

UNIVERSITY OF CALIFORNIA, SANTA CRUZ
BOARD OF STUDIES IN COMPUTER ENGINEERING



CMPE-242:
APPLIED FEEDBACK CONTROL

HOMEWORK #6
DUE 18-FEB-2015 @ 6:00PM

1. *Z-plane Root Locus*: Consider controlling the system $G(s)$ with a discrete controller (in the usual unity feedback configuration) with a sample time of $\Delta T = 0.2\text{sec}$:

$$G(s) = \frac{1}{s^2 + \frac{1}{5}s + 1}$$

- a. Use root locus techniques to design a compensator of the form $K(z) = \frac{K(z+a)}{z+b}$ that will yield closed loop roots in the z-plane that correspond to two poles at $s = -1$. What is your $K(z)$?
- b. What is the DC gain of your closed loop system (work in the Z-domain)?
- c. Calculate the first three terms of the step response of your closed loop system. Do this by hand (i.e.: use long division), then use MATLAB's `dstep` to plot the step response as a check.

Note: there are an infinite number of controllers that will generate closed loop poles at the desired locations. If you have time, experiment with a few of these designs to get a feeling for working in the z-domain.

2. *Discrete Bode*: Consider the same system $G(s)$ as before. This time the goal is to design a lead compensator using bode techniques (in the z-domain) that will yield an $\omega_{x0} = 5 \text{ rad/s}$ with a phase margin of 36° :

$$G(s) = \frac{1}{s^2 + \frac{1}{5}s + 1}$$

- a. What $K(s)$ would work? (Place the pole and zero such that the peak phase of $K(s)$ is at ω_{x0}).

- b. If $\Delta T = 0.2$, and $K(z) = K$ (*constant*), what value of K will make the system go unstable? Sketch the Nyquist diagram.
- c. If $\Delta T = 0.01$, how much phase lead is necessary to meet the 36° phase margin spec? How much is necessary if $\Delta T = 0.2$?
- d. What $K(z)$ will meet the specs if $\Delta T = 0.01$? Use `dbode.m` to plot the magnitude and phase of $G(z)K(z)$.
- e. What $K(z)$ will meet the specs if $\Delta T = 0.2$? Use `dbode.m` to plot the magnitude and phase of $G(z)K(z)$.

Hint: Convert plant to $G(z)$, then use `gain` to get ω_{x0} ; from there, design your lead in s , and convert to $K(z)$ using Tustin w/prewarping. Make sure to plot bode plots of both compensated and uncompensated plants.