CMPE-242
Applied Feedback Control

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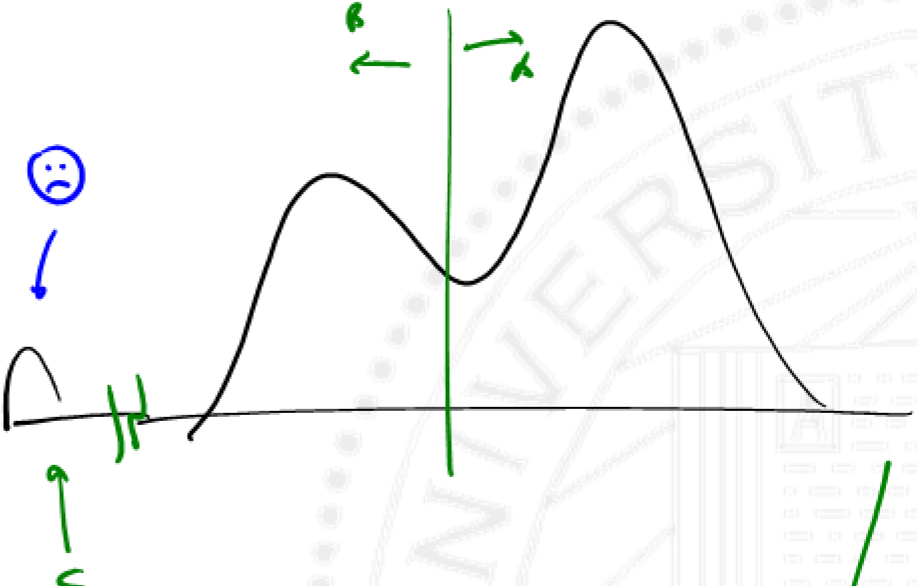
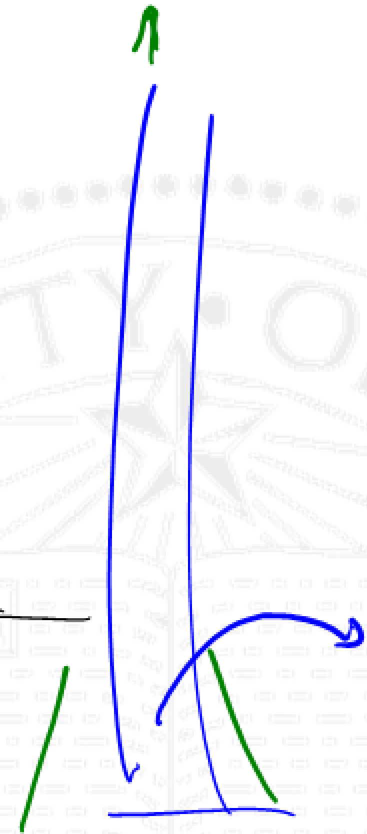
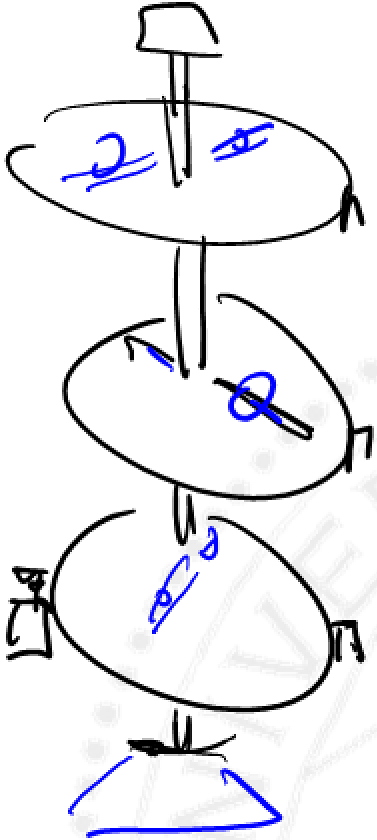
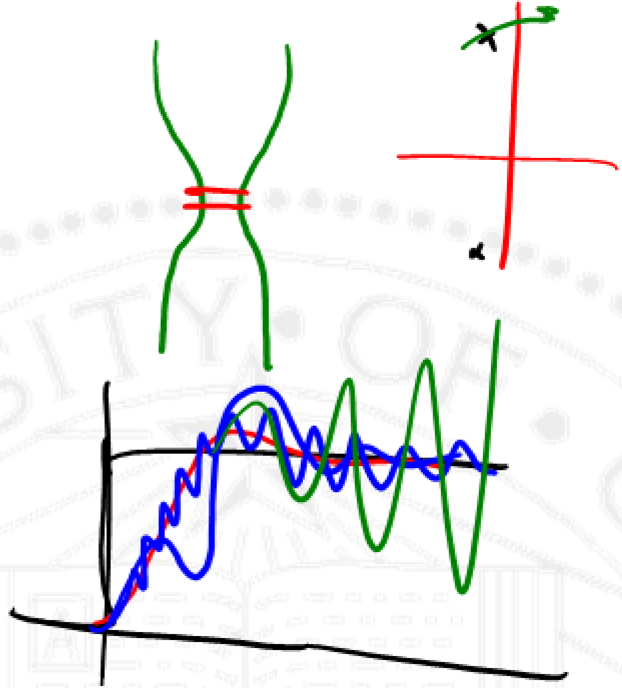
MIDTERM IS DONE

