CMPE-242
Applied Feedback Control

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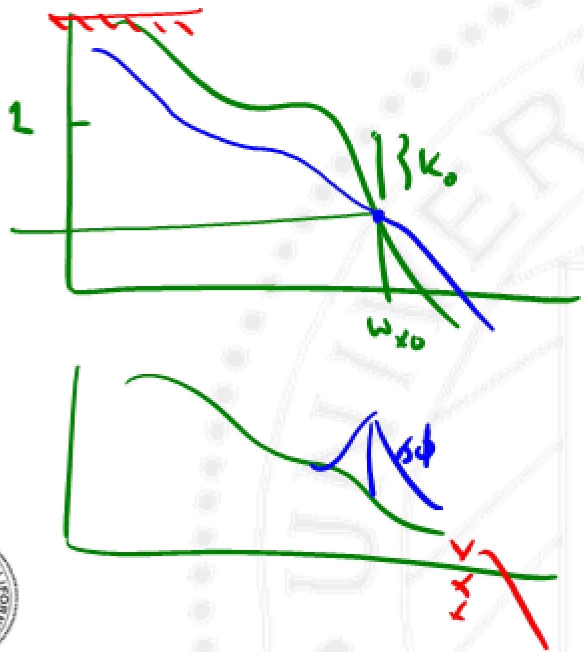
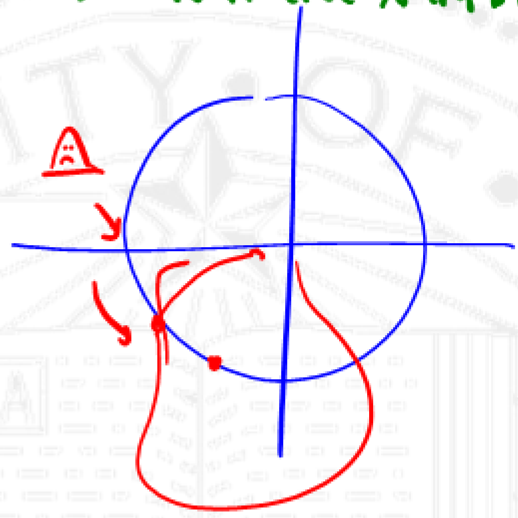
DIGITAL BODE DESIGN

$KG(j\omega) \rightarrow$ $| \; |$, $\angle$ — $\log \omega$

BODE PLOT ($G(j\omega)$)

STRAIGHT LINE ASYMPTOTES.



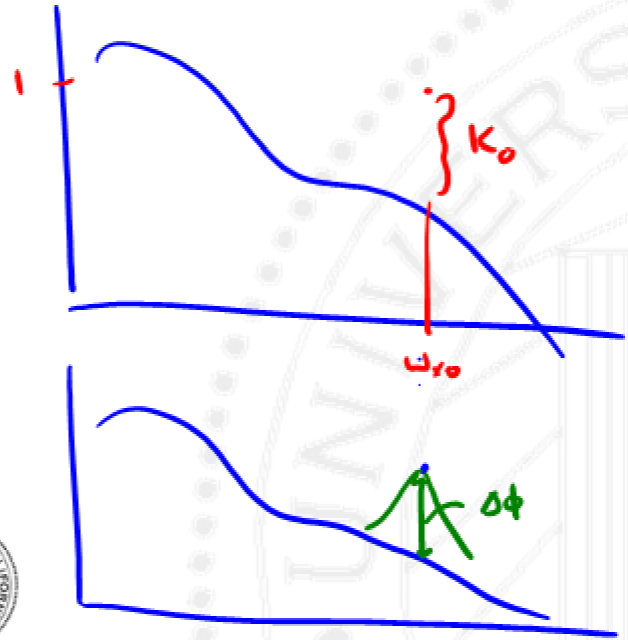


DIGITAL BODE

$G(e^{j\omega T}) < \begin{matrix} \lvert\,\rvert \\ \angle \end{matrix} \rightarrow \log \omega$

NO STRAIGHT LINE ASYMPTOTES.

dbode



DESIGN LEAD NETWORK in "S"

