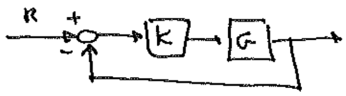
CMPE 242

Practice Midterm Solutions

Fall 2010

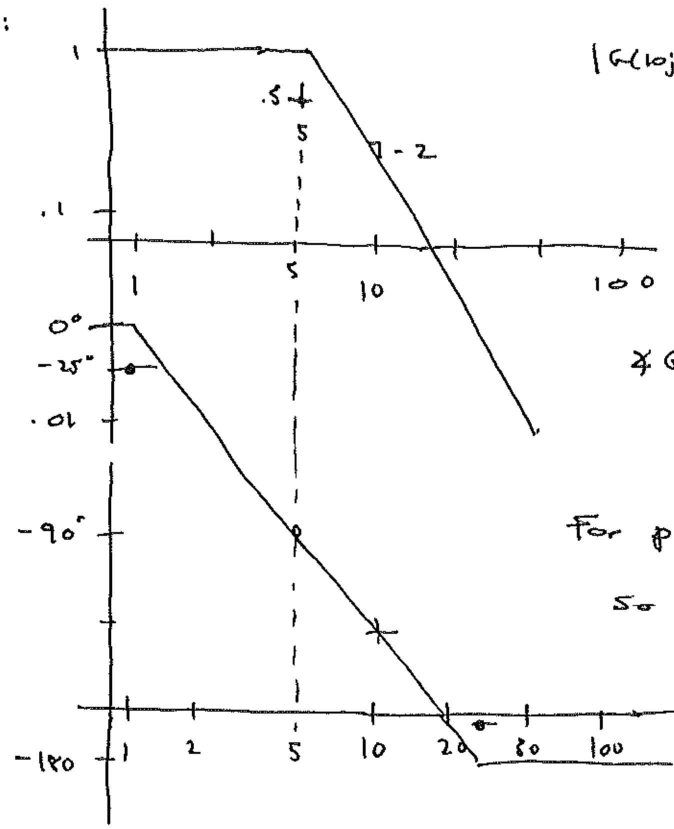
(1) $G(s) = \dfrac{25}{(s+5)^2}$



(a) Spec's: $\omega_{xo} = 10$ rad/sec, PM = 50°.

$G(s) = \dfrac{25}{s^2+10s+25}$

→ Do Bode plot:



$|G(10j)| = \dfrac{25}{-100+100j+25} = \dfrac{25}{100j-75}$

$= \dfrac{25}{\sqrt{100^2+75^2}} = \dfrac{25}{125} = \dfrac{1}{5} = 0.2$

$\boxed{\therefore K_0 = 5}$

$\measuredangle G(10j) = 0° + \tan^{-1}\left(\frac{100}{75}\right) - 180°$

$= -180 + \tan^{-1}\left(\frac{100}{75}\right) = \underline{-126.9°}$

For PM = 50° → need 130°

So close... add the smallest amount of lead...

Note: from asymptotes, $\measuredangle G(10j)$ → -90 @ $5j$

-180 @ $25j$

$5 \to 50$ → one decade

$5 \to 10$ → one fifth a decade

$\measuredangle G(10j) \Rightarrow 108°$

I can leave it with just a gain or I can add a small amount of lead → $\frac{b}{a} = 2$ $\sqrt{ab} = 10$ → $b = 2a \to \sqrt{2}\,a = 10 \to a = 7.07$

$b = 14.14$

$K\sqrt{\frac{a}{b}} = 1$ $K = \sqrt{\frac{b}{a}} = \sqrt{2}$

Additional:

$$\boxed{D(s) = 5\sqrt{2}\,\frac{s+7.07}{s+14.14}}$$

① (con'd)

(b) $D(z) = D(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$ $T = \frac{1}{100}$

$$= 5\sqrt{2}\left[\frac{200\frac{(z-1)}{z+1} + 7.07}{200\frac{(z-1)}{(z+1)} + 14.14}\right] = 5\sqrt{2}\left[\frac{200(z-1) + 7.07(z+1)}{200(z-1) + 14.14(z+1)}\right]$$

$$= 5\sqrt{2}\left[\frac{207.07z - 192.93}{214.14z - 185.86}\right] = \frac{6.838z - 6.371}{z - 0.868} = \boxed{6.838\,\frac{z - 0.932}{z - 0.868}}$$

(c) In bode, additional phase loss is $-\frac{\omega T}{2}$. $T = 0.01$ $\omega = 10$ $\Delta\phi = -\frac{10(0.01)}{2}$

$$\Delta\phi = -\frac{1}{2}\text{ rad} = -28.65^\circ.$$

So, @ 10 rad/sec — now we need an additional 25° of phase...

