CMPE-242
Applied Feedback Control

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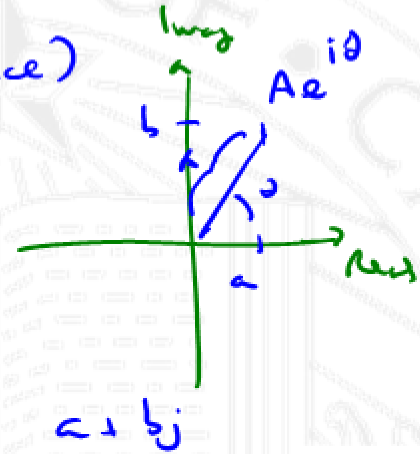
Winter 2016

Announcements

3 handouts:
- Course Info (Matlab tips)
- Syllabus
- Homework #1.

Pre-req's:
- Differential Equations (Laplace)
- Laplace transforms
- Complex numbers
- Linear Algebra
  - eigenvalues
  - determinants
  - inverse
- Fourier analysis



SISOTOOL

Intro

Classical Controls
- Transfer functions
- Root locus
- Bode
- Nyquist

} 3 weeks

Digital Control
- z-transforms
- equivalence

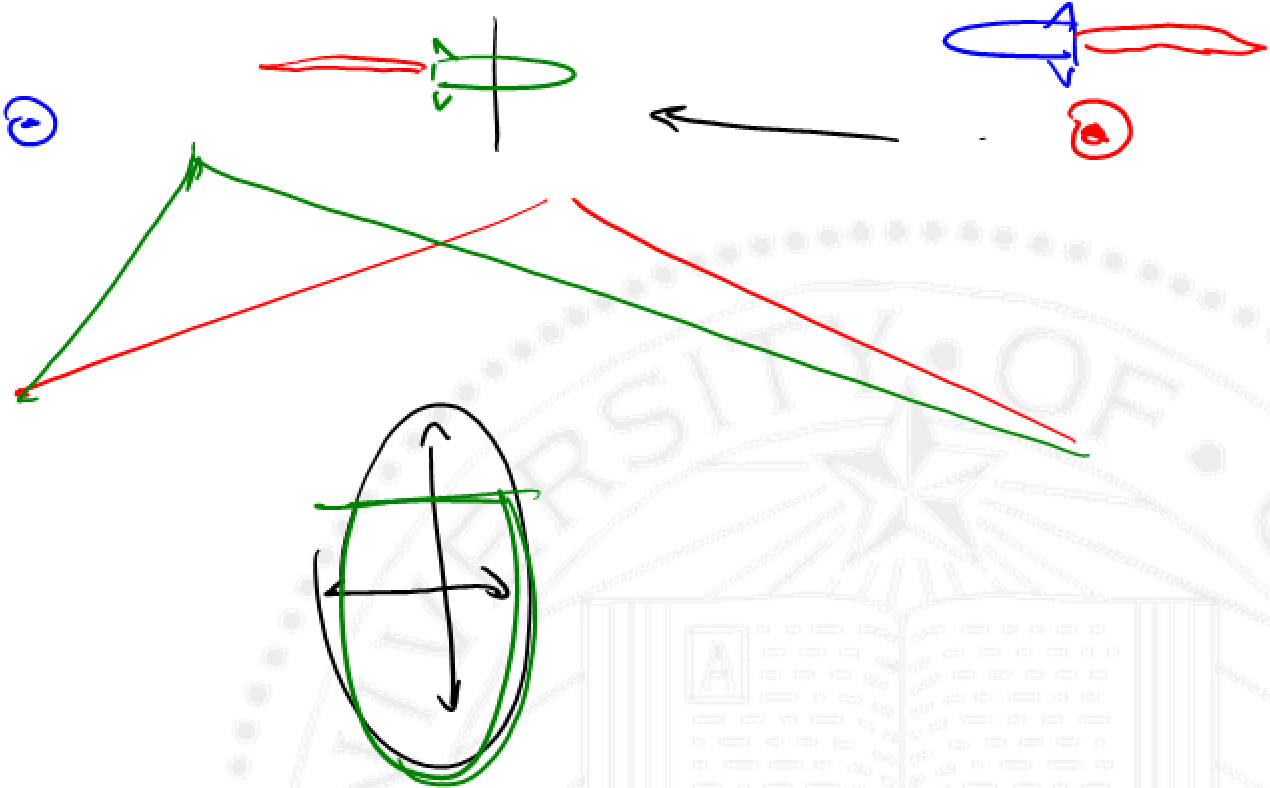
} 3 weeks

State Space
- "Modern Controls" - 1960
- Linear Algebra
- LQ(.) R, E, G

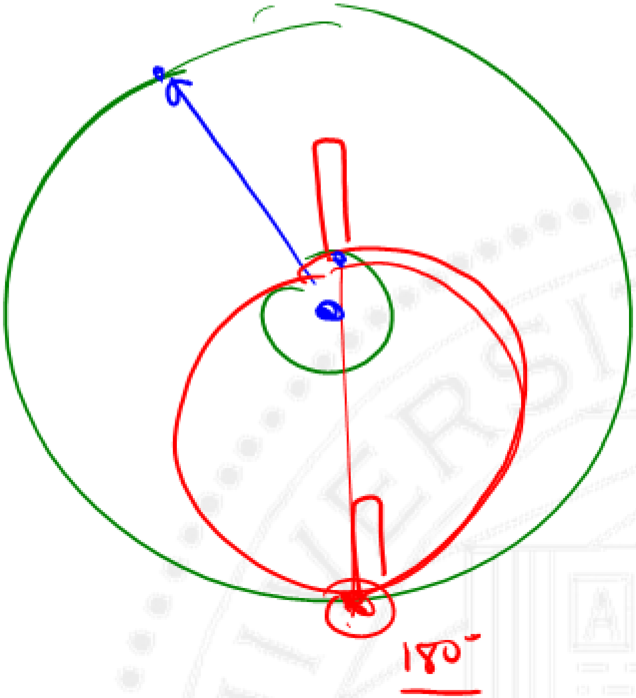
} rest of the class

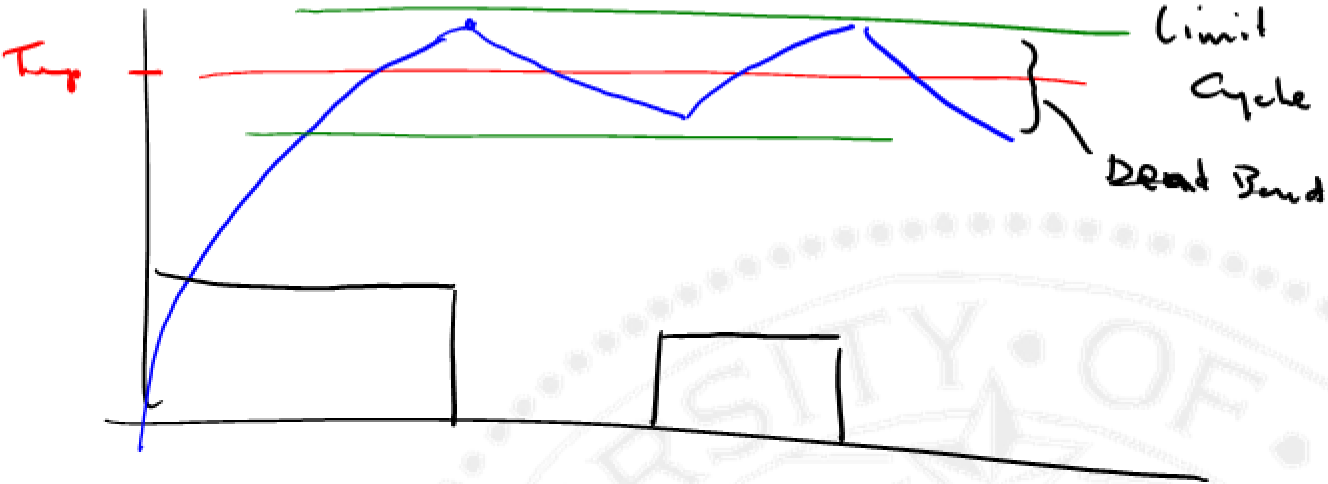
Non-linear control — Lyapunov Stability

Bang-Bang control +/0/−



Nonminimum Transfer





Linear Optimal Control (LQ)

$$u = -K\underline{x}$$

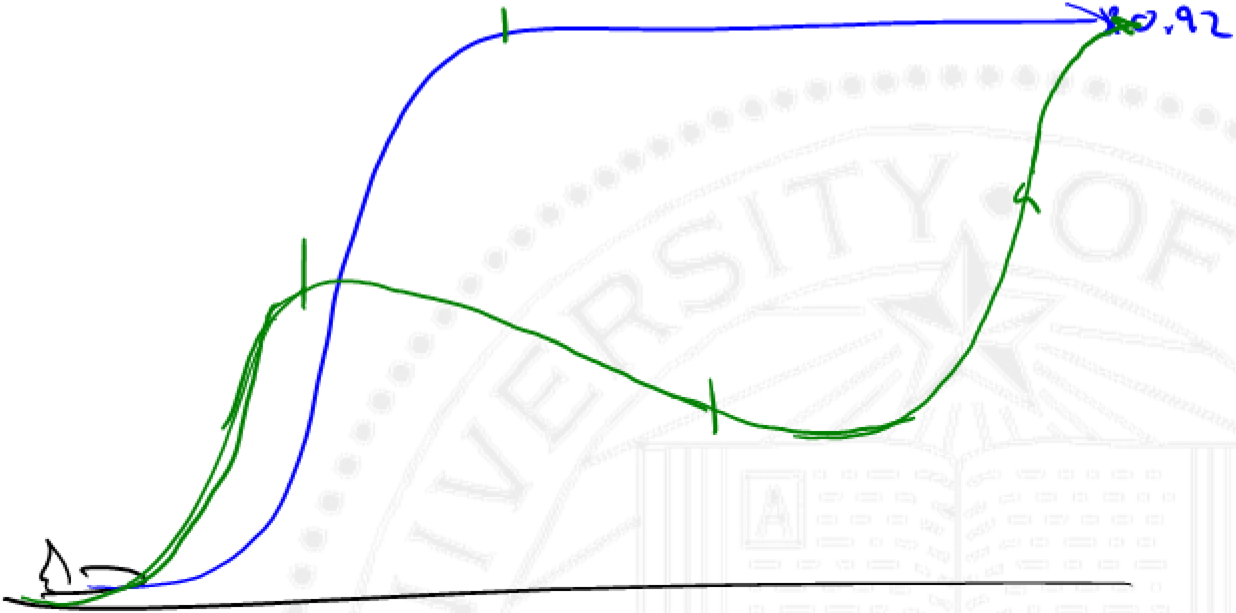
$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