$\omega_{co} = \sqrt{ab}$

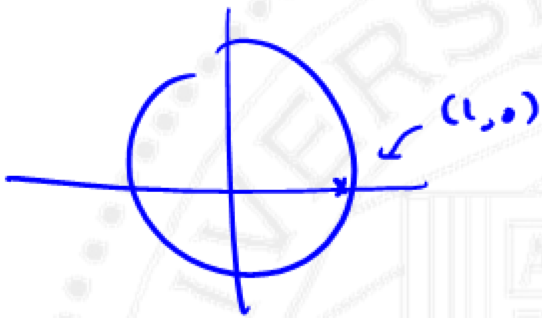
$\frac{b}{a} \rightarrow \Delta\phi$

$K_{LEAD}(s) \longrightarrow K_{LEAD}(z)\big|_{TUSTIN}$

PRE-WARP @ ω_{co}.

lnc → "s" → DC gain $\frac{b}{a}$ $\quad \frac{s+b}{s+a}$

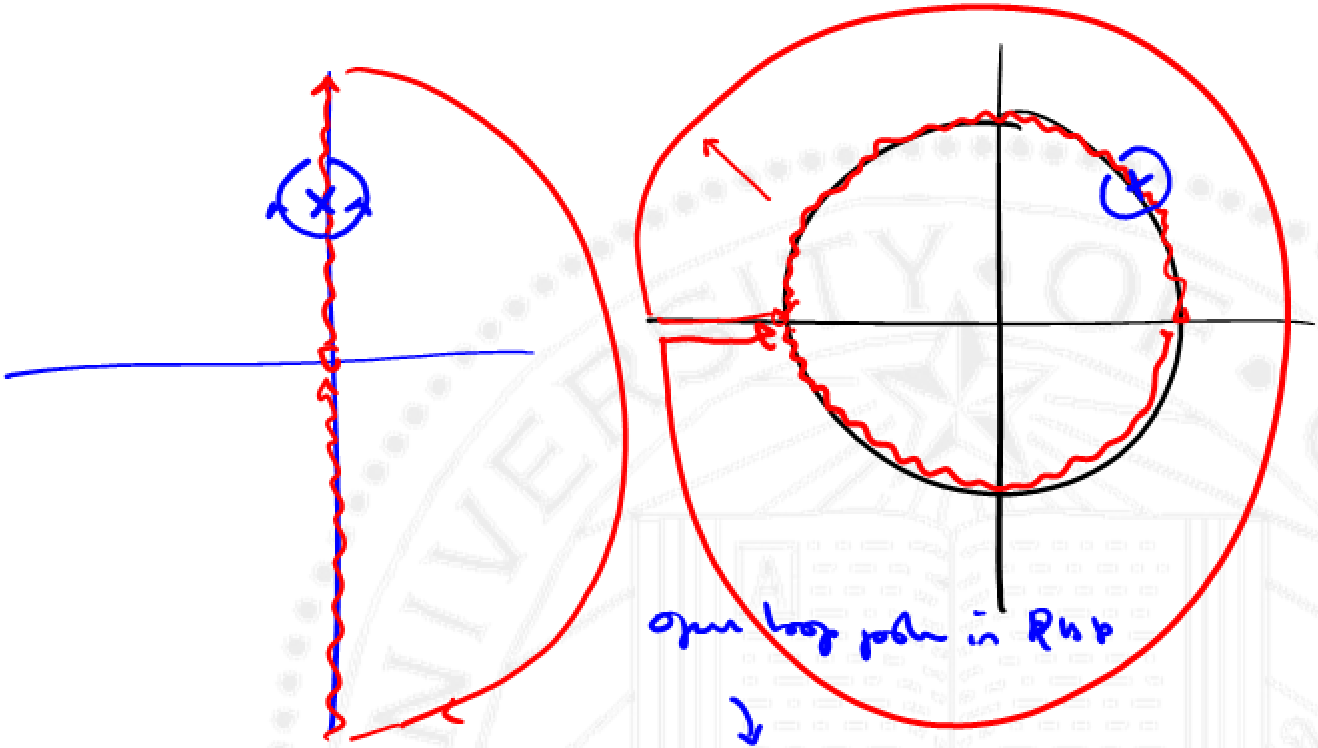
lnc → "z" → DC gain $\frac{1-b}{1-a}$ $\quad \frac{z+b}{z+a}$