Go to $\frac{b}{a} = 4 \longrightarrow K = \sqrt{\frac{b}{a}} = 2$ $2a = 10 \therefore a = 5$
$b = 20$.

$$\boxed{D_2(s) = 10\,\frac{s+5}{s+20}}$$

(d) To convert $G(s)$ to $G(z)$ — $G(z) = \frac{z-1}{z}\mathcal{Z}\left\{\frac{G(s)}{s}\right\} = \frac{z-1}{z}\mathcal{Z}\left\{\frac{25}{s(s+5)^2}\right\}$

This is $\mathcal{Z}\left\{\frac{a^2}{s(s+a)^2}\right\} \xrightarrow{\text{table \#16}} \frac{z\left[\left(1 - e^{-aT} - aTe^{-aT}\right)z + e^{-2aT} - aTe^{-aT} - e^{-aT}\right]}{(z-1)(z-e^{-aT})^2}$

$a = 5$ $T = 0.01$ $e^{-aT} = 0.9512$

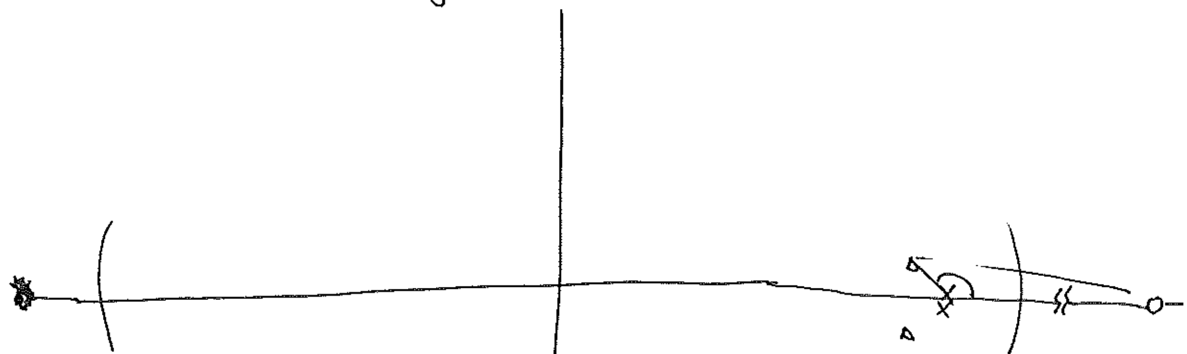
$$G(z) = \frac{\cancel{z-1}}{\cancel{z}}\cdot\frac{\cancel{z}}{\cancel{z-1}}\,\frac{\left[1.209\times10^{-3}z + (0.9048 - 0.0476 - 0.9512)\right]}{(z-0.9512)^2}$$

$$\boxed{G(z) = 1.209\times10^{-3}\,\frac{\left[z - 77.75\right]}{(z-0.9512)^2}}$$

① (cont)

ⓔ Design in z-domain $s_{des} = -7 \pm 7j \longrightarrow z_{des} = e^{-7(0.01)} e^{\pm 7(0.01)j} = 0.932\, e^{\pm 0.07j}$

$z_{des} = 0.9297 \pm 0.0652j$



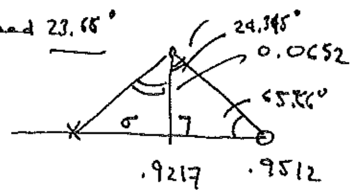
Ignore the zero @ +77, adds zero phase.. (0.04°)

at $z_\Delta \rightarrow$ 

$$2\left(-180 + \tan^{-1}\left[\frac{0.0652}{0.9512 - 0.9217}\right]\right) = -360 + 131.31 = -228.69$$

want −180 → Need 48° of lead.