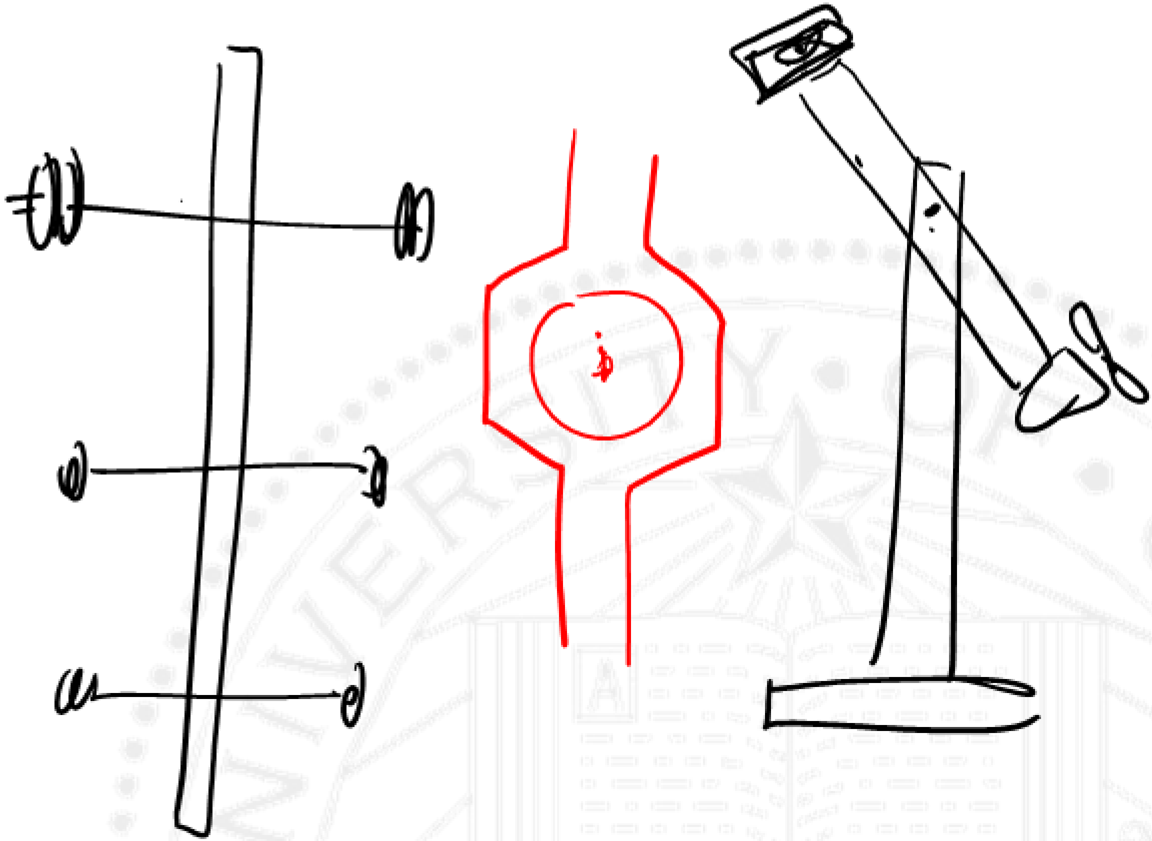
NOT LOOKED AT THEM, YET.