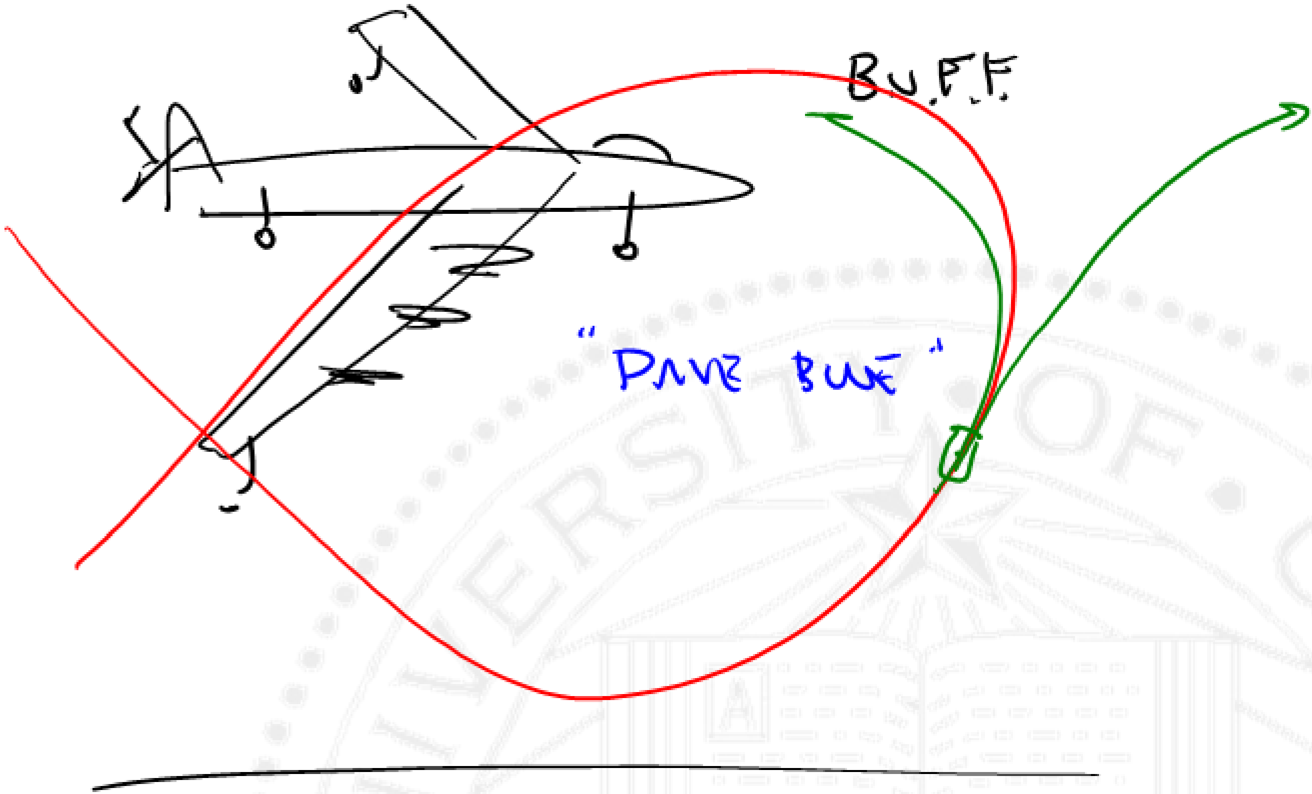
Optimal Control — calculus of variations (Art Bryson)