Let's plop the zero on top of the pole... $K_0 \dfrac{z - 0.9512}{z - 0.8931}$



$\tan 23.65° = \dfrac{\sigma}{0.0652} \quad \therefore \sigma = 0.0286$

$D_z(z) = K_0 \dfrac{z - 0.9512}{z - 0.8931}$ $\qquad \left| D(z) G(z) \right|_{z=\Delta} = 1$

$$\left| K_0 \left[\frac{\cancel{0.9297 + 0.0652j - 0.9512}}{0.9297 + 0.0652j - 0.8931}\right] \cdot 1.209\times10^{3} \left[\frac{0.9297 + 0.065j - 77.75}{(0.9297 + 0.065j - 0.9512)^2}\right] \right| = 1$$

$$= \left| 1.209\times10^{3} K_0 \cdot \left[\frac{-76.82 + 0.0652j}{(0.036 + 0.0652j)(-0.022 + 0.0652j)}\right] \right| = \left[\frac{1.209\times10^{-3} K_0 \cdot 76.82}{(0.0745)(0.0688)}\right] = 18.12\, K_0$$

$K_0 = 0.055 \quad \therefore \boxed{D_z(z) = 0.0552 \dfrac{z - 0.9512}{z - 0.8931}}$

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① (cont)

ⓔ Poles in s-domain: (Note — not closed loop poles, but compensator poles)

$D_1(s) \longrightarrow$ zero @ -7.07

pole @ -14.14

$D_1(z) \longrightarrow$ zero @ $0.932 \longrightarrow s = \frac{1}{\Delta T}\ln(z) = -7.04$

pole @ $0.868 \longrightarrow s = \frac{1}{\Delta T}\ln(z) = -14.16$

$D_2(s) \longleftrightarrow$ zero @ -5

pole @ -20

$D_3(z) \longrightarrow$ zero @ $0.9512 \longrightarrow s = \frac{1}{\Delta T}\ln(z) = -5$

pole @ $0.8931 \longrightarrow s = \frac{1}{\Delta T}\ln(z) = -11.31$

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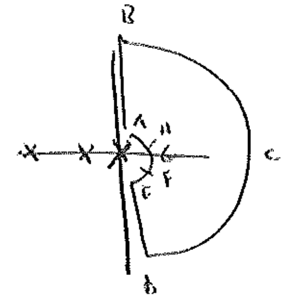
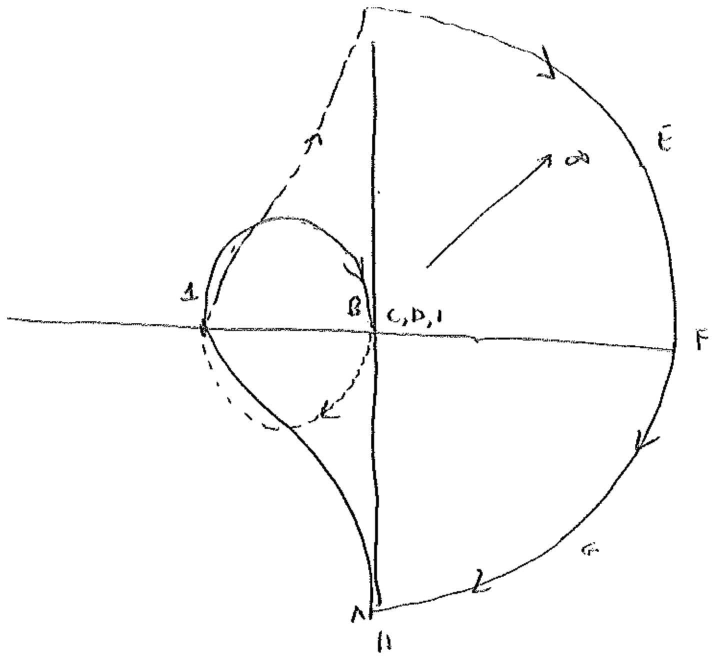
(2) $G(s) = \dfrac{20}{s(s+1)(s+4)}$

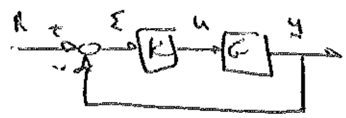


(a) What can you say about Poles / stability → Nyquist



@ $E \to \infty$, ∡ 98°

Range of Stability



180° (Neg Feedback)

$K < 1$ STABLE

$K > 1$ UNSTABLE 2 POLES

($N = 2$, $P = \phi \to Z = 2$).