<u>Nyquist</u> → $1 + GK = 0$ $\quad \underline{GK = -1}$

S-PLANE

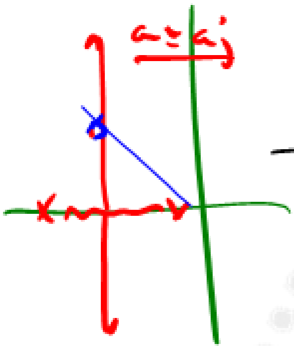
Z- PLANE

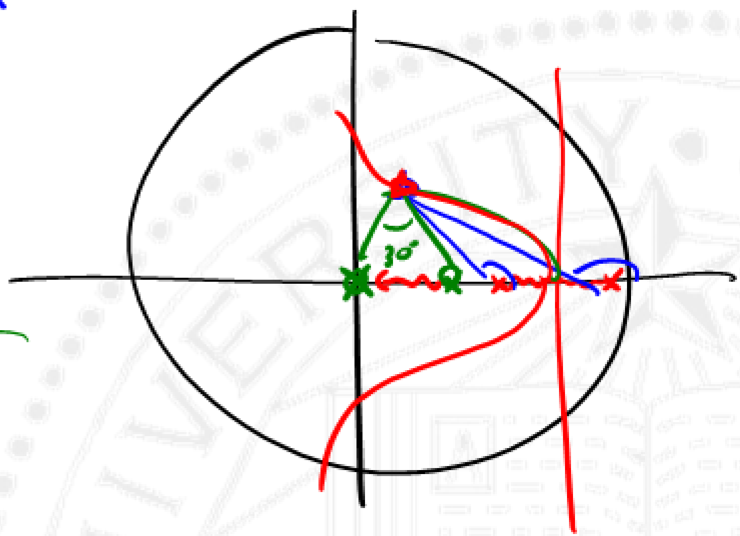


$Z = N + P$ open loop poles outside unit circle

$$G(s) \longrightarrow \frac{z-1}{z}\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\} \longrightarrow G(z)$$

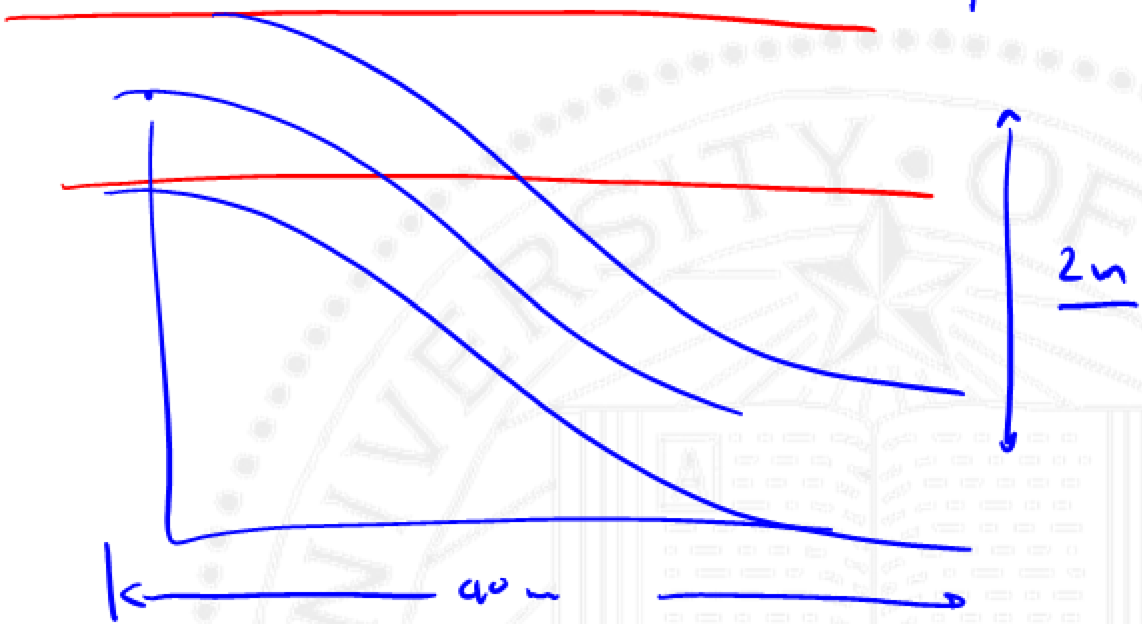
$$z_m = e^{s_m T}$$





2 sec / 20 m/s

$s^2 y$



$$z = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ (x_1 - x_2) \\ (\dot{x}_1 - \dot{x}_2) \end{bmatrix}$$

