CMPE-242
Applied Feedback Control

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Winter 2016

Midterm Schedule

~2pm – 5pm Today Thursday 11/Feb

~1pm – 4pm Friday 12/Feb.

Open Formula Sheet / Calculator.

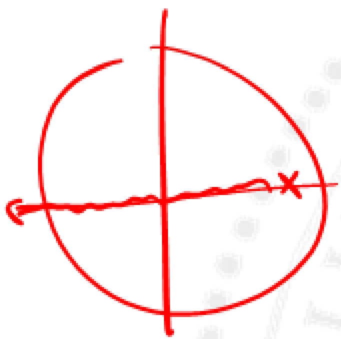
Specifically NO LAPTOPS, NO MATLAB

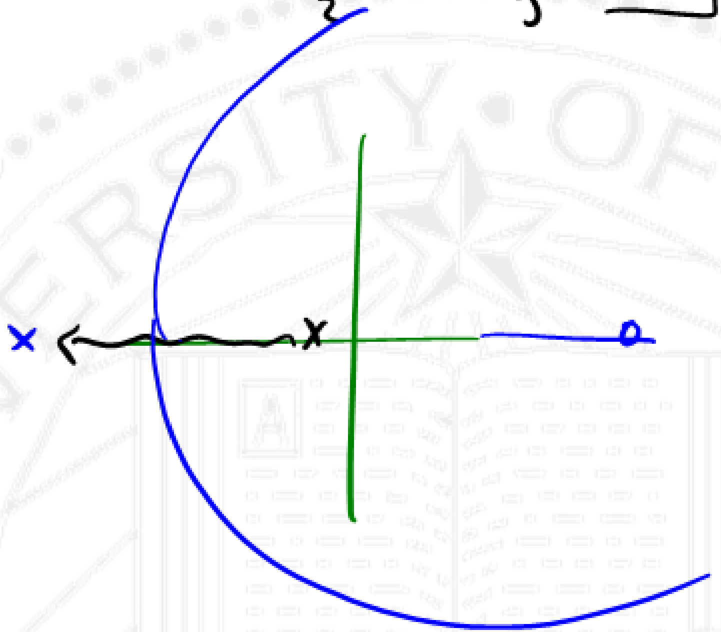
Root Locus

c2d

PADE

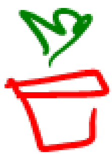
simulink sim $\frac{sT}{2}$ delay $\boxed{\text{ZOH}}$

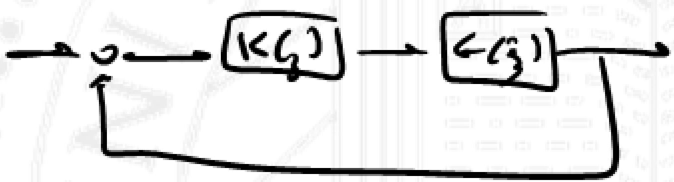




Design in RL &  + PADÉ

Convert $K(s) \rightarrow K(z)$.

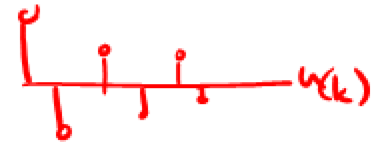
 $G(s) \longrightarrow$  $G(z)$.



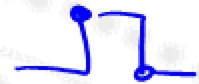
$$\frac{KG(z)}{1+KG(z)}$$

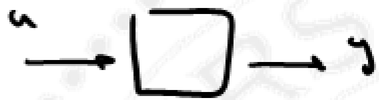
Convolution

$$u_k = u_0\delta_0 + u_1\delta_1 + u_2\delta_2 \cdots$$



$H(z)$ – pulse response of system from rest.

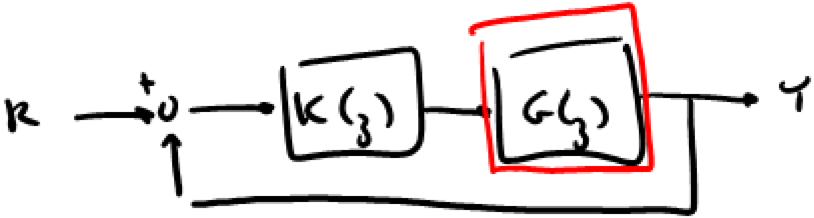




$$y_k = u_0 H(z) + z^{-1} u_1 H(z) + z^{-2} u_2 H(z) \cdots$$

$$Y(z) = \underbrace{\left[u_0 z^{-0} + u_1 z^{-1} + u_2 z^{-2} + \cdots\right]}_{U(z)} H(z)$$