For $K < 0$ (0° ROOT LOCUS, POSITIVE FEEDBACK)

$N = 1 \; \forall K$, UNSTABLE w/ 1 POLE IN RHP.

(b) $\omega_{co} = 2$ rad/sec PM = 30° $E_{ss} \le 0.01$

Strategy: (1) Lag for E_{ss}

(2) Keep gain = 1 @ ω_{co}

(3) Lead for PM.

$\dfrac{E}{R} = \dfrac{1}{1+KG}$

$$E_{ss} = \lim_{s\to 0} \cancel{s}\,\frac{1}{1+KG}\cdot\frac{1}{s^{\cancel{2}}} = \lim_{s\to 0}\frac{1}{s}\,\frac{1}{1+KG}$$

$$= \lim_{s\to 0}\frac{1}{s}\cdot\frac{1}{1+\frac{20k}{s(s+1)(s+4)}} = \lim_{s\to 0}\frac{1}{\cancel{s}}\,\frac{\cancel{s}(s+1)(s+4)}{s(s+1)(s+4)+20k} = \frac{1\cdot 4}{20K} \le 0.1$$

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(2) (con'd)

$$E_{ss} = \frac{4}{20K} \leq 0.1 \therefore K > \frac{40}{20} \rightarrow K > 2 \text{ @ DC!!}$$

→ Put Lead in to get PM → Then adjust DC gain w/ Lag.

$\angle \omega_{xo} = -180°$. → Need at least 30° of additional phase

Choose $\frac{b}{a} = 6$ to get ~ 45° of phase

$\frac{b}{a} = 6 \rightarrow b = 6a \quad \sqrt{6}\,a = 2 \therefore a = \frac{2}{\sqrt{6}} \quad b = 2\sqrt{6}$

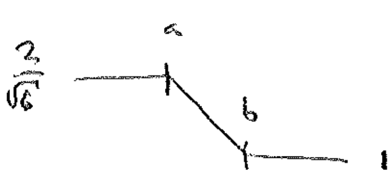
$K = \sqrt{6}$ so that gain = 1 @

$$\boxed{D_{LEAD} = \sqrt{6}\,\frac{(s + 2/\sqrt{6})}{(s + 2\sqrt{6})}}$$

@ DC, gain of system is: $\sqrt{6}\cdot\frac{2}{\sqrt{6}}\,\frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{6}$ → Need this to be > 2

add in a lag → Need DC gain of lag to be $\frac{2}{\sqrt{6}}$

gain @ $\omega_{xo} = 1$



$\frac{s+b}{s+a}$

$\frac{b}{a} = \frac{2}{\sqrt{6}}$

Want phase @ ω_{xo} to be normal, so make $5b = \omega_{xo}$

$5b = 2 \therefore b = 2.5 = \frac{5}{2}$

$a = \frac{\sqrt{6}\,b}{2} = 5\sqrt{8}$

$K = 1$

$a < b$

want $5b = 2 \rightarrow b = 2.5$

want $\frac{b}{a} = \frac{2}{\sqrt{6}}$

$a = 2.041$

$b = 2.5$

$$\boxed{D(s) = \sqrt{6}\,\frac{(s+2.5)(s+0.816)}{(s+2.041)(s+4.899)}}$$

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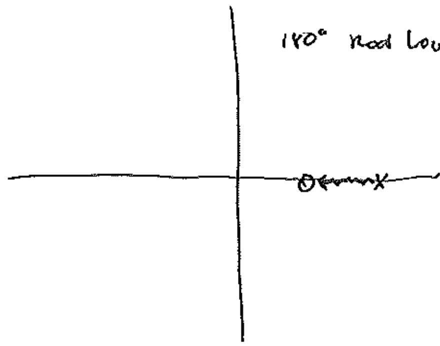
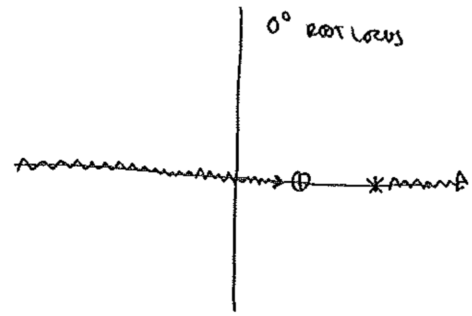
(3)

