x}} = \underbrace{(A-BK)}_{\text{stable}}\hat{x} + L(y-\hat{y})$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu} - L(y-\hat{y})$$

$$= A\tilde{x} - Ly + L\hat{y} = A\tilde{x} - LCx + LC\hat{x}$$

$$= A\tilde{x} - LC(\underbrace{x-\hat{x}}_{\tilde{x}}) = \underbrace{(A-LC)}_{\text{stable}}\tilde{x}.$$

For using estimator

desired poles

$(A-BK)$ control gain $\qquad K = \text{place}(A, B, P_{des})$

$$\begin{bmatrix} B \end{bmatrix} \begin{bmatrix} K \end{bmatrix}$$

desired poles

$(A-LC)$ estimator $\qquad L^T = \text{place}(A^T, C^T, P_{des})$

$$\begin{bmatrix} L \end{bmatrix} \begin{bmatrix} C \end{bmatrix}$$

5-10x faster than control

$A^T - C^T L^T$ "dual" $A - BK$

For control:

$$\mathcal{C} \triangleq [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B]$$

full rank controllable

cond ($\mathcal{C}$)

$$\mathcal{C}_{Dual} = [C^T \;\; A^TC^T \;\; (A^2)^TC^T \;\; \cdots \;\; (A^{n-1})^TC^T]$$

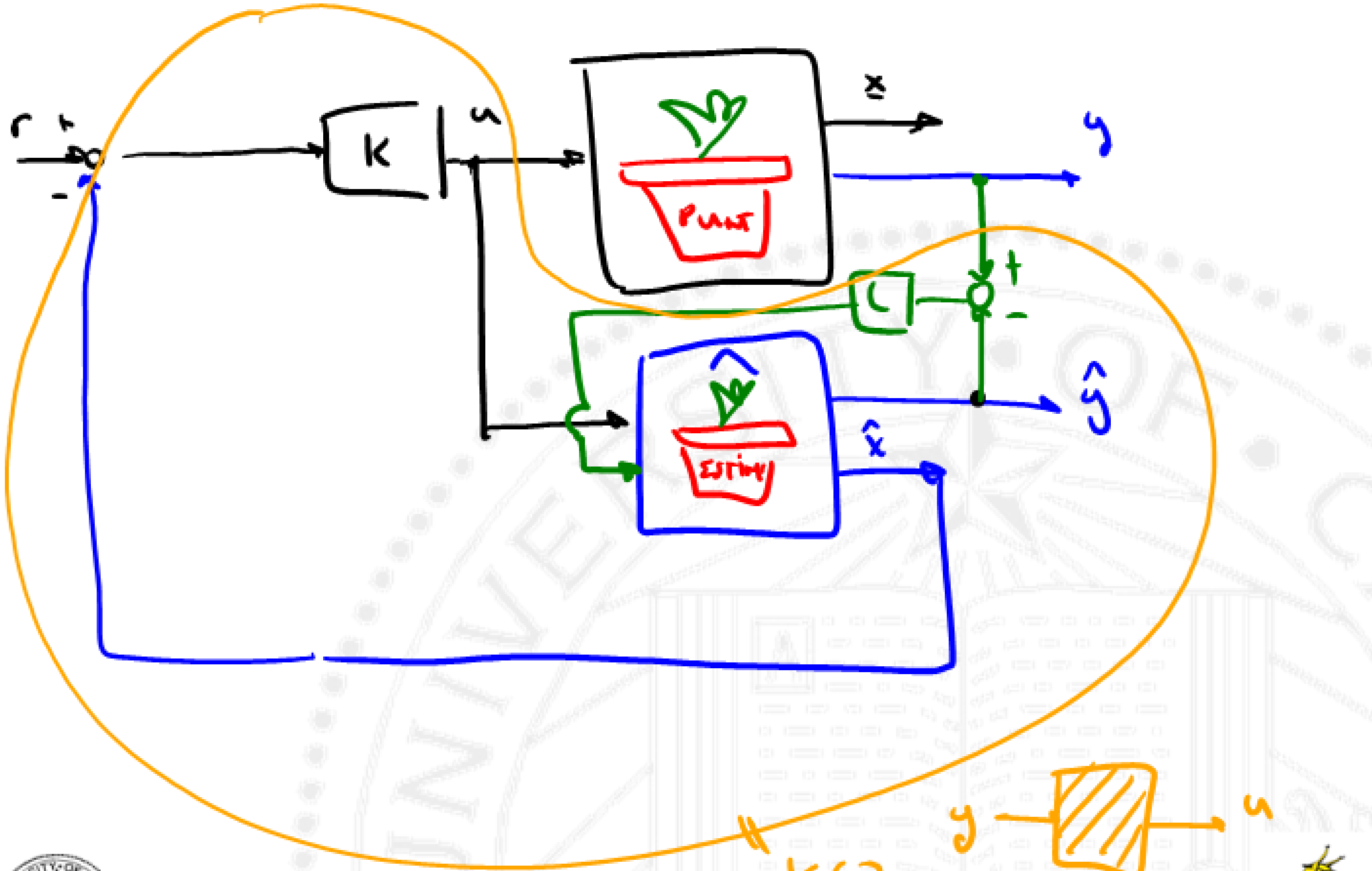
obsv(A,C)