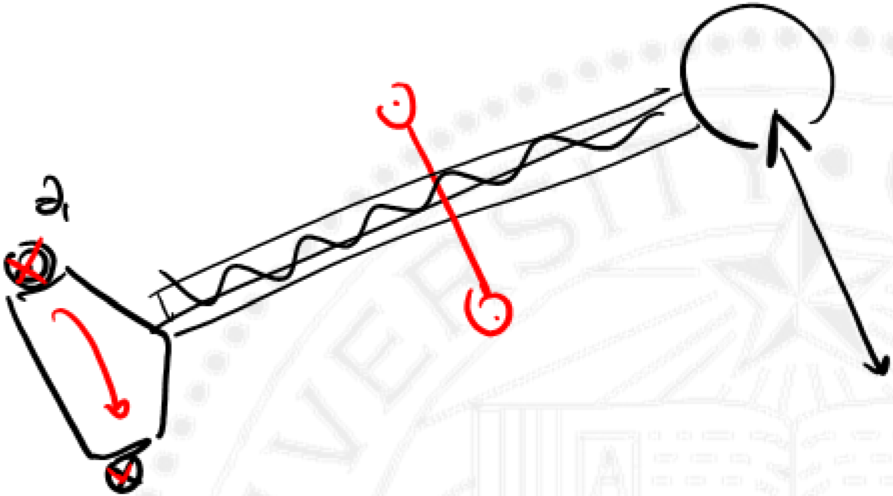






Actuate at Bottom measure at θ_1 $\rightarrow$ collocated

" " " " " θ_2 — θ_2 non-collocated.



State Space — Ch. 7 FPE

Root locus
Bode
Nyquist
Digital

State Space ← Linear Algebra

TF — n^{th} order [difference / differential] equation

N 1^{st} order differential/difference equations

$$\frac{Y}{U} = \frac{1}{s^2 + 2s + 3} \longrightarrow \ddot{y} + 2\dot{y} + 3y = u$$

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + [0]\, u$$

$$\underbrace{\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{u} \\ \underline{y} &= C\underline{x} + D\underline{u} \end{aligned}}_{\text{mixd coord/stanford}}$$

$$\underbrace{\begin{aligned} \dot{\underline{x}} &= F\underline{x} + G\underline{u} \\ \underline{y} &= H\underline{x} + D\underline{u} \end{aligned}}_{\text{ent coord/MIT}}$$

$$\underbrace{\begin{aligned} \underline{x}_{k+1} &= \Phi\underline{x}_k + \Gamma u_k \\ y_k &= H\underline{x}_k + D u_k \end{aligned}}_{\text{digital}}$$

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

$A \in \mathbb{R}^{n \times n}$ ← dynamics matrix
state transition matrix $[\Phi]$

$B \in \mathbb{R}^{n \times m}$ ← input matrix (not square)

$C \in \mathbb{R}^{r \times n}$ ← output matrix

$\underline{x} \in \mathbb{R}^{n}$ $n \times 1$ vector "states" $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ <u>not unique</u>
n^{th} order system

$\underline{u} \in \mathbb{R}^{m}$ $m \times 1$ vector "inputs" $\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$ nth order, m-input

$\underline{y} \in \mathbb{R}^{r}$ $r \times 1$ vector "outputs" $\begin{bmatrix} y_1 \\ \vdots \\ y_r \end{bmatrix}$ r-output.