Lagrange Multipliers $\int_0^\infty J + \lambda_i\,(\text{constraints})$

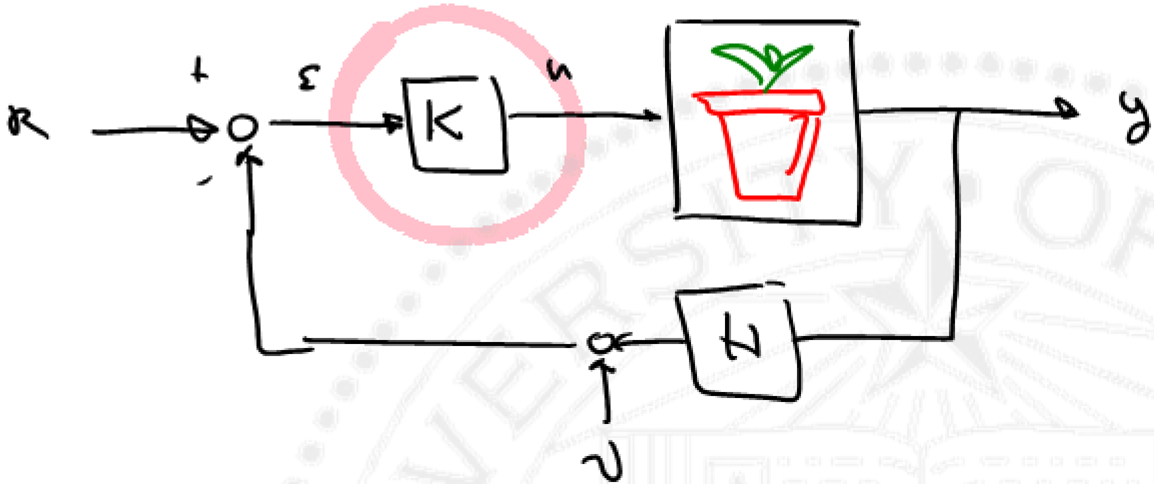
Open loop trajectory $\begin{cases} \text{Min time} \\ \text{Min Energy} \\ \text{Min Fuel Burn} \end{cases}$

Time to climb   0, 0 mph ⟶ 60,000 Mch .92




B.U.F.F.
"PAVE BUF"

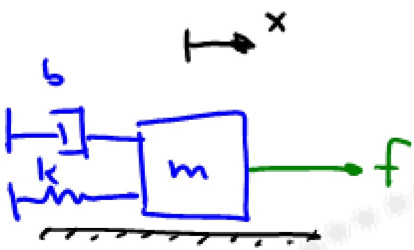
CONTROLS $\neq$ APPLIED MATH

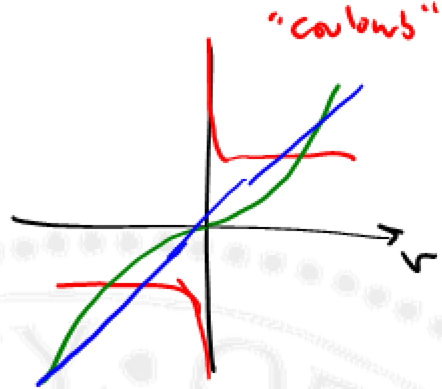


10 - 20% of your time is designing "K".

Ch. 1-3, Appendices, & F.P.W.

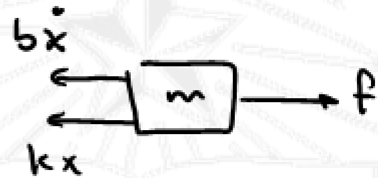
Equation of Motion





$$f = m\ddot{x} \quad \Big\} \quad f = \frac{d\vec{p}}{dt}$$

$$m\dot{v}$$



$$f = b\dot{x}$$ "viscous friction"

$$bv$$

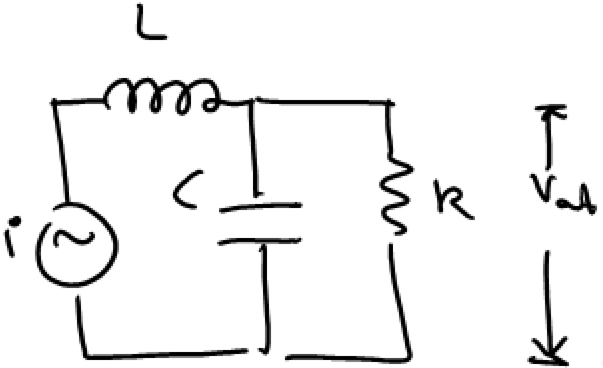
$$f = kx$$ spring constant

$$k\int v$$

$$\Sigma F_x = m\ddot{x}$$

$$f - b\dot{x} - kx = m\ddot{x}$$

$$\boxed{m\ddot{x} + b\dot{x} + kx = f}$$



Kirchov's Laws

$$\Sigma i = \phi$$

$$\Sigma v = \phi$$

$$\boxed{i = C\dot{v} \quad \text{"mass"}}$$

$$i = v/R \quad \text{"damper"}$$

$$i = \frac{1}{L}\int v\,dt \quad \text{"spring"}$$

E.O.M

$$f = m\ddot{x} + b\dot{x} + kx$$

Linear Constant Coefficient ordinary Differential Equation

(LCCODE) –

Linear Time Invariant (LTI)