$$\mathcal{O} \triangleq \begin{bmatrix} C \\ AC \\ A^2C \\ \vdots \\ A^{n-1}C \end{bmatrix}$$

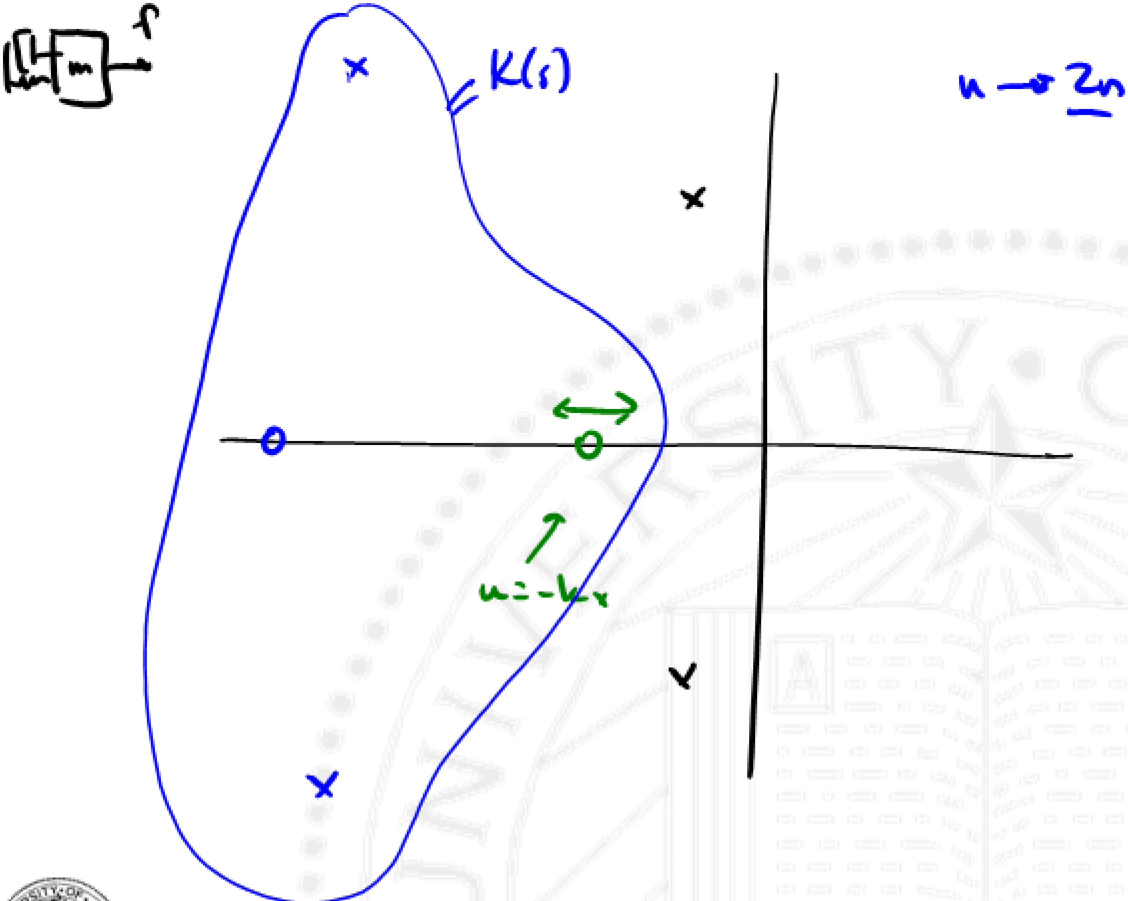
observability matrix

full rank "observable"

cond ($\mathcal{O}$)



r
+
-
K
u
x
y
Plant
L
+
-
ŷ
x̂
Estimate
K(s)
y
u


K(s)
u → 2u
u = -kx

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$u = -K\hat{x}$$

$$\hat{y} = C\hat{x}$$

$$\longrightarrow \left[\begin{array}{l} \dot{\hat{x}} = \overbrace{(A - BK - LC)}^{\tilde{A}}\hat{x} + L\underset{\text{input}}{y} \\ \underset{\text{output}}{u} = -K\hat{x} \end{array} \right]$$

$$\frac{U}{Y}(s) = \underbrace{-K\left[sI - A + BK + LC\right]^{-1}L}$$

$$\underline{\qquad}$$

$$K(s)$$

$u = -Kx$ place poles @ P_c (choose K).

L — place poles @ P_e

$\tilde{x} \triangleq x - \hat{x}$

$\hat{x} = x - \tilde{x}$

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

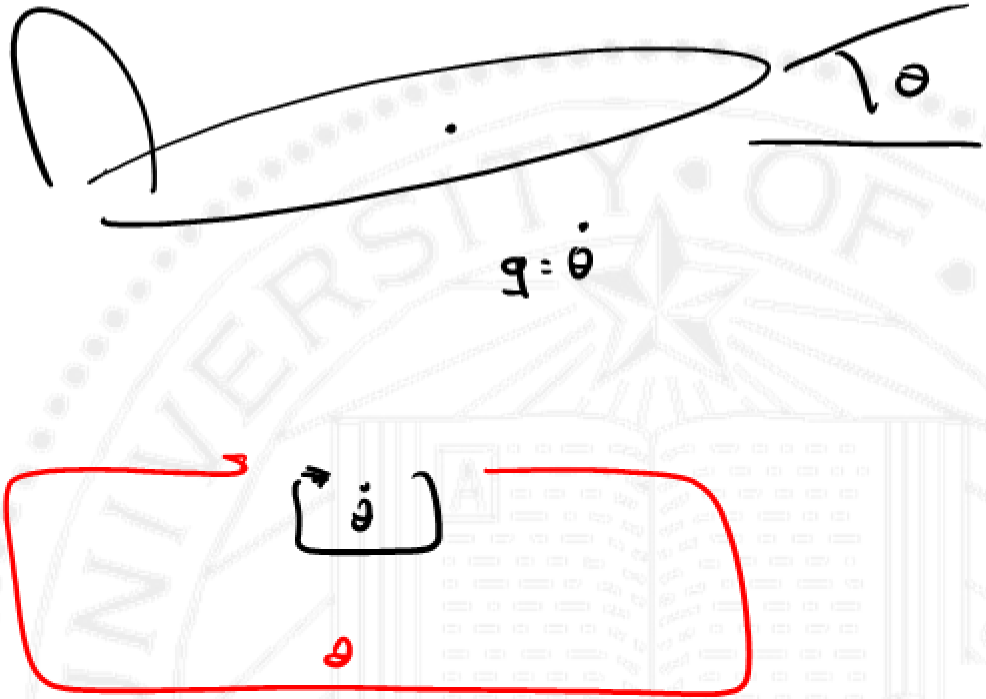
$$\dot{x} = Ax - BK\hat{x} = Ax - BK(x - \tilde{x})$$