Similarity Transform

T (T^{-1} exists)

$$x = Tz \qquad z = T^{-1}x$$

$$\dot{z} = T^{-1}\dot{x}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$(T\dot{z}) = ATz + Bu \rightarrow T^{-1}T\dot{z} = T^{-1}(ATz + Bu)$$

$$y = CTz + Du$$

$$\dot{z} = T^{-1}ATz + T^{-1}Bu$$

$$y = CTz + Du$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \longleftrightarrow \left[\begin{array}{c|c} T^{-1}AT & T^{-1}B \\ \hline CT & D \end{array}\right]$$

center of mass: $\dfrac{m_1}{m_1+m_2}x_1 + \dfrac{m_2}{m_1+m_2}x_2 \triangleq \underline{x_{cm}}$.

x_{cm}

$\downarrow$

$$y = \left[\begin{array}{c|c|c|c} 0 & \frac{m_1}{m_1+m_2} & 0 & \frac{m_2}{m_1+m_2} \\ \hline \frac{m_1}{m_1+m_2} & 0 & \frac{m_2}{m_1+m_2} & 0 \end{array}\right] \begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ (x_1 - x_2) \\ (\dot{x}_1 - \dot{x}_2) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix}$$

$$T^{-1}T = I$$

ζ

T^{-1}

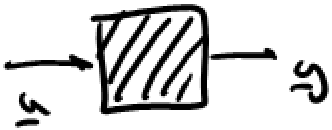
x

$$\begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_1 - x_2 \\ (\dot{x}_1 - \dot{x}_2) \end{bmatrix}$$

T

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

System Identification



"Black Box"

$$\underline{x}_{k+1} = \Phi \underline{x}_k + \Gamma \underline{u}_k$$

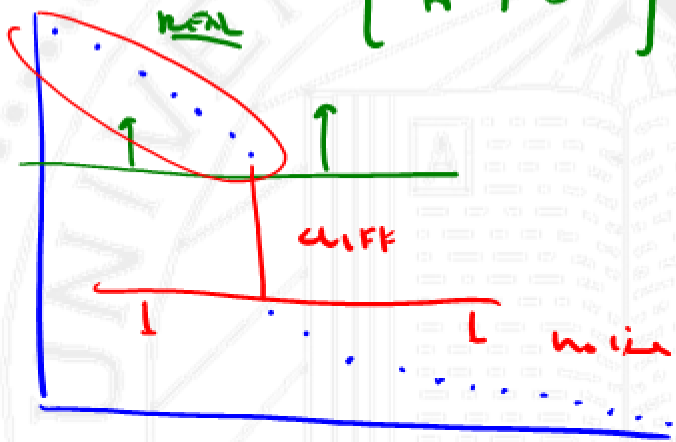
$$y_k = H \underline{x}_k + D \underline{u}_k$$

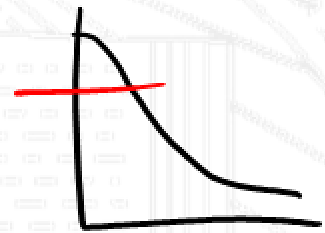
$$\left[\begin{array}{c|c} \Phi & \Gamma \\ \hline H & D \end{array}\right]$$

$380^{\underline{n}}$

BALREAL

SVD $\Phi^T Q$





$$\left.\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}\right\}\ \text{Time Domain Equations}$$

$$\mathcal{L}\left\{\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}\right\} = \quad \begin{aligned} sX(s) - \cancelto{0}{x_0} &= AX(s) + BU(s) \\ Y(s) &= CX(s) + DU(s)\end{aligned}$$

$$sX(s) - AX(s) = BU(s)$$

$$[sI - A]X(s) = BU(s) \qquad \therefore \frac{X(s)}{U(s)} = [sI-A]^{-1}B$$

$$\frac{Y(s)}{U(s)} = \underbrace{C[sI-A]^{-1}B + D}_{\text{Transfer function}}$$