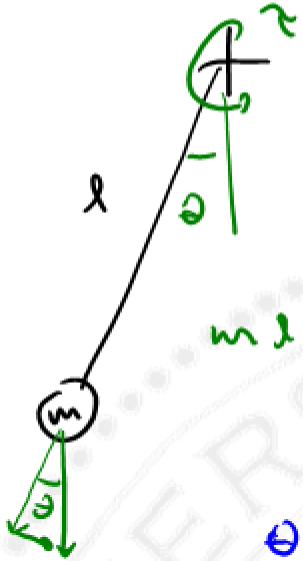
$f_1(t) \longrightarrow x_1(t)$

$f_2(t) \longrightarrow x_2(t)$

$$\Rightarrow (f_1 + f_2)(t) \Rightarrow x_1(t) + x_2(t)$$

$$\alpha f_1(t) \longrightarrow \alpha x_1(t)$$

Non-linear



$$m l^2 \ddot{\theta} = -mgl\sin\theta + \tau$$

$$m l^2 \ddot{\theta} + mgl\,\underline{\sin}\,\theta = \tau$$

$$\theta \triangleq \theta_0 + \delta\theta$$

$$\tau_0 = mgl\sin\theta_0 \qquad \ddot{\theta} = \delta\ddot{\theta}$$

$$\sin(\theta_0 + \delta\theta) = \sin\theta_0 \overset{1}{\cancel{\cos\delta\theta}} + \cos\theta_0 \underbrace{\sin\delta\theta}_{\delta\theta}$$

$$ml^2\ddot{\delta\theta} + mgl\left[\sin\theta_0 + \cos\theta_0\,\delta\theta\right] = \tau_0 + \delta\tau$$

$$ml^2\ddot{\delta\theta} + \cancel{mgl\sin\theta_0} + mgl\cos\theta_0\,\delta\theta = \cancel{\tau_0} + \delta\tau$$

$$\tau_0 = mgl\sin\theta_0$$

$$ml^2\,\ddot{\delta\theta} + mgl\cos\theta_0\,\delta\theta = \delta\tau$$

$$ml^2\ddot{\theta} + \cancel{mgl\sin\theta} = \tau_c + \cancel{mgl\sin\theta}$$

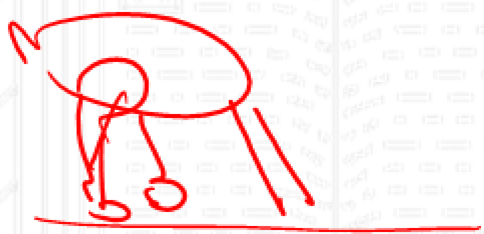
feedback linearization

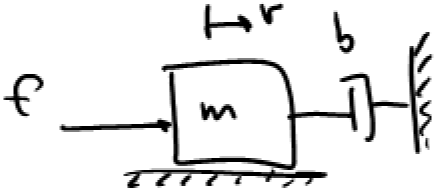
$$ml^2\ddot{\theta} = \tau_c$$

(1) Linearize about a trim point $(\delta\theta, \delta\tau)$

(2) Apply feedback linearization

(3) Ignore it.





$$F - bv = m\dot{v}$$

$$\boxed{m\dot{v} + bv = f + \phi}$$

forced   natural / homogeneous

$$\left.\begin{aligned} v &= Ae^{st} \\ \dot{v} &= sAe^{st} \end{aligned}\right\} \quad ms\cancel{Ae^{st}} + b\cancel{Ae^{st}} = \phi$$

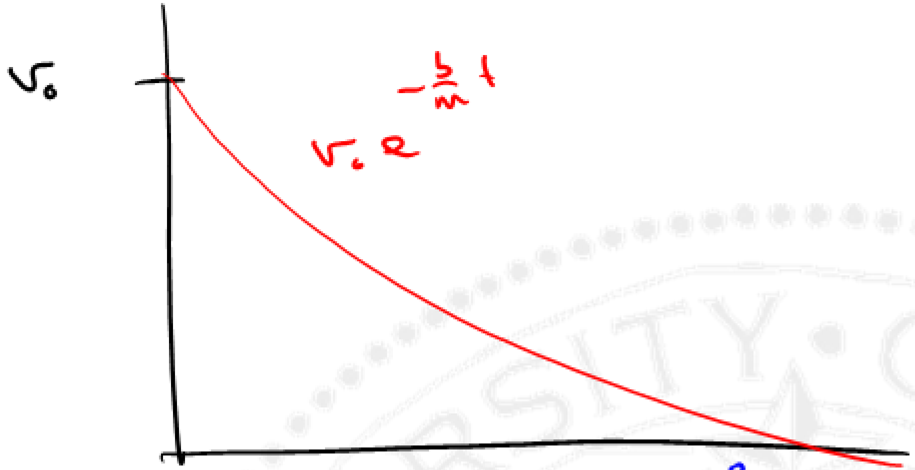
$$ms + b = \phi$$

$$\therefore \; s = -\frac{b}{m}$$

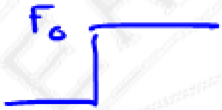
$$v = Ae^{-\frac{b}{m}t}$$

$$v\Big|_{t=0} \triangleq v_0$$

$$v(t) = v_0 e^{-\frac{b}{m}t}$$



Forced response:



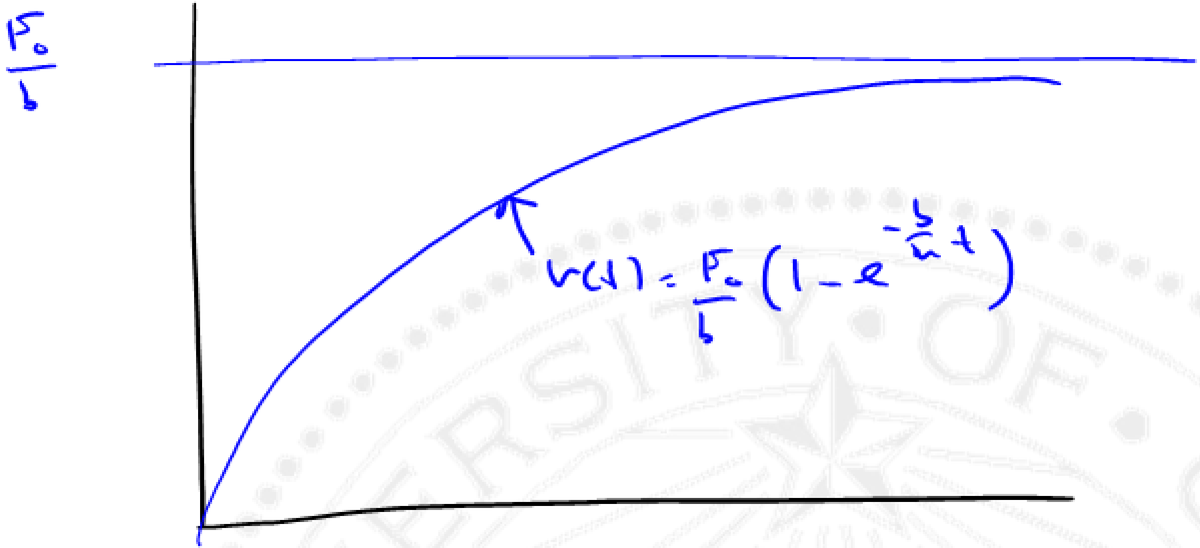
$$m\cancel{\dot{v}}^{\,0} + bv = F_0 \quad \therefore \; v = \frac{F_0}{b}$$

$$v = \frac{F_0}{b} + Ae^{-\frac{b}{m}t}$$

$$v_0 = \phi = \frac{F_0}{b} + A(1)$$

$$A = -\frac{F_0}{b}$$

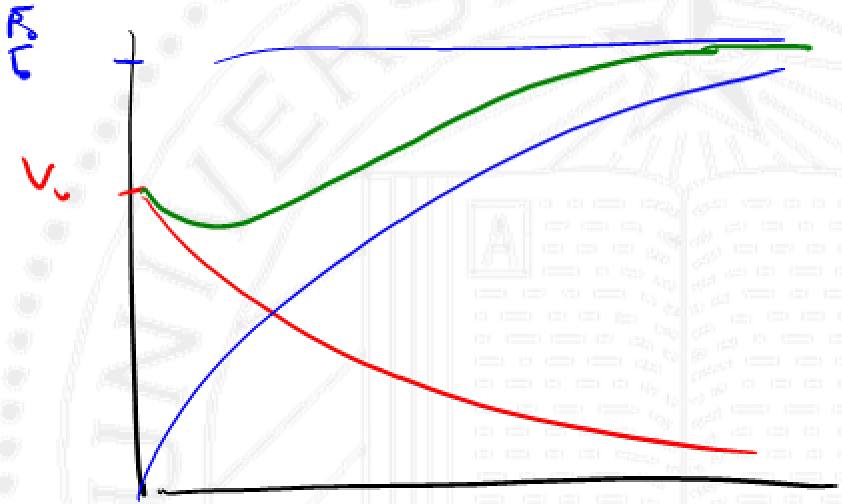
$$v(t) = \frac{F_0}{b}\left[1 - e^{-\frac{b}{m}t}\right]$$


$\frac{F_0}{b}$
$v(t) = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)$

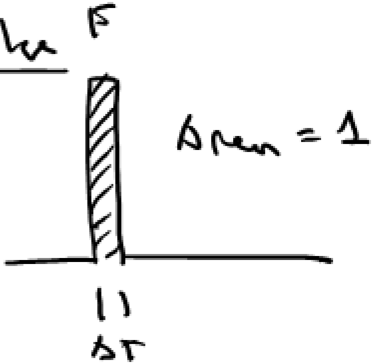
$$v_N = v_0 e^{-\frac{b}{m}t}$$

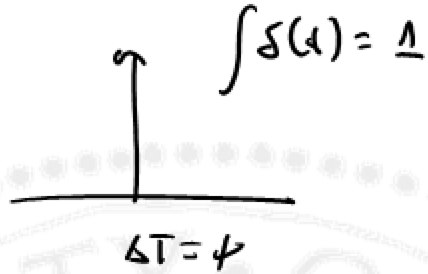
$$v_F = \frac{F_0}{b}\left(1 - e^{-b/m\,t}\right)$$