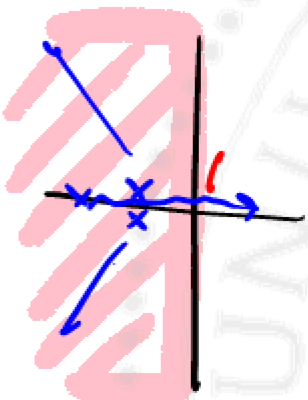
$$\boxed{Y(z) = U(z)H(z)}$$

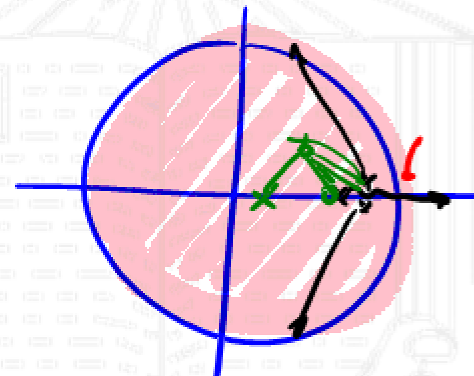


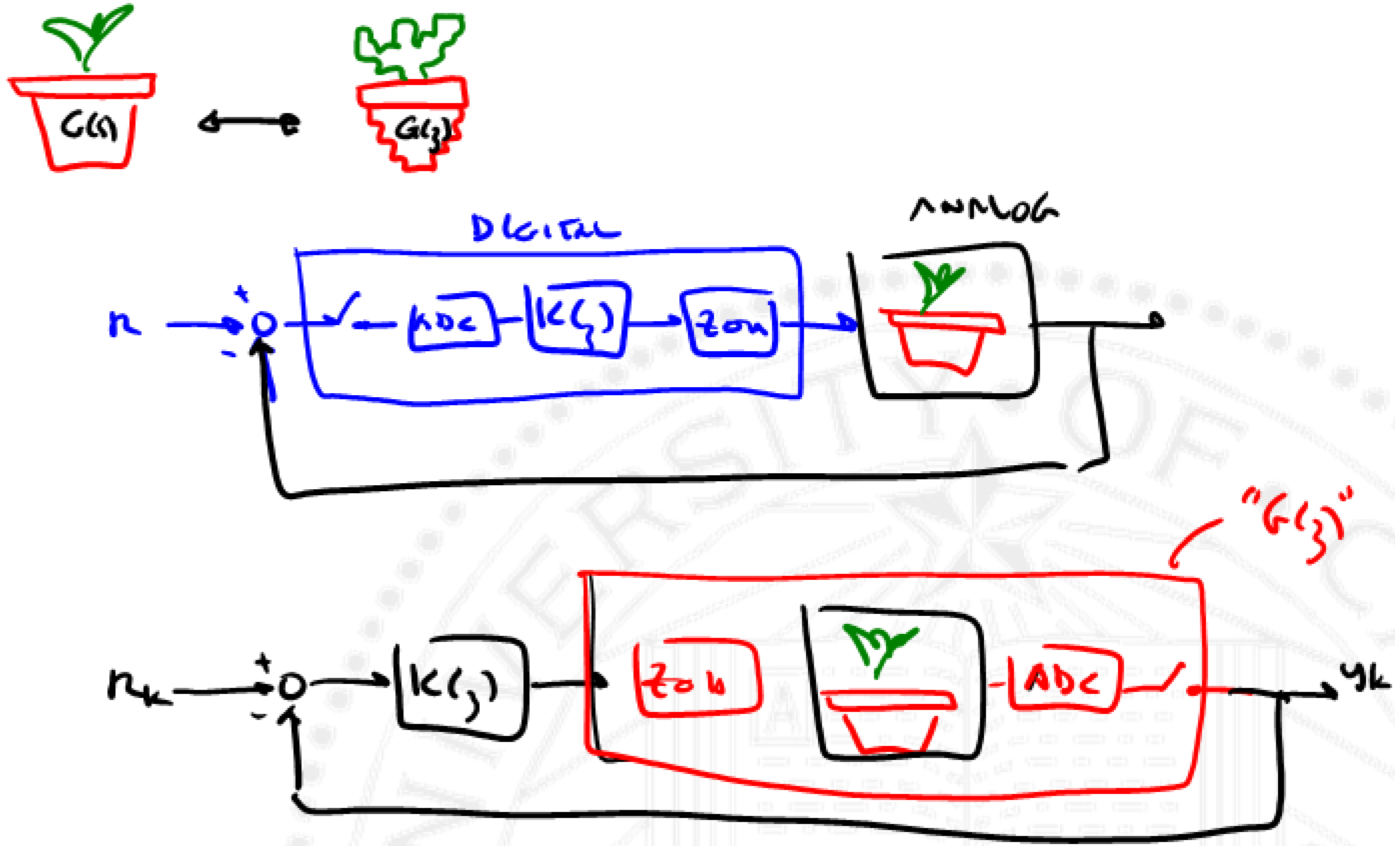
$$\frac{Y}{R}(z) = \frac{KG}{1+KG} \leftarrow \Delta(z)$$

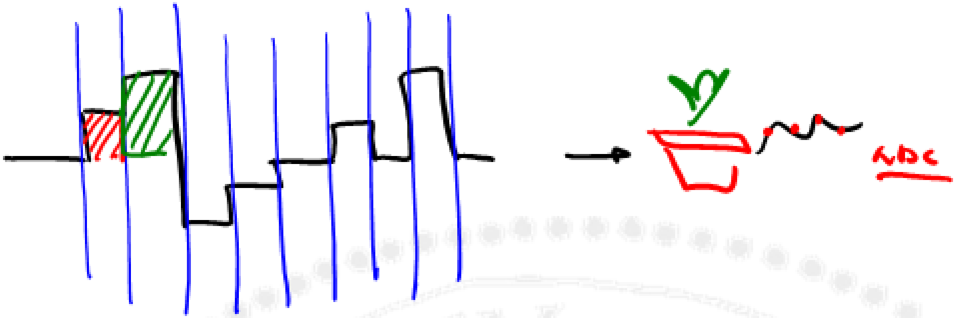
$$1+KG=0 \quad KG=-1 \begin{cases} \angle 180^\circ \\ \|\ 1 \end{cases}$$

Root locus is unchanged

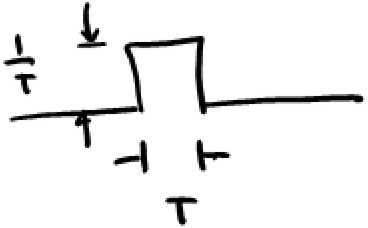




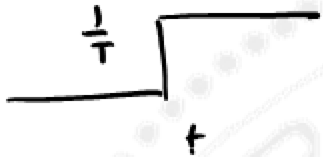

G(s)
G(z)
DIGITAL
ANALOG
r
ADC
K(z)
ZOH
"G(z)"
r_k
y_k

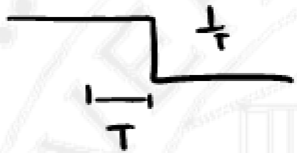



0
1
=
+
2



unit area.