$$\frac{Y}{U} = C[sI - A]^{-1}B + D$$

$$= CT[sI - T^{-1}AT]^{-1}T^{-1}B + D$$

$$T^{-1}IT$$

$$= CT[sT^{-1}IT - T^{-1}AT]^{-1}T^{-1}B + D$$

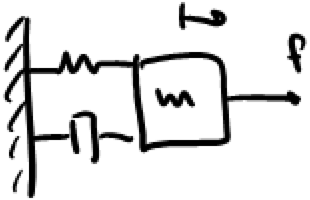
$$= CT[T^{-1}\{sI - A\}T]^{-1}T^{-1}B + D$$

$$= \underbrace{CTT^{-1}}_{I}\{sI - A\}^{-1}\underbrace{TT^{-1}}_{I}B + D$$

$$= C\{sI - A\}^{-1}B + D.$$

$$\left.\begin{aligned} \tilde{A} &= T^{-1}AT \\ \tilde{B} &= T^{-1}B \\ \tilde{C} &= CT \\ \tilde{D} &= D \end{aligned}\right\}$$ Similarity Transform

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$



$$A = \begin{bmatrix} -\frac{b}{m} & -\frac{k}{m} \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix} \quad \underline{x} \triangleq \begin{bmatrix} \dot{x} \\ x \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$\frac{Y}{U} = C\left[sI - A\right]^{-1} B + \cancel{D}^{\,0}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \Bigg[\underbrace{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{b}{m} & -\frac{k}{m} \\ 1 & 0 \end{bmatrix}}_{sI \; - \; A} \Bigg]^{-1} \underbrace{\begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}}_{B} + \underbrace{\phi}_{D}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \left(s + \frac{b}{m}\right) & K/m \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix} + \phi$$

$$\underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\frac{\left[\begin{array}{c|c} s & -\frac{k}{m} \\ \hline 1 & s+\frac{b}{m} \end{array}\right]}{s(s+\frac{b}{m}) + \frac{k}{m}}}_{(sI-A)^{-1}} \underbrace{\begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}}_{B} + \underbrace{\phi}_{D}$$

$$C\vert\Psi = \begin{bmatrix} 0 & 1 \end{bmatrix} \frac{\begin{bmatrix} s/m \\ \hline \frac{1}{m} \end{bmatrix}}{\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)} = \boxed{\frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}} = \frac{Y}{U}$$

$C[sI-A]^{-1}B$

$M^{-1} \triangleq \frac{\text{ADJOINT}(M)}{\text{DET}(M)}$

$C\left[\frac{\text{ADJOINT}(sI-A)}{\text{DET}(sI-A)}\right]B$

$\text{DET}(sI-A)=\phi \quad \Delta(s)=\phi.$

Eigenvalues: $Ax_0=\lambda x_0 \longrightarrow (\lambda I-A)x_0=0$

trivial solution: $x_0=\phi.$

if $x_0\neq 0. \;-\; \det(\lambda I-A)=\phi.$

λ's are the eigenvalues of A corresponding to the eigenvectors x_0

$$C\left[\frac{\text{ADJOINT}(sI-A)}{\text{DET}(sI-A)}\right]B$$

$$\text{DET}(sI-A) = \phi \leftarrow \Delta(s)$$

Poles of the system $\longleftrightarrow$ eigenvalues of A.

eigenvalues of A $\longleftrightarrow$ Poles of the TF.

$\dot{x} = Ax + Bu$ choose $u = -Kx$.

$y = Cx + Du$

$\dot{x} = (A-BK)x$

$y = (C+DK)x$

$\boxed{\text{closed loop poles are the eig}(A-BK)}$

CHOOSE "K" TO PUT EIG(A-BK) where you want them.

BACK TO THE S.S. TOOLS

$-Ax_0 \quad Ax_0 = \lambda x_0 - Ax_0$

$0 = \lambda x_0 - Ax_0$

$0 = \underbrace{[\lambda I - A]}\,x_0 \quad \phi = [\lambda I - A]^{-1}[\lambda I - A]\,x_0$

Controller Canonical Form

$$\frac{Y}{U} = \frac{1}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n}$$

choose for $\underline{x}$: $\begin{bmatrix} y^{(n-1)} \\ \vdots \\ \dddot{y} \\ \ddot{y} \\ \dot{y} \\ y \end{bmatrix}$

$$\frac{y}{u} = \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\dddot{y} = -a_1 \ddot{y} - a_2 \dot{y} - a_3 y + u$$

