

UNIVERSITY OF CALIFORNIA, SANTA CRUZ  
BOARD OF STUDIES IN COMPUTER ENGINEERING



CMPE-242:  
APPLIED FEEDBACK CONTROL

---

HOMEWORK #2  
DUE 21-JAN-2016

---

1. *Root Locus*: Sketch the  $0^\circ$  and the  $180^\circ$  root loci vs. gain,  $K$ , for each of the following transfer functions. Show asymptotes and indicate arrival and departure angles for any complex poles or zeros. Check your results in MATLAB only *after* sketching them out by hand.

a.  $G(s) = \frac{(s^2+1)}{s(s^2+4)}$

b.  $G(s) = \frac{(s+3)}{s^3(s+4)}$

c.  $G(s) = \frac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$

d.  $G(s) = \frac{(s+2)}{s(s-1)(s+6)^2}$

2. *Root Locus*: Given the plant below, where  $\alpha$  is a parameter that can be both positive and negative:

$$G(s) = \frac{1}{s^2 + (1 + \alpha)s + (1 + \alpha)}$$

- a. Rewrite the transfer function to be in Evan's Form ( $1-GK=0$ ).
- b. Sketch the  $0^\circ$  and the  $180^\circ$  root loci vs. parameter,  $\alpha$ .
- c. Comment on the limits of stability for the plant.
3. *Lag Compensation*: Given a plant with three poles (at 0, -3, and -6) and no zeros in a conventional feedback configuration, design a lag compensator to meet the following specifications: settling time ( $t_s$ ) < 5 seconds, maximum overshoot ( $M_p$ ) < 17%, and a steady state error to a unit ramp less than 10%.

$$G(s) = \frac{1}{s(s+3)(s+6)}$$

4. *Root Locus Design*: Given the plant below, design a compensator to meet the following specifications:

$$G(s) = \frac{10}{s(s+1)(s+10)}$$

- i. Overshoot ( $M_p$ ) < 16%
  - ii. Rise time ( $t_r$ ) < 0.4 sec
  - iii. Steady state error to a unit ramp ( $e_{ss}$ ) < 0.02
- a. Design a lead compensator that will cause the system to meet the dynamic response specs (i and ii).
  - b. If  $K(s)$  is proportional control,  $K(s) = K_p$ , what is the velocity constant,  $K_v$ ?
  - c. Design a lag compensator to be used in series with your lead compensator from part (a) that will cause the system to meet the error spec (iii).
  - d. Give a MATLAB root locus plot of your total compensator (a + c).
  - e. Give a MATLAB step response of the closed loop system

Include MATLAB root locus plots, and step response plots of the final closed loop system.

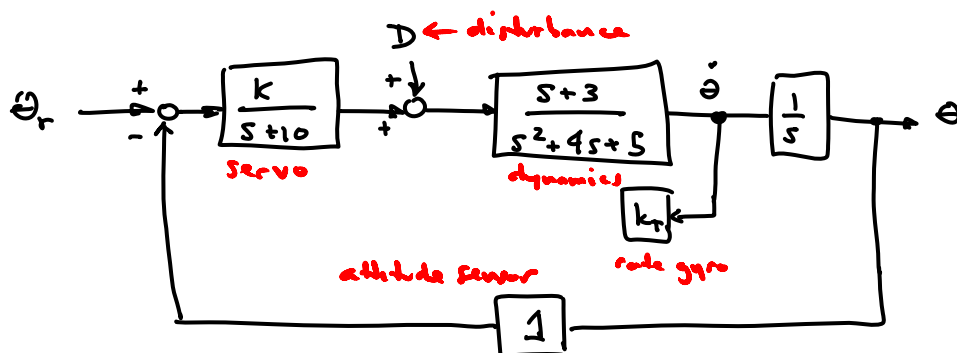
**Hint:** you might need to add a very low frequency lag to hit the steady state error spec.

5. *Non-minimum phase plant*: Given the plant below (with a zero in the RHP), determine the feedback gain,  $K_p$ , required to put the damping ratio ( $\zeta$ ) at 0.707. Use MATLAB to show the root locus with the closed loop poles indicated, and plot the step response of the closed loop system. Add in a feedforward "pre-gain" to fix the step response.

$$G(s) = \frac{4 - 2s}{s^2 + s + 9}$$

**Hint:** Be careful, this one is in positive feedback or  $0^\circ$  root locus. Make sure you understand why.

6. *Rate Feedback*: Given the aircraft pitch autopilot below, with a disturbance input of a step with a maximum magnitude of 0.6,  $[D(s) = 0.6/s]$ , answer the following questions:



- What value of  $K$  is required to keep the steady state error to this disturbance at less than 1 degree (assume system is stable).
- Sketch the root locus with respect to the gain,  $K$ .
- Based on the above, at what  $K$  does the system go unstable.
- Assume the  $K$  required is 600, what are the closed loop poles of the system (and are they stable)?
- Add in where the feedback from the rate gyro should go on the block diagram (that is, you have a rate gyro which gives a signal:  $K_{rate}\dot{\theta}$ , where to you put the feedback).
- Assume  $K$  is 600, draw the root locus with respect to  $K_{rate}$ , the rate gyro gain.
- What can you do with the complex roots using  $K_{rate}$ ? Performance, Damping, etc.
- What  $K_{rate}$  maximizes the damping.
- Let's say you are not satisfied with the design, comment on what would happen if you added integral control, or extra lead networks (show what happens on a root locus).

**Hint:** Take a close look at the "servo" block. A better way to think of this is as a gain of  $K/10$  followed by a servo actuator with a bandwidth of 10 rad/s. The servo actuator is what actually pushes the control surface around. The command to the servo actuator is generated by the control system (in the figure above, the control system is just a proportional gain).