$$\mathcal{L}\{\_\_\lceil\overline{\ \ }\ @\ t=0\} = \int_0^\infty 1(t)\, e^{-st}\, dt = \int_0^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_0^\infty$$

$$= 0 - \frac{-1}{s}(1) = \frac{1}{s}$$

$$\mathcal{L}\{\_\_\lceil\overline{\ \ }\ @\ t=T\} = \int_0^\infty 1(t-T)\, e^{-st}\, dt = \int_T^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_T^\infty$$

$$= 0 - \left(-\frac{e^{-sT}}{s}\right) = \underline{\frac{1}{s} e^{-sT}}.$$

e^{-Ts} – pure time delay.

$$\mathcal{L}\{z_{oh}\} = \mathcal{L}\left\{\tfrac{1}{t}\int_0 + \tfrac{1}{t}\int_T\right\} = \frac{1}{s} - \left(\frac{e^{-sT}}{s}\right) = \boxed{\frac{1-e^{-sT}}{s}}$$

$$\boxed{\mathcal{L}\{z_{oh}\} = \frac{1-e^{-sT}}{s}}$$

exact

approx. $e^{-\frac{Ts}{2}}$

$$e^{-\frac{T}{2}\lambda} = 1 - \frac{T}{2}\lambda + \left(\frac{T}{2}\right)^2\frac{\lambda^2}{2!} - \left(\frac{T}{2}\right)^3\frac{\lambda^3}{3!} + \cdots$$

$$\frac{1}{T}\mathcal{L}\{h(t)\} = \frac{1}{T}\,\frac{1-e^{-T\lambda}}{\lambda} = \frac{1 - \left(1 - \cancel{T\lambda} + \frac{T^2\lambda^2}{2!} - \frac{T^3\lambda^3}{3!} + \cdots\right)}{T\lambda}$$

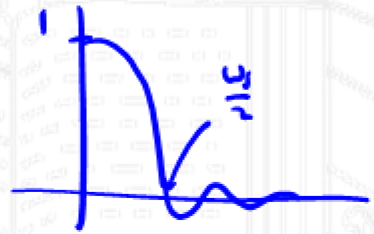
$$= 1 - \frac{T\lambda}{2!} + \frac{T^2\lambda^2}{3!} - \frac{T^3\lambda^3}{4!} + \cdots$$

$e^{-\frac{T}{2}s}$ in bode $\begin{cases} |\,| = 1 \ \forall \omega \\ \angle = -\frac{\omega T}{2} \end{cases}$

$$\mathcal{L}\{ZOH\}\Big|_{s=j\omega} = \frac{1}{T}\,\frac{1-e^{-j\omega T}}{j\omega} = \frac{1}{T}\left[\frac{e^{-\frac{j\omega T}{2}}e^{+\frac{j\omega T}{2}} - e^{-j\omega T}}{j\omega}\right]$$

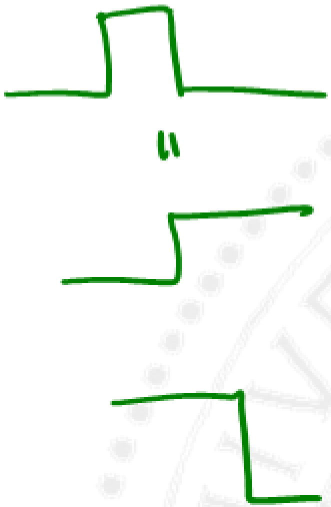
$$= e^{-j\frac{\omega T}{2}}\left[\frac{e^{j\frac{\omega T}{2}} - e^{j\frac{\omega T}{2}}}{2j}\right]\frac{2}{\omega T}$$

$$= e^{-j\frac{\omega T}{2}}\underbrace{\left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}\right]}_{\text{sinc}}$$



$\phi = -\frac{\omega T}{2}$

$\frac{G(s)}{s}$ step response of my plant.

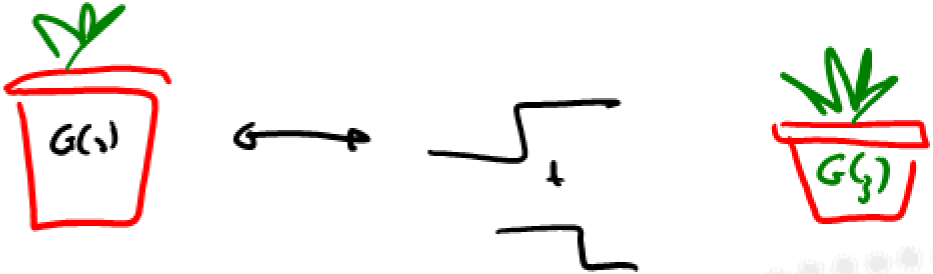


$\frac{1}{T}\frac{G(s)}{s}$

$-\frac{z^{-1}}{T}\frac{G(s)}{s}$

$\Rightarrow \frac{1}{T}\left[1 - z^{-1}\right]\frac{G(s)}{s}$

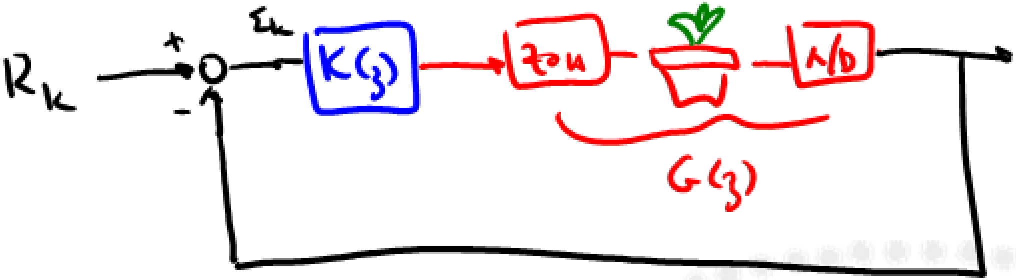
$$\boxed{\left[\frac{z-1}{Tz}\right]\frac{G(s)}{s}}$$



