UNIVERSITY OF CALIFORNIA, SANTA CRUZ
BOARD OF STUDIES IN COMPUTER ENGINEERING

CMPE-242:
APPLIED FEEDBACK CONTROL

HOMEWORK #6
DUE 19-FEB-2014

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1. *Z-plane Root Locus*: Consider controlling the system *G(s)* with a discrete controller (in the usual unity feedback configuration) with a sample time of $\Delta T = 0.2$sec:

$$G(s) = \frac{1}{s^2 + \frac{1}{5}s + 1}$$

   a. Use root locus techniques to design a compensator of the form $K(z) = \frac{K(z+a)}{z+b}$ that will yield closed loop roots in the z-plane that correspond to two poles at *s = -1*. What is your *K(z)*?

   b. What is the DC gain of your closed loop system (work in the Z-domain)?

   c. Calculate the first three terms of the step response of your closed loop system. Do this by hand (i.e.: use long division), then use MATLAB's dstep to plot the step response as a check.

   Note: there are an infinite number of controllers that will generate closed loop poles at the desired locations. If you have time, experiment with a few of these designs to get a feeling for working in the z-domain.

2. *Discrete Bode*: Consider the same system G(s) as before. This time the goal is to design a lead compensator using bode techniques (in the z-domain) that will yield an $\omega_{xo} = 5$ rad/s with a phase margin of 36°:

$$G(s) = \frac{1}{s^2 + \frac{1}{5}s + 1}$$

   a. What *K(s)* would work? (Place the pole and zero such that the peak phase of *K(s)* is at ω_{xo}).

b. If $\Delta T = 0.2$, and *K(z) = K* (*constant*), what value of *K* will make the system go unstable? Sketch the Nyquist diagram.

c. If $\Delta T = 0.01$, how much phase lead is necessary to meet the 36° phase margin spec? How much is necessary if $\Delta T = 0.2$?

d. What K(z) will meet the specs if $\Delta T = 0.01$? Use dbode.m to plot the magnitude and phase of G(z)K(z).

e. What K(z) will meet the specs if $\Delta T = 0.2$? Use dbode.m to plot the magnitude and phase of G(z)K(z).

Hint: Convert plant to G(z), then use gain to get $\omega_{xo;}$ from there, design your lead in s, and convert to K(z) using Tustin w/prewarping. Make sure to plot bode plots of both compensated and uncompensated plants.