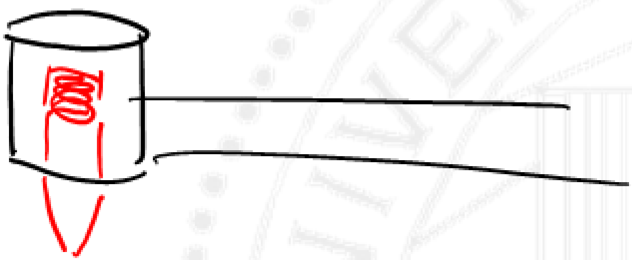
$$v_{TOT} = v_N + v_F = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right) + v_0 e^{-\frac{b}{m}t}$$

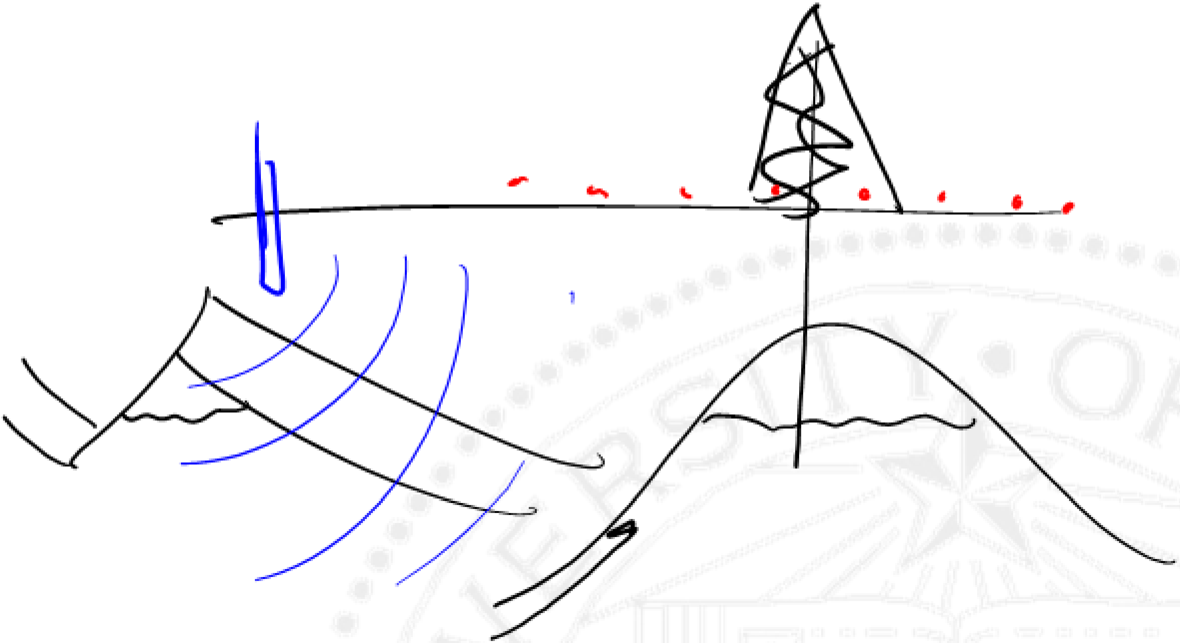


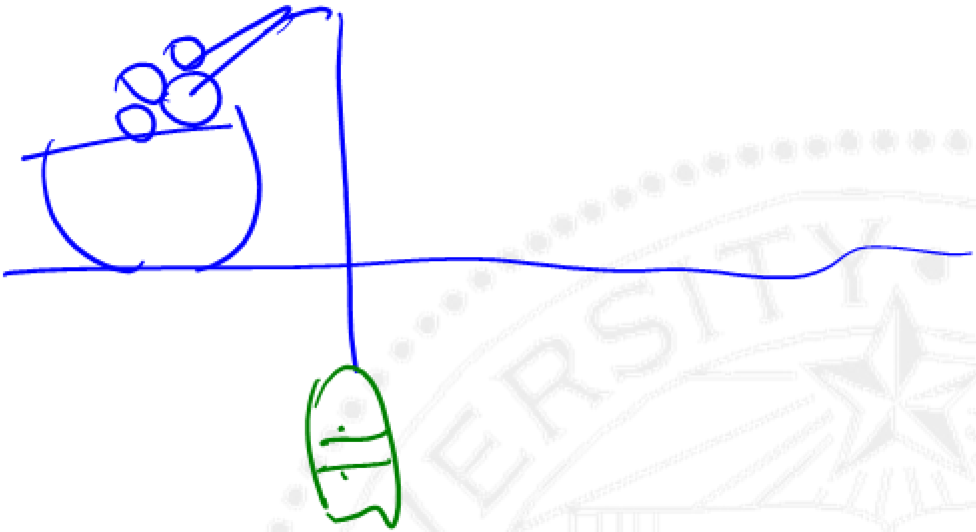
Impulse

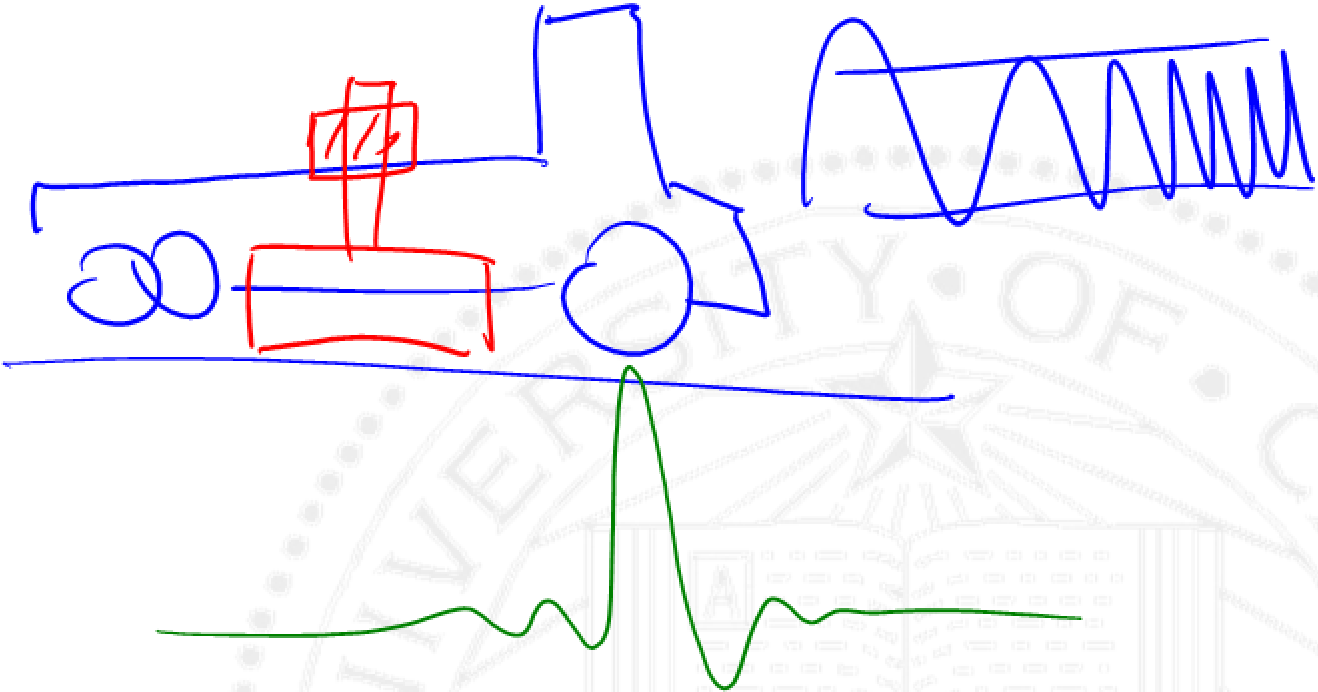




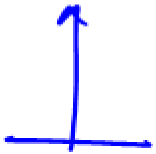






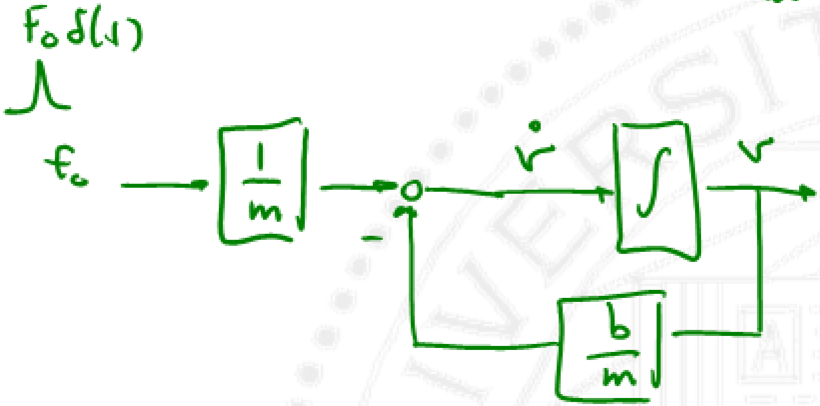


Impulse



$\delta(t)$ $\quad \int \delta(t)\,dt = 1$ $\quad F_0\,\delta(t)$

$m\dot{v} = bv = F$ $\qquad \dot{v} = -\frac{b}{m} v + \frac{F}{m}$



0^-

$0^+ : v_0 = \frac{F_0}{m}$

$$v(t) = \frac{F_0}{m} e^{-\frac{b}{m}t}$$

//

$$v_{step} = \frac{F_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

$$v_{imp} = \frac{F_0}{m} e^{-\frac{b}{m}t}$$

$$\frac{d}{dt}\left(v_{step}\right) = v_{imp}$$

$$\frac{d}{dt}\left[\cancel{\frac{F_0}{b}}^{\,0} - \frac{F_0}{b} e^{-\frac{b}{m}t}\right] = -\frac{F_0}{\cancel{b}}\left(\frac{-\cancel{b}}{m}\right) e^{-\frac{b}{m}t}$$

$$= \frac{F_0}{m} e^{-\frac{b}{m}t}$$

✓

$$F_0 = 1$$

$$v_{imp}(t) = \underbrace{\frac{1}{m} e^{-\frac{b}{m}t}}$$

$h(t) \triangleq$ impulse response