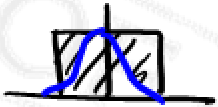
$$G(z) = \mathcal{Z}\left\{ \frac{z-1}{z} \left[\frac{G(s)}{s}\right] \right\}$$

$$\boxed{G(z) = \frac{z-1}{z}\, \mathcal{Z}\left\{ \frac{G(s)}{s} \right\}}$$

c2d(sys, Ts, 'zoh')



$$G(z) = \frac{z-1}{z}\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

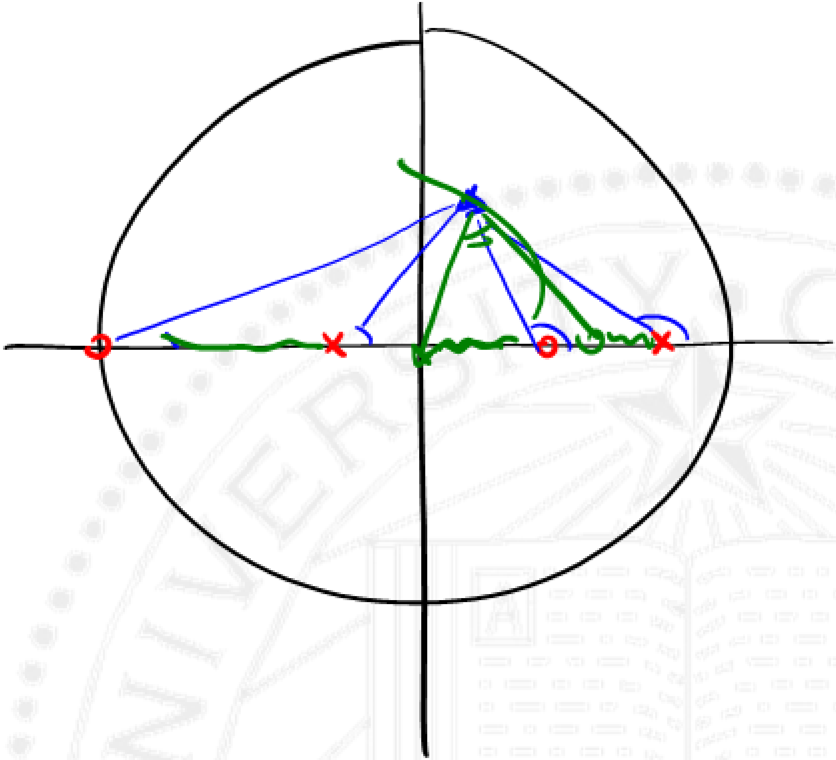


$\pm\frac{1}{2}$ LSB.

$$\frac{Y_k}{R_k} = \frac{GK(z)}{1+GK(z)} \leftarrow D(z) = 0.$$

$$GK(z) = -1 \quad \begin{cases} |\,| = 1 \\ \angle = 180 \end{cases} \quad \underline{\text{RL}}\ !!$$

<u>s-dom</u> $\longrightarrow$ <u>z-dom</u>



STEP: $\frac{z}{z-1} H(z)$.

FVT: $\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$

$$\lim_{k \to \infty} f(kT) = \lim_{z \to 1} \frac{z-1}{z} F(z)$$

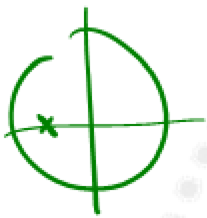
DC GAIN: $\lim_{s \to 0} s \frac{F(s)}{s} = F(s)\Big|_{s=0}$

$$\lim_{z \to 1} \frac{z-1}{z} \cdot \frac{z}{z-1} F(z) = F(z)\Big|_{z=1}$$

$$H(z) = \frac{3z^2+2z+1}{4z^3+5z^2+2z+9} \longrightarrow DC(H(z)) = \frac{6}{15}.$$

$$F(z) = \frac{3/2}{z+1/2} \longrightarrow u_{k+1} = \frac{3}{2}e_k - \frac{1}{2}u_k$$

e_k	u_k
0	0
1	0
0	$3/2$
0	$-3/4$
0	$+3/8$
0	$\vdots$



In MATLAB include the leading zeros

$$\frac{z+3}{z^2+2z+1}$$

$N = [~~1~~\ 3]$ $D = [1\ 2\ 1]$

$N = [0\ 1\ 3]$

$$z = e^{j\omega T} \Rightarrow \cos(\omega T) + j\sin(\omega T)$$

$$G(z) = \frac{z-1}{z} \mathcal{Z}\left\{ \frac{G(s)}{s} \right\}$$

$G(z)$ is exact for zoh.

Analysis / Design entirely in z-domain.

DIGITAL CONTROL

STAY IN ANALOG WORLD

ZOH ~ $e^{-\frac{Ts}{2}}$

BODE ~ $\Delta\phi = -\frac{\omega T}{2}$

ROOT LOCUS ~ PADÉ

$$e^{-\frac{Ts}{2}} \simeq \frac{-(s-\frac{4}{T})}{(s+\frac{4}{T})}$$

$0°$

use ANALOG tools —

$K(z)$ mapped from $K(s)$

use tustin $s = \frac{2}{T}\frac{z-1}{z+1}$

GO FULL DIGITAL

$$G(z) = \frac{z-1}{z} \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$G(z)$ — exact

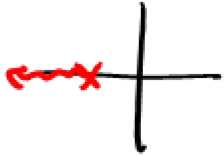
$$\frac{Y}{R}(z) = \frac{GK(z)}{1+GK(z)} \to D_{cl}$$

ROOT LOCUS IN z-plane

DESIGN IN z.