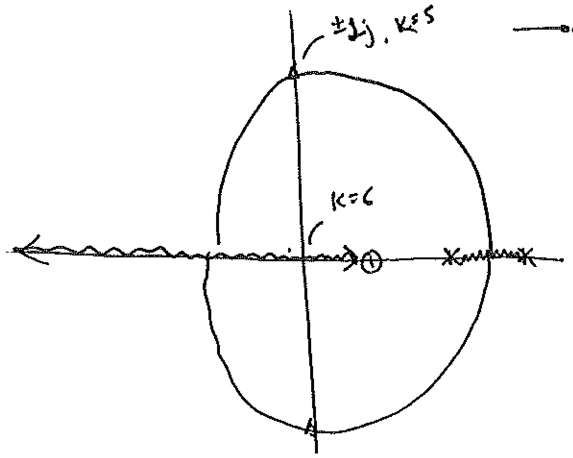
$$G(s) = \frac{(s-1)}{(s-2)}$$

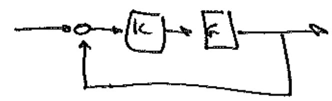
(a) 180° Root Locus





(b) $K(s) = \frac{1}{(s-3)}$





Find K for $j\omega$ crossing

$$\frac{Y}{R} = \frac{KG}{1+KG}$$

$$1 + KG = 0$$

$$1 + \frac{K(s-1)}{(s-2)(s-3)} = 0$$

$$(s-2)(s-3) + K(s-1)\Big|_{s=j\omega} = 0 + 0j \longrightarrow s^2 - 3s - 2s + 6 + Ks - K\Big|_{s=j\omega} = 0 + 0j$$

$$s^2 + (-5 + K)s + 6 - K\Big|_{s=j\omega} = 0 + 0j$$

$$s^2 + (K-5)s + (6-K)\Big|_{s=j\omega} = 0 + 0j$$

$-\omega^2 + 6 - K = 0$ $\quad K - 5 = 0$

$\therefore \underline{K = 5}$ $\quad \omega = 0$

$+\omega^2 = 6 - 5$

$= 1$

$\therefore \omega = \pm 1$ $\quad \omega = 0 \rightarrow K = 6$

Stabilized @ $\underline{5 < K < 6}$

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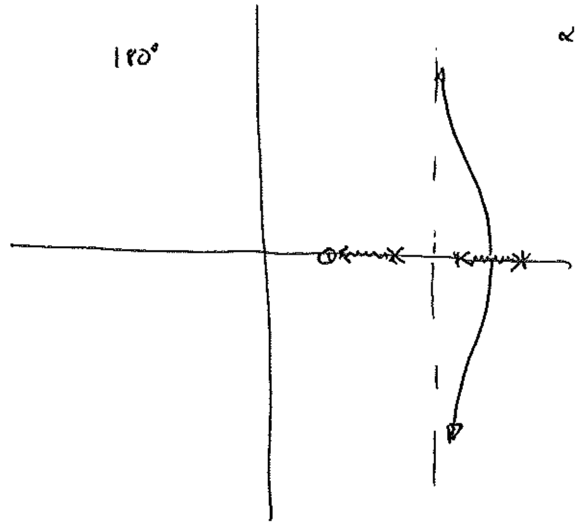
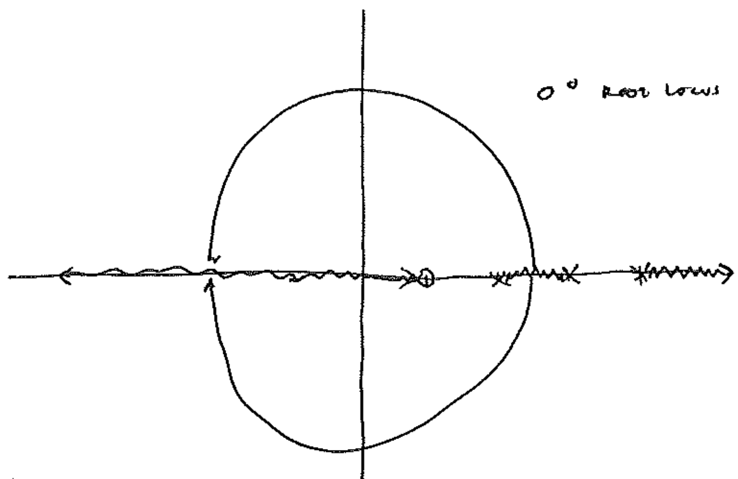
(3) (cont'd)