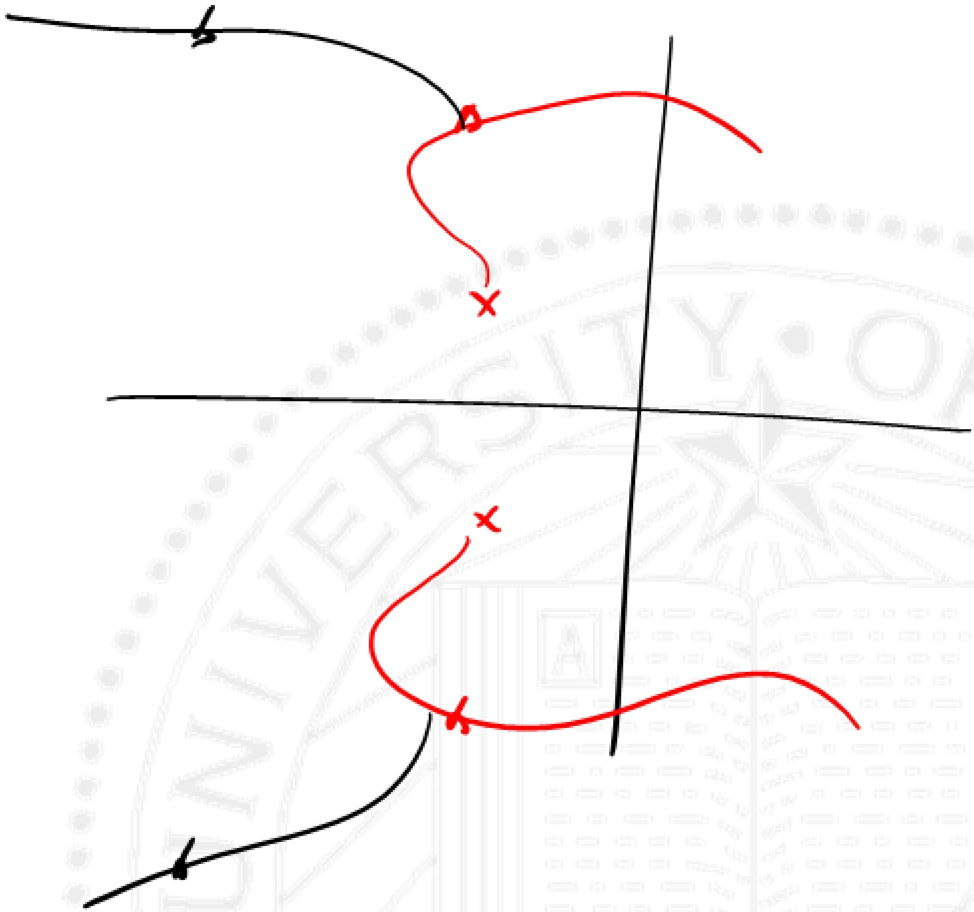
$$= (A - BK)x + BK\tilde{x}$$

$$\begin{bmatrix} \dot{x} \\ \cdots \\ \dot{\tilde{x}} \end{bmatrix} = \left[\begin{array}{c|c} A-BK & BK \\ \hline 0 & A-LC \end{array}\right] \begin{bmatrix} x \\ \cdots \\ \tilde{x} \end{bmatrix}$$