$$\underline{z_{DES} = e^{s_{des} \delta T}}$$

$$G(s) = \frac{a}{(s+a)}$$

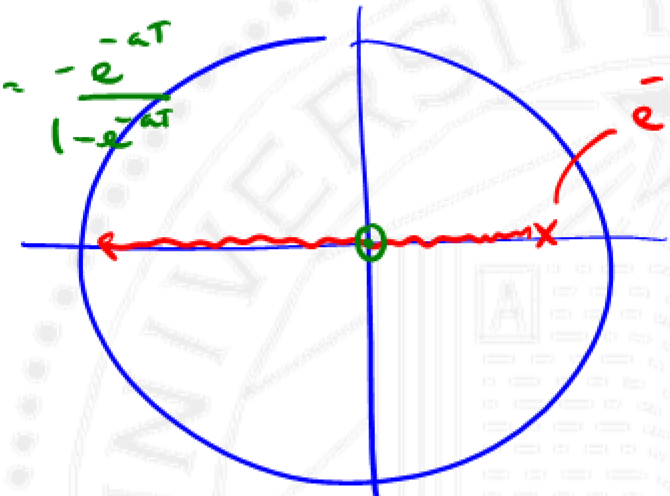


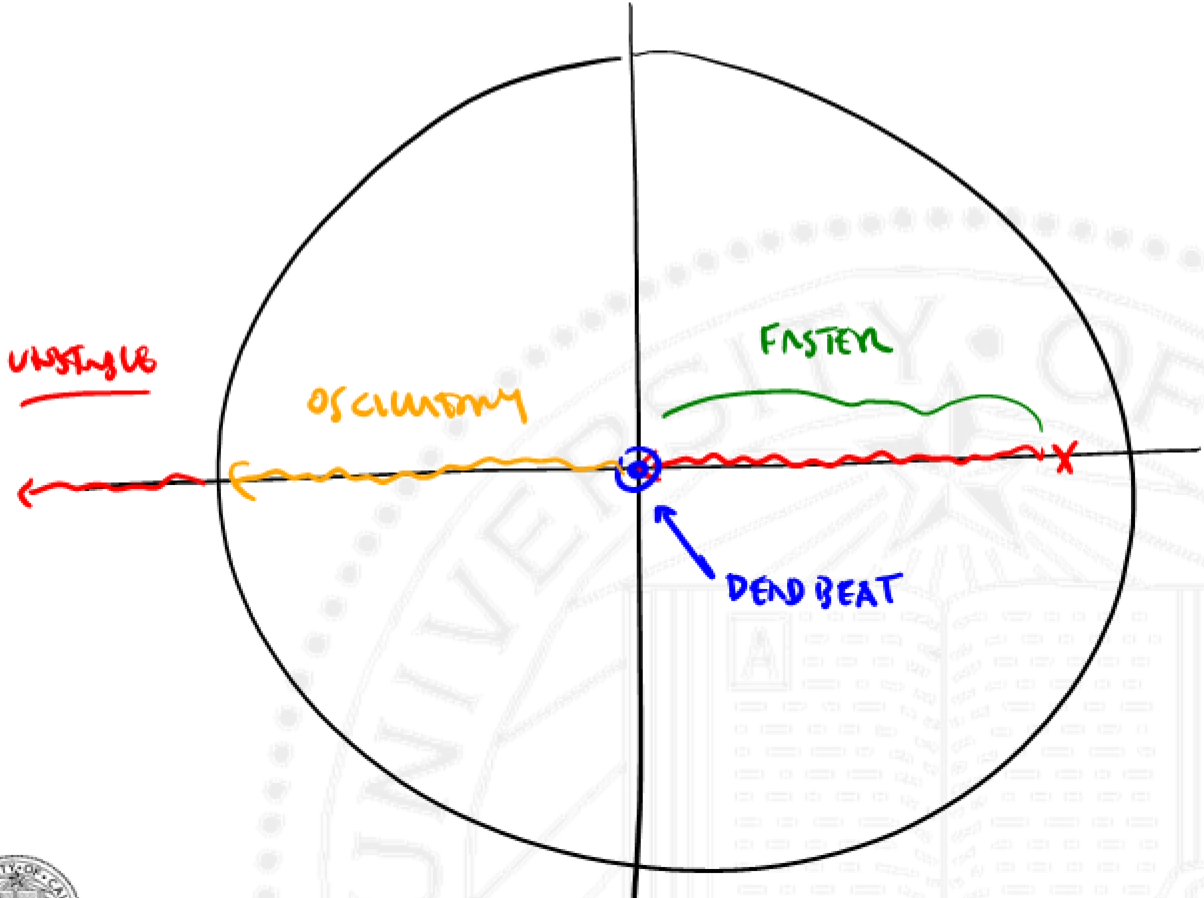
$$\mathcal{Z}\left\{\frac{a}{s(s+a)}\right\} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$$

$$G(z) = \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{\cancel{z}}{\cancel{z-1}} \frac{1-e^{-aT}}{z-e^{-aT}} = \frac{1-e^{-aT}}{z-e^{-aT}}$$

$KG=1$ @ $(0,0)$

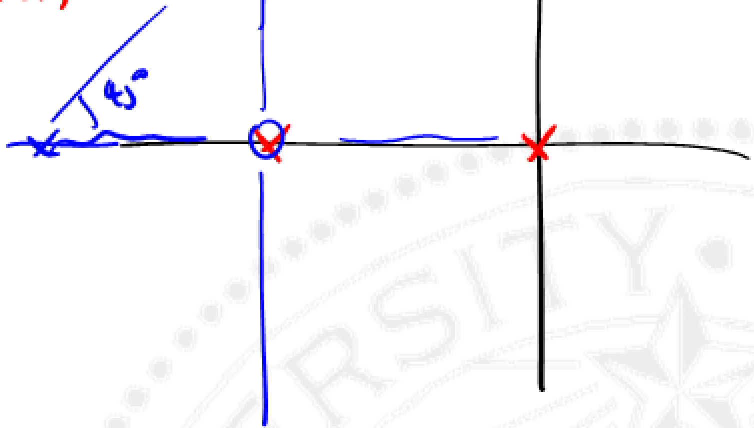
$$K = \left(\frac{1-e^{-aT}}{-e^{-aT}}\right) = \frac{-e^{-aT}}{1-e^{-aT}}$$