$D \in \mathbb{R}^{r \times m}$ — direct feedthrough matrix

Proper TF : more x's than o's.

$D = 0$

$$\frac{s^2 \cdots}{s^3 \cdots}$$

Strictly Proper TF : x's equal o's.

$D \neq 0$.

$$\frac{s^3 \cdots}{s^3 \cdots}$$

Improper TF :

more o's than x's.

$\dot{x} = Ax + Bu$ $\underline{u = -Kx}$. control.

$y = Cx$

$\dot{x} = Ax + B(-Kx) = Ax - BKx = \underline{(A - BK)x}$

eigenvalues of $(A-BK)$ $\longleftrightarrow$ poles of a system

$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$

$\min_u J$

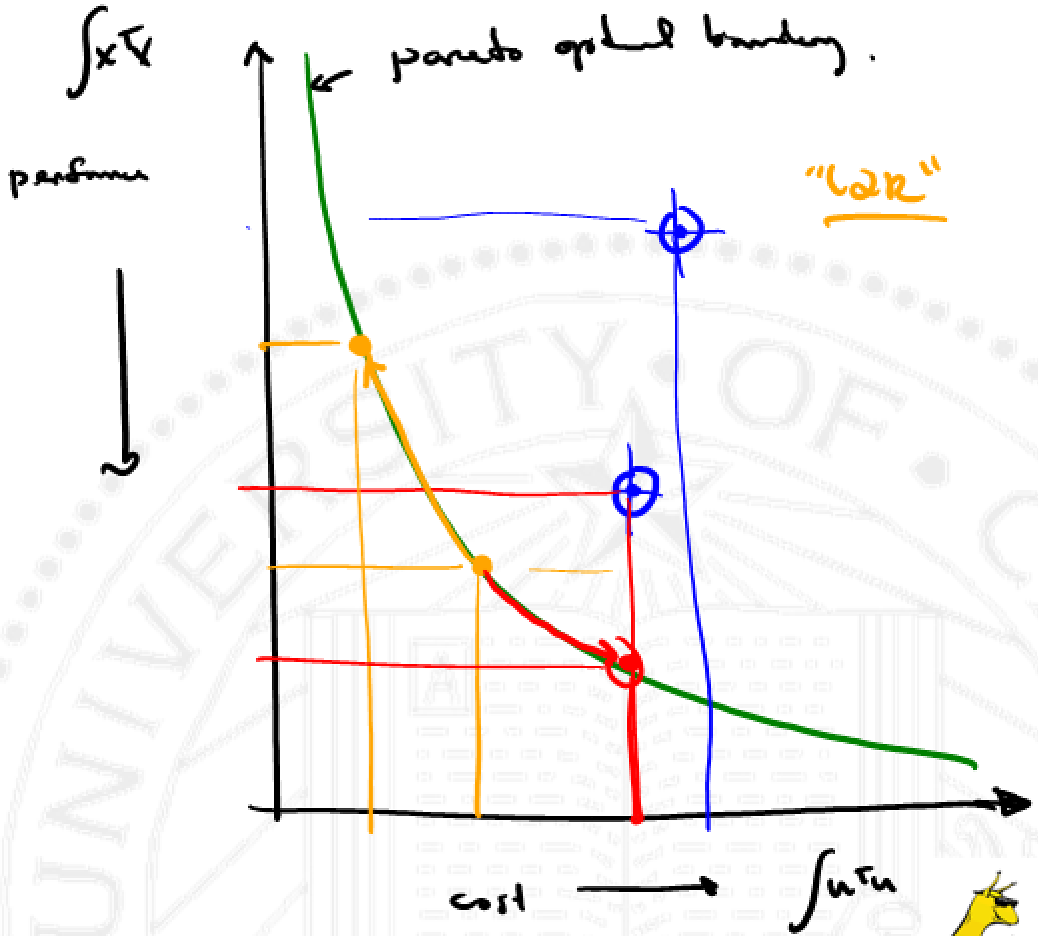
subj. $\dot{x} = Ax + Bu$

$\downarrow$

$\underline{u = -Kx}$

$\underline{\text{A.R.E.}}$

- Stable
- > 50° pm
2) GM > ∞.