"Separation Principle"

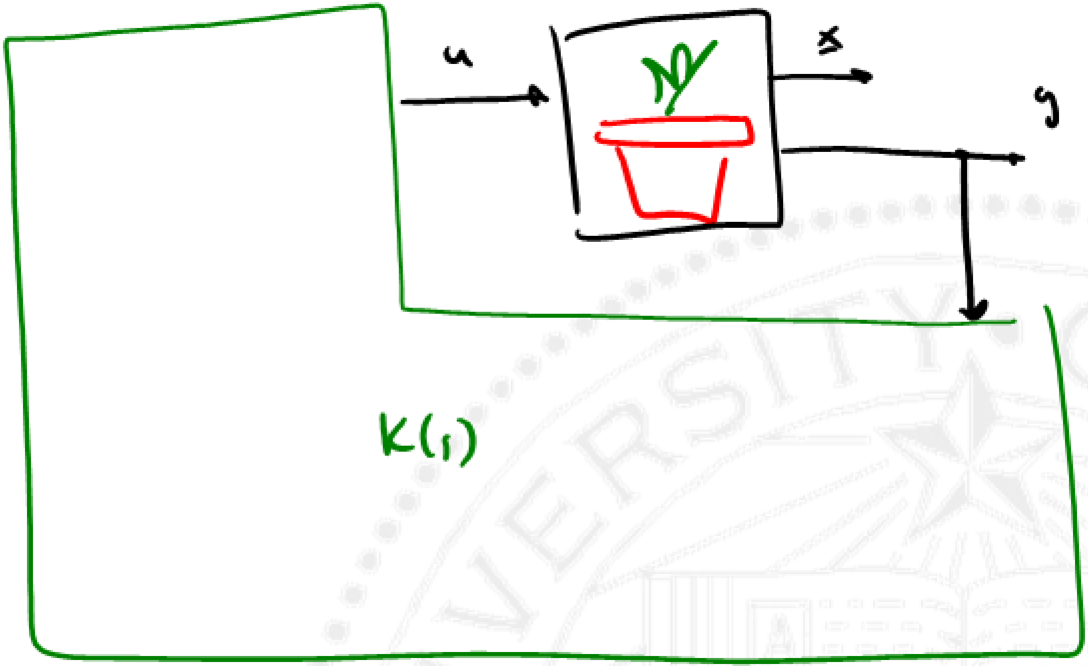
$$\det\left[\begin{array}{c|c} & \\ \hline 0 & \end{array}\right] = \underset{\sim}{\det(A-BK)}\,\widetilde{\det(A-LC)}$$