UNSTABLE
OSCILLATORY
FASTER
DEAD BEAT

$G(s) = \dfrac{1}{s(s+1)}$

$s_d = -1 \pm j$



$G(s) = \dfrac{1}{s(s+1)}$

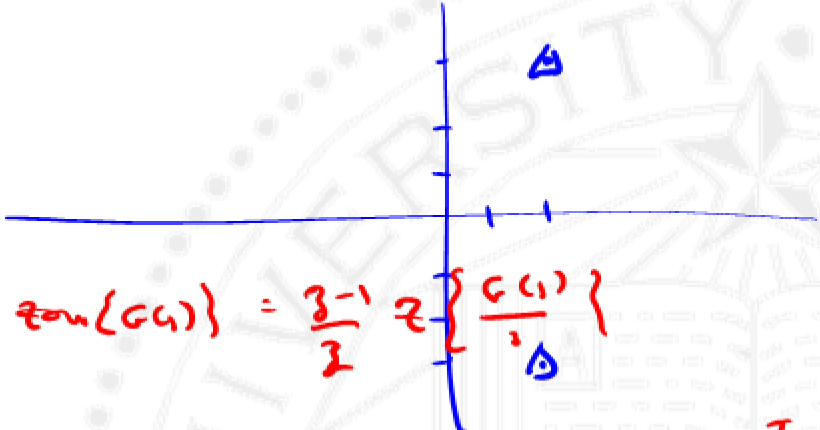
$T = 1$ sec.

$\omega_s = 1\ \text{Hz} \rightarrow 2\pi$ rad/sec.

$\dfrac{\omega_s}{2} = \pi$.

$$z_m = e^{\lambda_m T} = e^{(-1\pm j)T} \underset{1}{\uparrow} = e^{-1\pm j} = e^{-1}e^{\pm j}$$

$$z_m = 0.2 \pm 0.3j$$

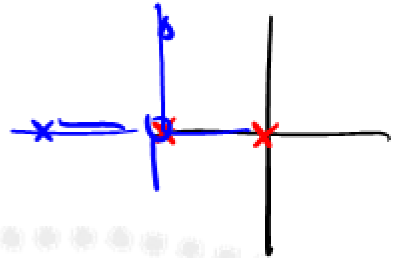


$$G(z) = \text{zoh}\{G(s)\} = \frac{z-1}{z}\, Z\left\{\frac{G(s)}{s}\right\}$$

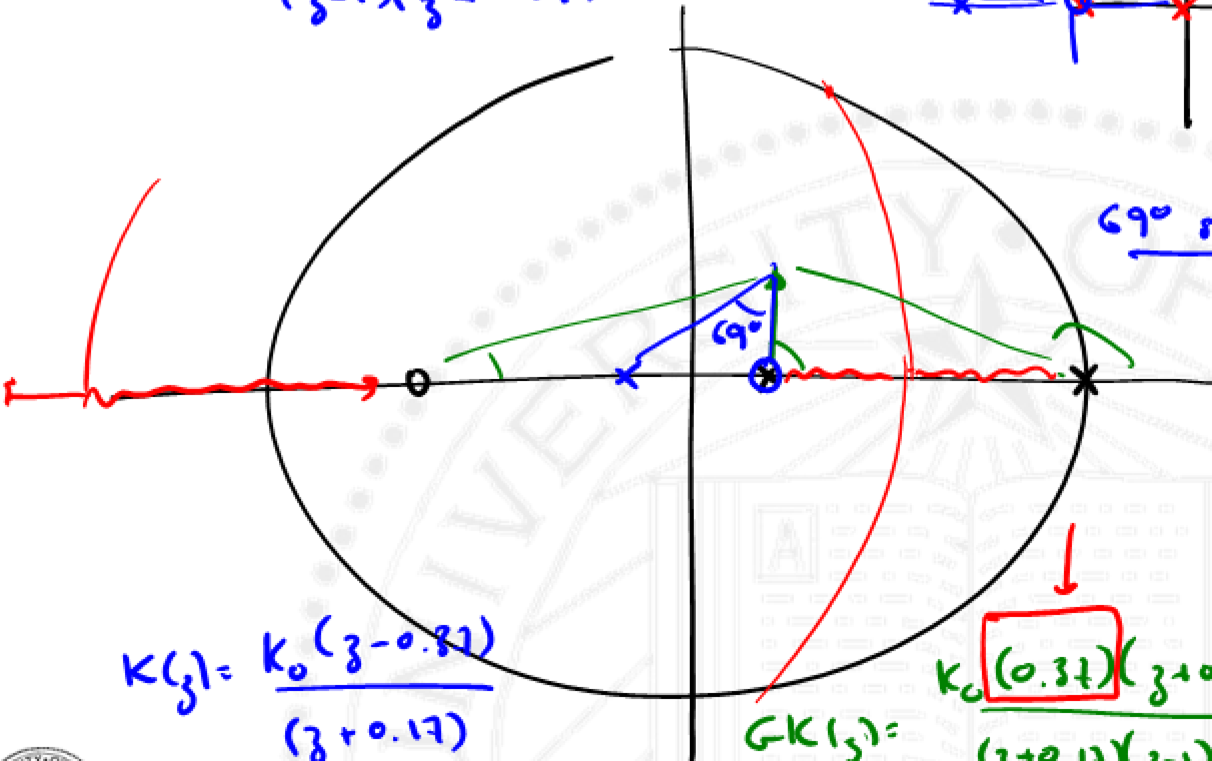
$$G(z) = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s^2(s+1)}\right\} \rightarrow \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{z\left[e^{-T}z + (1-2e^{-T})\right]}{(z-1)^2\,(z-e^{-T})}$$