(c) $K(s) = \frac{1}{(s-3)(s-4)}$

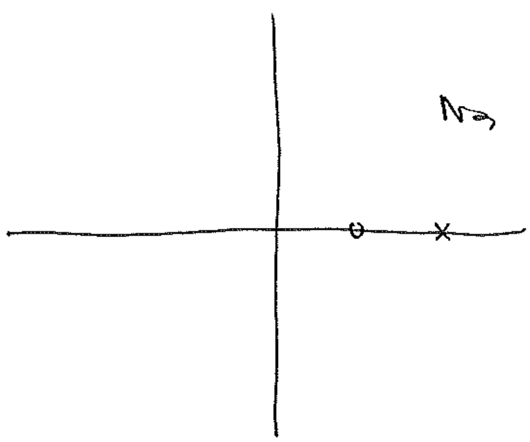
$\alpha = \frac{4+3+2-1}{3} = \frac{8}{3}$



Nope, cannot stabilize

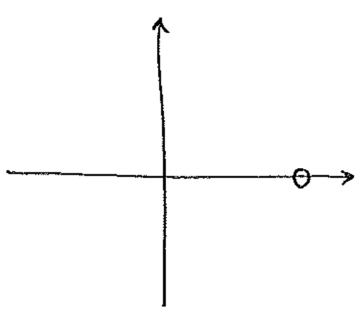


(d)



No, pole will always move towards zero in 180° root locus...

(e) $G(s)$



Can only be made stable with a stable controller if all poles are to either to left of zero, or in pairs to the right (even number to right).

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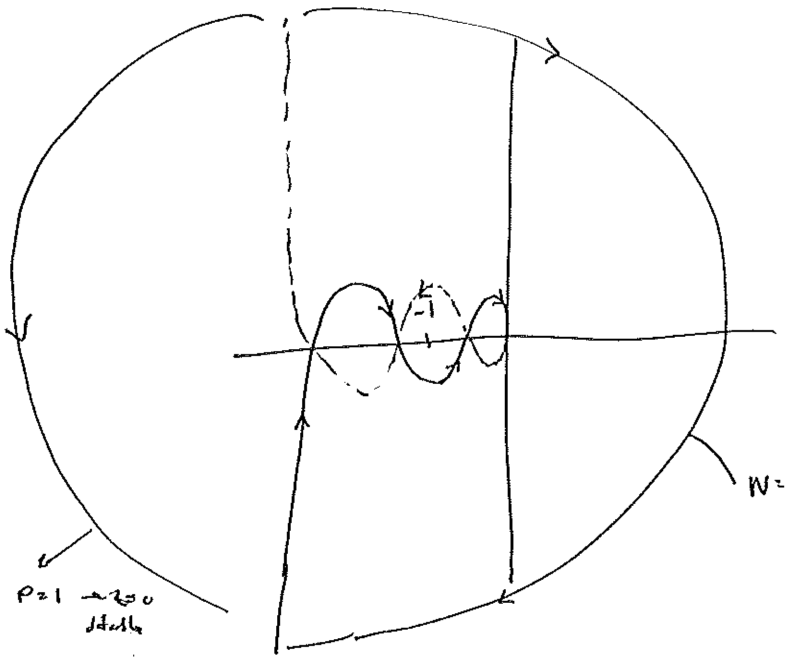
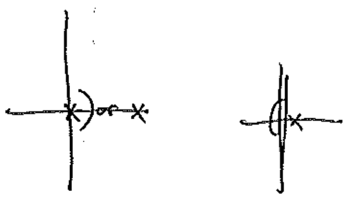
(4)



$N=1$ $P=1$ $\rightarrow Z=0$ stable

$N=\phi$, $P=\phi$ $Z=0$ $\rightarrow$ stable

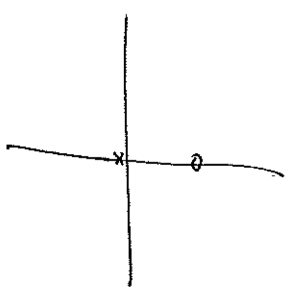
(a)



$N=-1$ $P=1$ $\rightarrow Z=\phi$

Yes, system is stable

(b)



$N: -1$ $P=\phi$ $\rightarrow Z=-1$

~~stable~~. unstable