$$\frac{Y}{U}(s) = \frac{1}{s^2+2s+3} \longrightarrow \left[s^2+2s+3\right]Y(s) = U(s)$$

$$\ddot{y} + 2\dot{y} + 3y = u$$

$$\underline{x} = \begin{bmatrix} \dot{y} \\ \hline y \end{bmatrix}$$

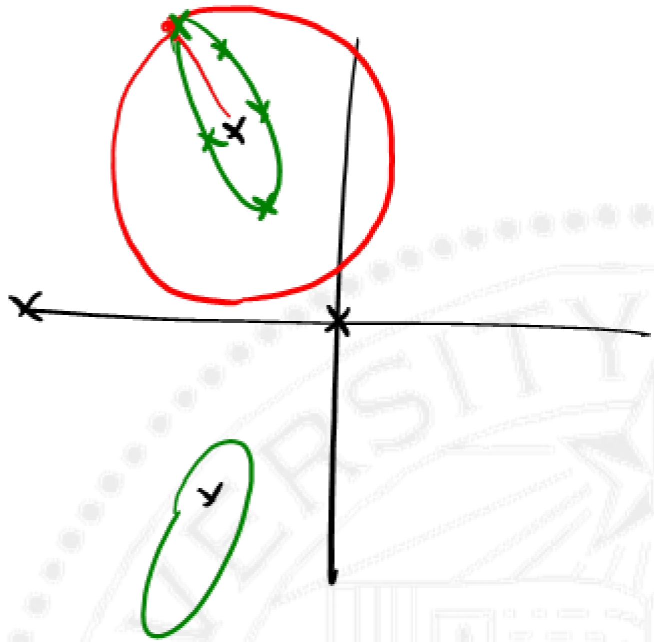
$$\underline{\dot{x}} = \begin{bmatrix} \ddot{y} \\ \hline \dot{y} \end{bmatrix}$$

$$\underline{\dot{x}} = \begin{bmatrix} \ddot{y} \\ \hline \dot{y} \end{bmatrix} = \overset{A}{\left[\begin{array}{c|c} -2 & -3 \\ \hline 1 & 0 \end{array}\right]} \begin{bmatrix} \dot{y} \\ \hline y \end{bmatrix} + \overset{B}{\begin{bmatrix} 1 \\ \hline 0 \end{bmatrix}} u$$

$$\underline{y} = [y] = \underset{C}{\left[\begin{array}{c|c} 0 & 1 \end{array}\right]} \begin{bmatrix} \dot{y} \\ \hline y \end{bmatrix} + \underset{D}{\begin{bmatrix} 0 \end{bmatrix}} u$$

$$\ddot{y} = -2\dot{y} - 3y + u$$

TF2SS $\longleftrightarrow$ SS2TF



State Space

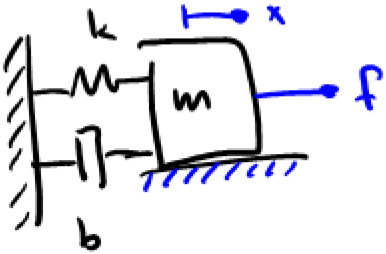
(1) Higher Order Systems — easy to deal with

(2) Multiple Input / Multiple Output systems — easy to handle.