#13

$$G(z) = \frac{0.37(z+0.72)}{(z-1)(z-0.37)}$$



69° short



$z = 0.2 \pm 0.5j$

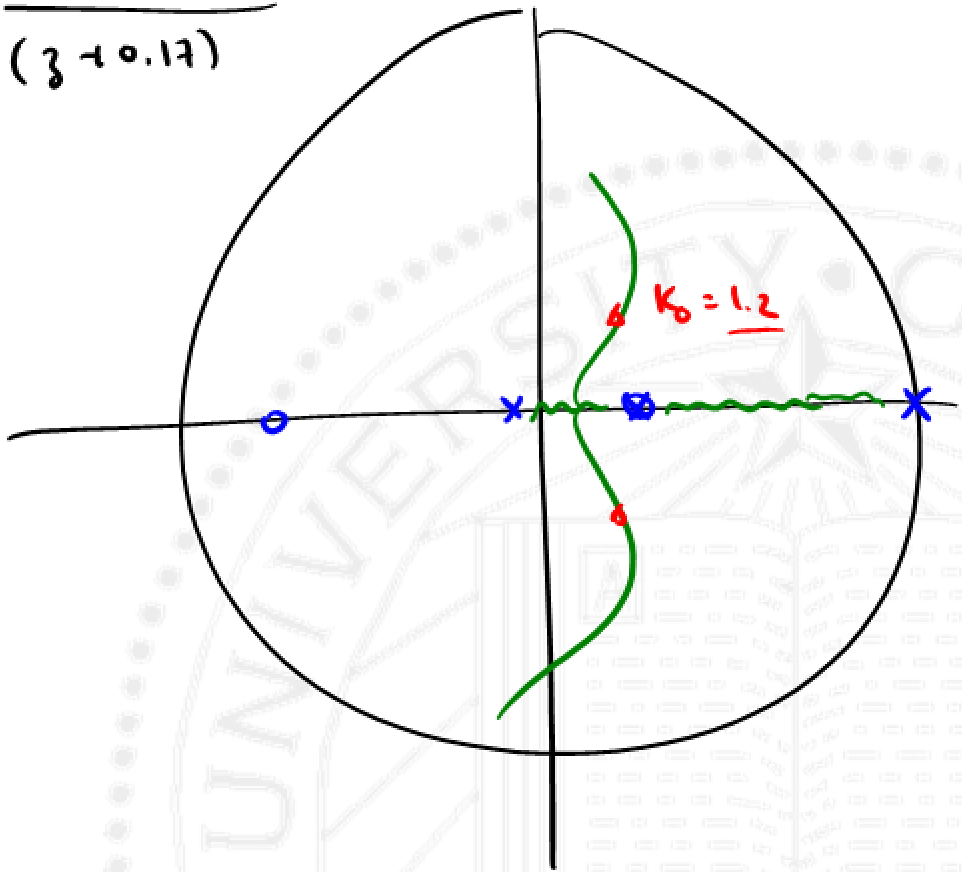
$z = e^{-1} e^{j}$

$$K(z) = \frac{K_0 (z-0.37)}{(z+0.17)}$$

$K_0 = 1.2$

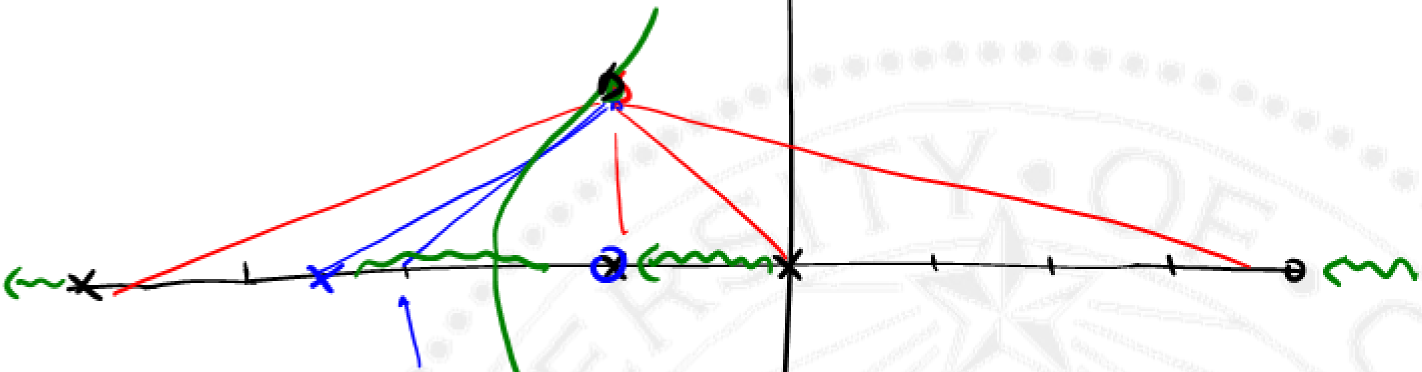
$$GK(z) = \frac{K_0 \boxed{(0.37)} (z+0.72)}{(z+0.17)(z-1)}$$