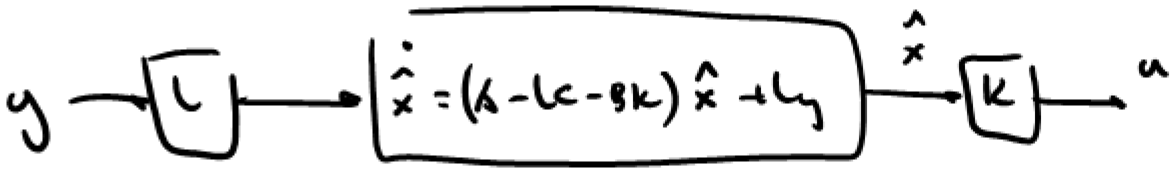

θ
$q = \dot{\theta}$
$\dot{q}$
θ

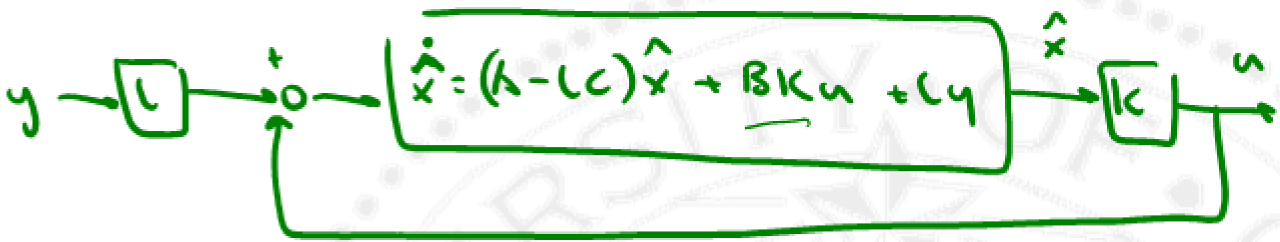


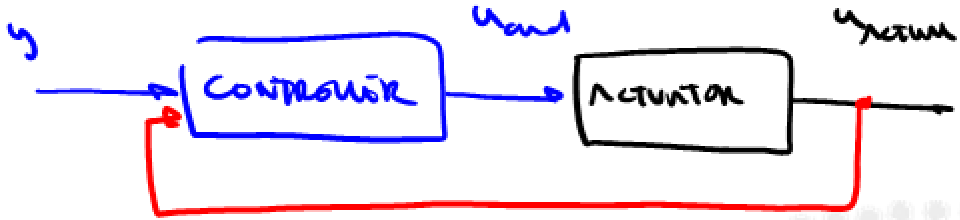
$$\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \longrightarrow \quad u = -kx$$

use estimator to get $\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \leftarrow \dot{\theta}$


u
x
y
K(s)


y
L
$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly$
$\hat{x}$
K
u


y
L
+
$\dot{\hat{x}} = (A - LC)\hat{x} + BKu + Ly$
$\hat{x}$
K
u



(1) measure y

(2) $\times L \longrightarrow$ form $\dot{\hat{x}}$

(3) $\dot{\hat{x}} = [\text{measurement}] + [\cdots]\hat{x} + [u_{actual}]$

(4) $\hat{x}^+ = \hat{x}^- + \dot{\hat{x}}\,\Delta T \longleftarrow$ integrate

(5) $u^+_{cmd} = -K\hat{x}^+$

$$\dot{\hat{x}} = \overbrace{[A - LC]}^{\text{compute \& store offline}}\,\hat{x} + \overbrace{[B \quad L]}^{\text{don't change}}\begin{bmatrix} u \\ y \end{bmatrix}$$

$u = -Kx$ choose K — place(A, B, P_c)

$\mathcal{C}$ full rank

choose L — place(A^T, C^T, P_e)

$\mathcal{O}$ full rank

$$|P_{est}| > 5 \times |P_{cont}|$$

$$G(s) = \frac{Y}{U} = \frac{1}{s^2} = \frac{1}{s^2 + 0s + 0}$$

$$s_{des} = -1 \pm j$$

$$\Delta_d = s^2 + 2s + 2$$

$$\underbrace{\begin{bmatrix} \dot{y} \\ y \end{bmatrix}}_{\dot{x}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \qquad \underline{x} \triangleq \begin{bmatrix} \dot{y} \\ y \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{y} \\ y \end{bmatrix} + [0] u$$

$$\mathbb{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{rank}(\mathbb{C}) = 2 \quad \text{cond}(\mathbb{C}) = 1.$$

$$\mathbb{O} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{rank}(\mathbb{O}) = 2 \quad \text{cond}(\mathbb{O}) = 1.$$

$P_D = -1 \pm j \qquad K = \text{place}(A, B, P_{des}) \longrightarrow K = [2 \quad 2]$

