

UNIVERSITY OF CALIFORNIA, SANTA CRUZ  
BOARD OF STUDIES IN COMPUTER ENGINEERING



CMPE-242:  
APPLIED FEEDBACK CONTROL

---

HOMEWORK #5  
DUE 11-FEB-2015 @ 11:59PM

---

1. *Discrete Filters*: The z-transform of a discrete time filter  $h(k)$  at a 1Hz sample rate is:

$$H(z) = \frac{1 + \left(\frac{1}{2}\right)z^{-1}}{\left[1 - \left(\frac{1}{2}\right)z^{-1}\right]\left[1 + \left(\frac{1}{3}\right)z^{-1}\right]}$$

- Let  $u(k)$  and  $y(k)$  be the discrete input and output of this filter. Find a difference equation relating  $u(k)$  to  $y(k)$ .
  - Find the natural frequency and damping of the filter's poles.
  - Is the filter stable?
2. *Discrete Bode*: The following transfer function is a lead network designed to add about 60° of phase at  $\omega_1 = 3$  rad/sec:

$$H(s) = \frac{s + 1}{(0.1s + 1)}$$

- Assume a sampling period of  $T = 0.25$  seconds, and compute and plot in the z-plane the pole and zero locations of the digital implementations of  $H(s)$  using:
  - Tustin (trapezoidal integration)
  - Euler (backwards difference)
  - Pole-Zero mapping ( $s = \frac{1}{\Delta T} \ln z$ )

For each case, compute the amount of phase lead provided by the network at  $z_1 = e^{-j\omega_1 T}$

- Using a log-log scale for the frequency range of  $\omega = 0.1$  to  $\omega = 100$  rad/s, plot the magnitude bode plots for each of the equivalent digital systems you found in the first part (a), and compare with  $H(s)$ . *Hint*: Magnitude plots are given by  $|H(z)| = |H(e^{j\omega T})|$ .

3. *Distortion of Lead Networks:* In the s-domain, the magnitude and phase introduced by a lead compensator depend only on the ratio of its zero and pole. This is independent of your choice of center frequency. Things are different in the z-domain.

Consider the lead compensator  $K(s)$  below. This has a peak phase and a gain of one at  $\omega_c = \sqrt{ab}$ . Let the center frequency  $\omega_c = 1$  and let  $b/a = 4$ . This will yield  $36^\circ$  of phase lead at  $\omega = 1$ .

$$K(s) = \sqrt{\frac{b}{a}} \frac{(s + a)}{(s + b)}$$

- a. Overplot the magnitude and phase vs. frequency of  $K(s)$  and  $K(z)$  where  $K(z)$  is found using the Tustin transformation ( $s = \frac{2z-1}{Tz+1}$ ). Do this for both of the sampling times,  $T = 0.1 * \pi / \omega_c$  and  $T = 0.8 * \pi / \omega_c$  (Nyquist frequency is  $w = \pi / T$ ). What is your  $K(z)$ ?
- b. Redo part (a), using the Tustin Transformation with pre-warping. That is:  
 $s = \frac{\omega_c}{\tan(\omega_c T / 2)} \frac{z-1}{z+1}$ . *Hint:* it is OK to use `dbode.m` and `c2d.m` on this problem.

4. Consider the system:

$$G(s) = \frac{1}{(s^2 + 1)}$$

- a. Design a compensator,  $D_1(s)$ , that will place the closed-loop roots of this system at  $s_d = -1 \pm 2j$  (use a unity gain feedback configuration). Use a simple lead compensator, and don't worry about DC gain. Plot the *impulse* response of your resulting closed loop system.
- b. Assume that you want to implement the compensator of (a) in a digital computer using a sample time of  $T = 0.5 \text{ sec}$ . Convert  $D_1(s)$  into  $D_1(z)$  using the Tustin mapping. Write the difference equation that would represent  $D_1(z)$ , and write some "pseudo-code" to show how you would implement it. *Hint:* The Tustin mapping is  $s = \frac{2z-1}{Tz+1}$ .
- c. Redo parts (a) and (b), but this time design a compensator,  $D_2(s)$  that "accounts for" the  $T/2$  time delay in the system. Do this by adding a first order Pade approximation to your plant (i.e.: design a compensator for the plant  $G(s) * \frac{-s+4/T}{s+4/T}$ ). Again, use Tustin to convert  $D_2(s)$  to  $D_2(z)$ .
- d. What is your phase margin for part (a)? How much phase margin did you lose when you implemented your design in part (b)? What is your phase margin in part (c)? Did you recover your lost phase margin? (Put these in a table).
- e. Prove that the ZOH equivalent of  $G(s)$  is  $G(z) = 0.1224 \frac{z+1}{z^2 - 1.7552z + 1}$ .

- f. Analyze the designs you created in parts (b) and (c). In particular, calculate the closed-loop z-plane roots using the ZOH-equivalent,  $G(z)$ , and your compensators  $D(z)$ . Then map these roots back to the s-plane using the relation  $s = (1/T) \ln(z)$ . How do the  $\omega_n$  and  $\zeta$  compare with what you were trying to achieve? Note that this analysis is exact.
- g. Create a new compensator  $D_3(z)$  by doing your design directly in the z-plane. First, pick your desired z-plane pole locations to be  $z_d = e^{s_d T}$ . Then, use root-locus in the z-plane to find a  $D_3(z)$  that will yield closed loop roots at  $z_d$ .
- h. Overplot the *impulse* responses of the four closed-loop systems:
  - i.  $D_1(s)$  and  $G(s)$  (this is the “ideal”)
  - ii.  $D_1(z)$  and  $G(z)$
  - iii.  $D_2(z)$  and  $G(z)$
  - iv.  $D_3(z)$  and  $G(z)$

Comment on how well each of the designs works. Try to draw some conclusions. Note: you might want to shift your time axes so they line up. Your continuous response will start at  $t = 0$  whereas your discrete responses will start at  $t = T = 0.5$ . *Hint:* check out IMPULSE.M, DIMPULSE.M, EXP.M, LOG.M, and DAMP.M