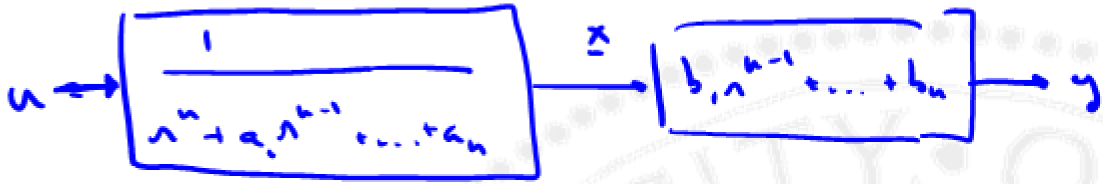
$$x = \begin{bmatrix} \ddot{y} \\ \dot{y} \\ y \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dddot{y} \\ \ddot{y} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \dot{y} \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ \hline 0 \\ 0 \end{bmatrix} u$$

$$\frac{Y}{U} = \frac{1}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n}$$

choose $\underline{x} = \begin{bmatrix} y^{(n-1)} \\ \vdots \\ \ddot{y} \\ \dot{y} \\ y \end{bmatrix}$

$$\underline{\dot{x}} = \left[\begin{array}{cccc} -a_1 & -a_2 & \cdots & -a_n \\ \hline 1 & & 0 & \\ & \ddots & & 0 \\ 0 & & 1 & \end{array}\right] \underline{x} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$\frac{Y}{U} = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$



$$\dot{\underline{x}} = \left[\begin{array}{cccc} -a_1 & -a_2 & \cdots & -a_n \\ \hline 1 & & & 0 \\ & \ddots & & \\ 0 & & & 1 \end{array}\right] \underline{x} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

Controller canonical form

$$y = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \underline{x} + [0] u$$

$$\frac{Y}{U} = \frac{b_0 s^n + b_1 s^{n-1} + \dots b_n}{s^n + a_1 s^{n-1} + \dots + a_n}$$

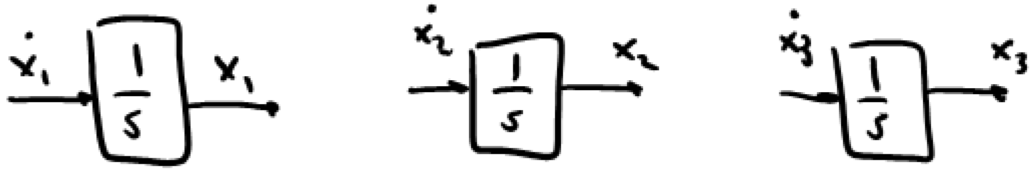
$$s^n + a_1 s^{n-1} + \dots + a_n \overline{\big)\, \begin{array}{l} \qquad\qquad b_0 \\ b_0 s^n + b_1 s^{n-1} + \dots + b_n \\ -b_0 s^n - b_0 a_1 s^{n-1} + \dots + b_0 a_n \\ \hline \underbrace{(b_1 - b_0 a_1)}_{c_1} s^{n-1} + \dots \underbrace{(b_n - b_0 a_n)}_{c_n} \end{array}}$$

$$\frac{Y}{U} = b_0 + \frac{c_1 s^{n-1} + c_2 s^{n-2} + \dots + c_n}{s^n + a_1 s^{n-1} + \dots + a_n}$$

$$y = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \underline{x} + \underset{\substack{\uparrow \\ b}}{\begin{bmatrix} b_0 \end{bmatrix}} u$$

$$\dot{\underline{x}} = \left[\begin{array}{cccc} -a_1 & -a_2 & \cdots & -a_n \\ \hline 1 & & & \\ & \ddots & & \\ & & 1 & \end{array}\right] \underline{x} + \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & \cdots & c_n \end{bmatrix} \underline{x} + \begin{bmatrix} b_0 \end{bmatrix} u$$



$$\dot{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \begin{bmatrix} -b & -c \\ d & -e \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} a \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u$$

$$\begin{array}{cc} A & B \\ \begin{bmatrix} -b & -c \\ d & -e \end{bmatrix} & \begin{bmatrix} a \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & f \end{bmatrix} & \begin{bmatrix} \phi \end{bmatrix} \\ C & D \end{array} \longleftrightarrow \begin{aligned} \dot{x}_1 &= -bx_1 - cx_2 + au \\ \dot{x}_2 &= dx_1 - ex_2 + \phi u \\ y &= fx_2 + \phi u. \end{aligned}$$