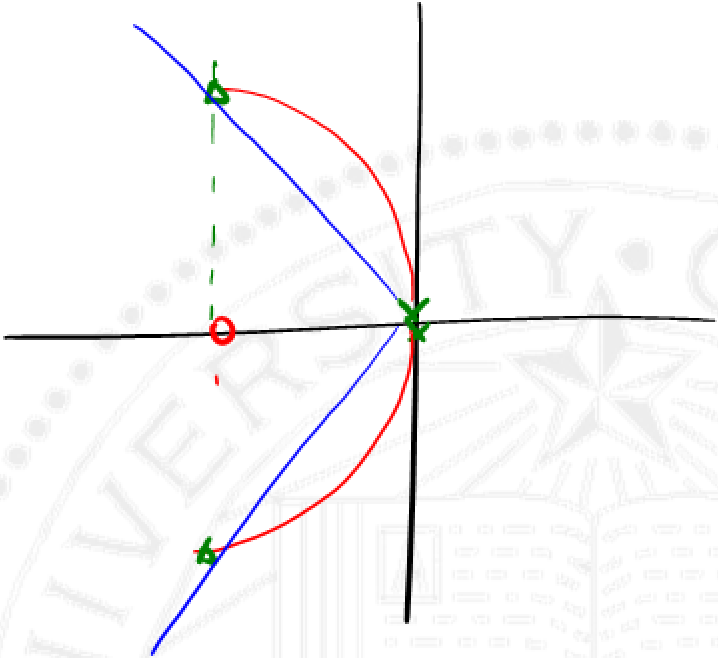
$$(A - BK) = \begin{bmatrix} -k_1 & -k_2 \\ 1 & 0 \end{bmatrix} \qquad \det(sI - (A-BK)) = \det\begin{pmatrix} s+k_1 & +k_2 \\ -1 & s \end{pmatrix}$$

$$s(s+k_1) + k_2 = 0 \qquad s^2 + \boxed{k_1} s + \boxed{k_2} = 0$$

$$s^2 + \boxed{2} s + \boxed{2} = 0$$

$$u = -Kx = -[2 \quad 2]\begin{bmatrix} \dot{y} \\ y \end{bmatrix} = -2\dot{y} - 2y = u$$

$u = -2(s+1)$ pure zero



$P_{des} = \underline{-10 \pm 10j}$ $\quad L^T = \text{place}(A^T, C^T, P_{des}) = \begin{bmatrix} 200 \\ 20 \end{bmatrix}$

$\det(sI - (A - LC)) \longrightarrow \Delta_{des} = s^2 + 20s + 200$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \underbrace{\hat{y}}_{C\hat{x}}) \longrightarrow \dot{\hat{x}} = (A - BK - LC)\hat{x} + Ly$$