$$K(z) = \frac{1.2\,(z-0.37)}{(z+0.17)}$$

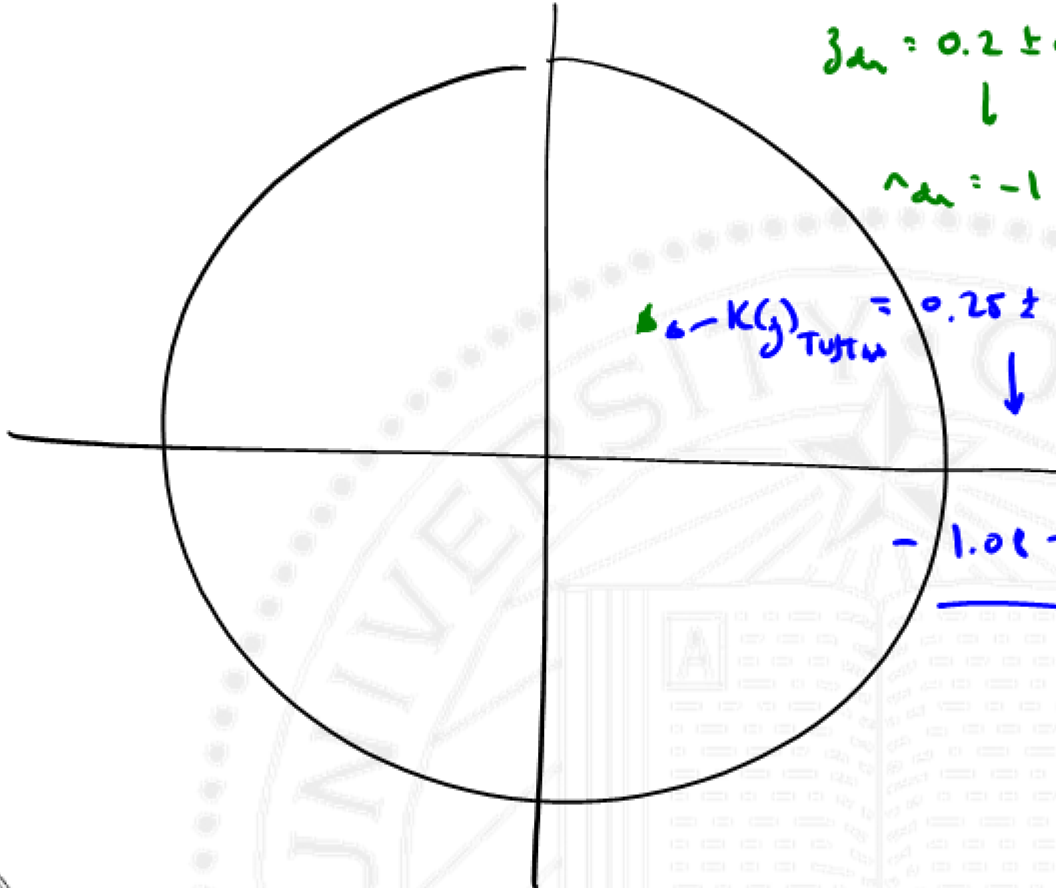


$$K(s) = \frac{2.5\,(s+1)}{(s+3.66)}$$

Padé: $\dfrac{-(s-4)}{s+4}$



$$K(z) = K(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} \longrightarrow \boxed{K(z) = \frac{1.325\,(z-0.334)}{z+0.2935}}$$



$z_m : 0.2 \pm 0.3j$

$\downarrow$

$s_m : -1 \pm j$

$s \leftarrow K(z)_{Tustin} = 0.25 \pm 0.22j$

$\downarrow$

$\underline{-1.01 \pm 0.32j}$

$\omega_s/2$
$-\omega_s/2$

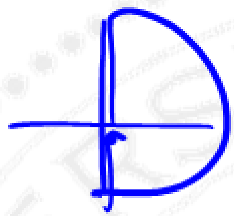
Root locus in z → Unchanged

$G(z) = \frac{z-1}{z} \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$

$z_m = e^{s_m \Delta T}$

Bode in z?

Nyquist



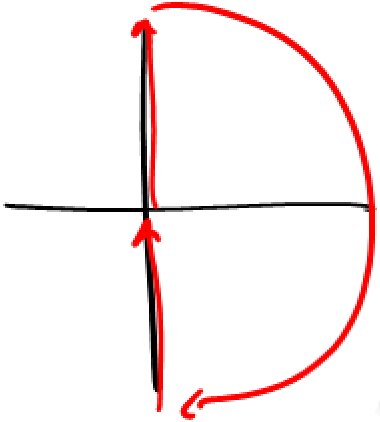
$s = j\omega$

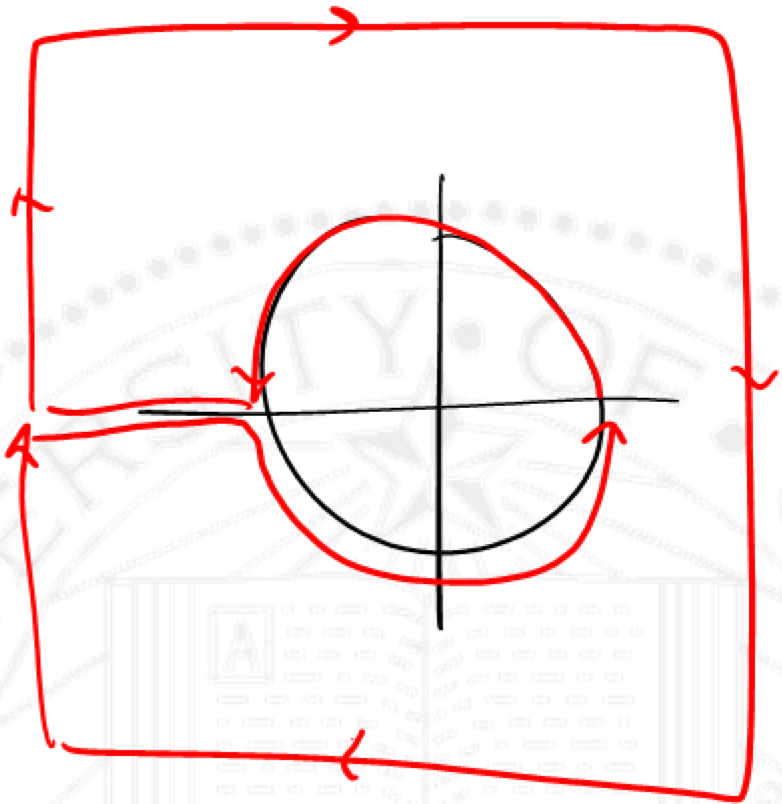
$z = e^{j\omega T}$ → lost my straight line asymptotes.

dbode (bode)

$s \rightarrow z \rightarrow \underline{w}$

Nyquist





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$$K_p = \lim_{z \to 1} K(z)G(z)$$

$$K_v = \lim_{z \to 1} \frac{(z-1)\,K(z)G(z)}{T_z}$$