(3) New set of tools (Based on Linear Algebra) for control design.

$$\underline{\dot{x}} = A\underline{x} + B\underline{u}$$
$$\underline{y} = C\underline{x} + D\underline{u}$$

$$\longrightarrow \quad \begin{bmatrix} \dot{x} \\ \hline y \end{bmatrix} = \begin{array}{c} \\ n \\ r \end{array} \underbrace{\left[\begin{array}{c|c} \overset{\leftarrow n \rightarrow}{A} & \overset{\leftarrow m \rightarrow}{B} \\ \hline C & D \end{array}\right]}_{\text{NOT SQUARE}} \begin{bmatrix} x \\ \hline u \end{bmatrix}$$



$$m\ddot{x} + b\dot{x} + kx = f$$

$$\ddot{x} = -\frac{b}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}f$$

$$\dot{x} = v$$
$$\dot{v} = \ddot{x}$$

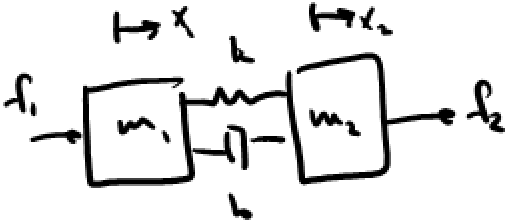
state

$$\begin{bmatrix} x \\ v \end{bmatrix}$$

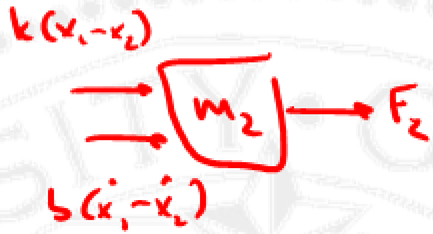
$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

$$y = \begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F$$

physics



$$\frac{X_1}{F_1} \quad \frac{X_1}{F_2} \quad \frac{X_2}{F_1} \quad \frac{X_2}{F_2}$$



$$m_1 \ddot{x}_1 = \Sigma F = F_1 + k(x_2 - x_1) + b(\dot{x}_2 - \dot{x}_1)$$

$$m_2 \ddot{x}_2 = \Sigma F = F_2 + k(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2)$$

$$\underline{x} = \begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix}$$

$$\underline{\dot{x}} = \begin{bmatrix} \ddot{x}_1 \\ \dot{x}_1 \\ \ddot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{b}{m_1} & -\frac{k}{m_1} & \frac{b}{m_1} & \frac{k}{m_1} \\ 1 & 0 & 0 & 0 \\ \frac{b}{m_2} & \frac{k}{m_2} & -\frac{b}{m_2} & -\frac{k}{m_2} \\ 0 & 0 & 1 & 0 \end{bmatrix}}^{A} \begin{bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\ddot{x}_1 = \frac{1}{m_1} F_1 + \frac{b}{m_1}(\dot{x}_2 - \dot{x}_1) + \frac{k}{m_1}(x_2 - x_1)$$

C

$$\ddot{x}_2 = \frac{1}{m_2} F_2 + \frac{b}{m_2}(\dot{x}_1 - \dot{x}_2) + \frac{k}{m_2}(x_1 - x_2)$$

b

$$\underline{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$z = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ (x_1 - x_2) \\ (\dot{x_1} - \dot{x}_2) \end{bmatrix}$$

$\boxed{\text{Similarity Transform}}$

T (T^{-1} exists)

$x = Tz \qquad z = T^{-1}x$

$\dot{z} = T^{-1}\dot{x}$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$(T\dot{z}) = ATz + Bu \qquad T^{-1}T\dot{z} = T^{-1}(ATz + Bu)$

$y = CTz + Du \qquad \dot{z} = T^{-1}ATz + T^{-1}Bu$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \longleftrightarrow \left[\begin{array}{c|c} T^{-1}AT & T^{-1}B \\ \hline CT & D \end{array}\right]$$

$y = CTz + Du$

center of mass: $\frac{m_1}{m_1+m_2} x_1 + \frac{m_2}{m_1+m_2} x_2 \triangleq \underline{x_{cm}}$.