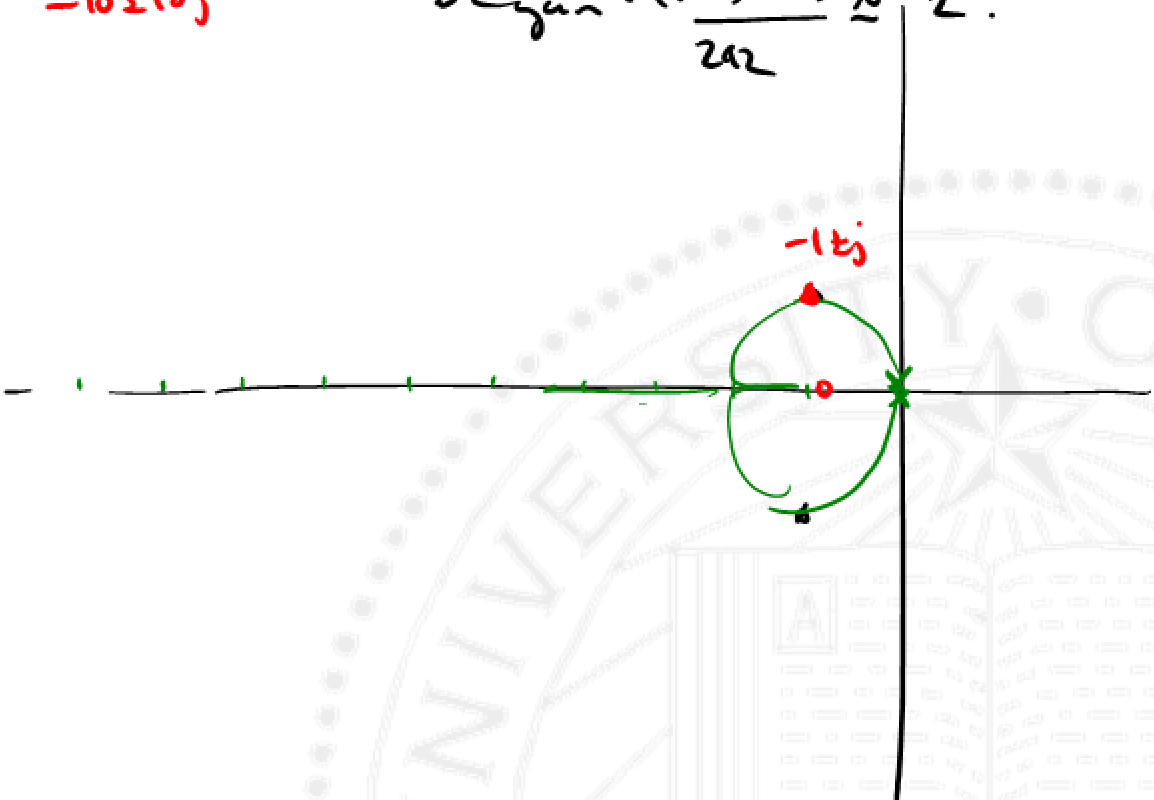
$$u = -K\hat{x} \qquad u = -K\hat{x}$$

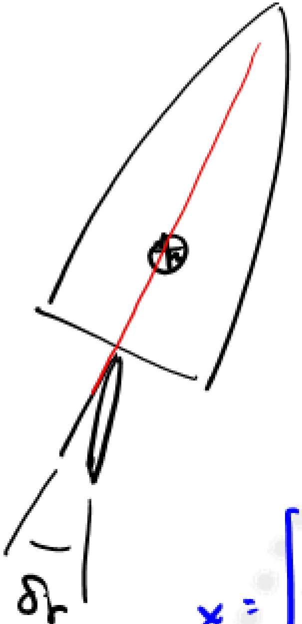
$$K(s) = -K[sI - (A - BK - LC)]^{-1}L$$

$$K(s) = \frac{440(s + 0.91)}{s^2 + 22s + 212}$$

$-10 \pm 10j$

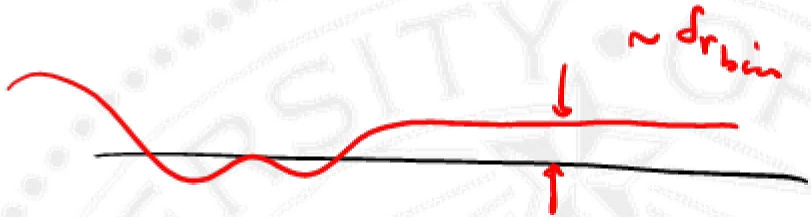
$$\text{DCgain} = \frac{(490)(.91)}{242} \approx 2.$$





$$\delta_{r_{True}} = \delta_{r_{meas}} + \underline{bias}$$

$$\cancel{u = -k\,\dot{\delta}_r} \qquad u = -k\,\delta_r + k\,\underline{b_r}.$$



$$\underline{x} = \begin{bmatrix} \psi \\ \dot{\psi} \\ \delta_r \\ \vdots \\ b_r \end{bmatrix} \qquad \dot{x} = \left[\begin{array}{c|c} A & \begin{matrix} 0 \\ 0 \\ -1 \end{matrix} \\ \hline 0 \text{---} 0 & 0 \end{array}\right] \qquad \delta_r = \underline{\delta_{r_m} - b_r}$$

$\mathcal{O}$ full rank.